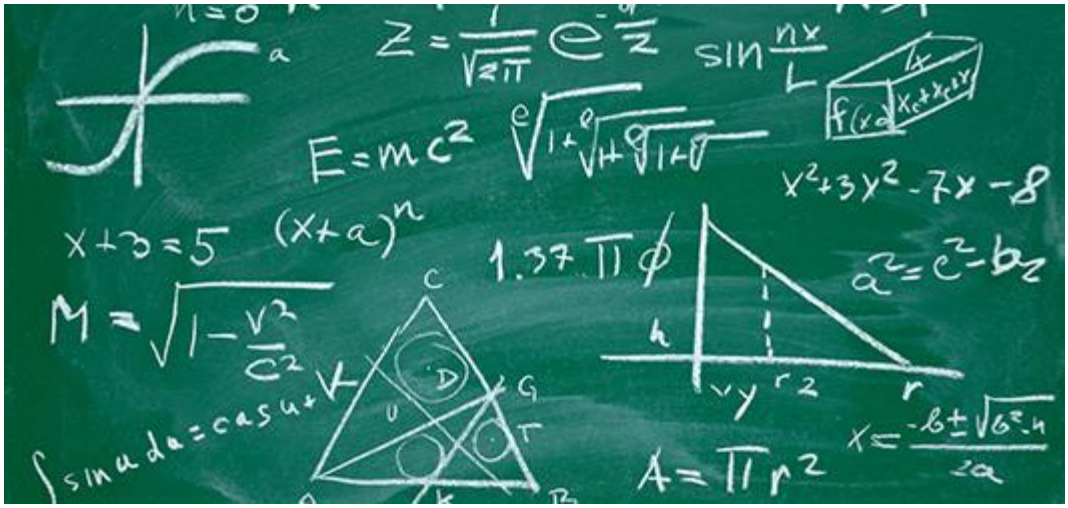
A level Maths

Revision sites and Examination Technique



Revision sites and resources

1. The Student Room

https://www.thestudentroom.co.uk/showthread.php?t=5860630&p=82408522

For the new linear A Level Maths students are tasked with revising all of the content across a 2 year course (1 year for AS). To assist with revision, we have produced a checklist in the posts below that contains every topic that could be in your AS or A Level exams. The checklist includes links to videos and exam practice questions, the majority of which have come from ExamSolutions, MathsGenie, Physics & Maths Tutor and TLMaths. All four of these are excellent revision websites.

☐ **Surds and indices**

Video1
Video2
Questions

☐ **Algebraic expressions**

☐ **Equations and inequalities**

☐ **The discriminant**

☐ **Sketching graphs**

☐ **Transformations of functions**

☐ **Coordinate geometry**

☐ **The equation of a circle and use of circle theorems**

☐ **The factor theorem and dividing polynomials**

☐ **Year 1 methods of proof & disproof**

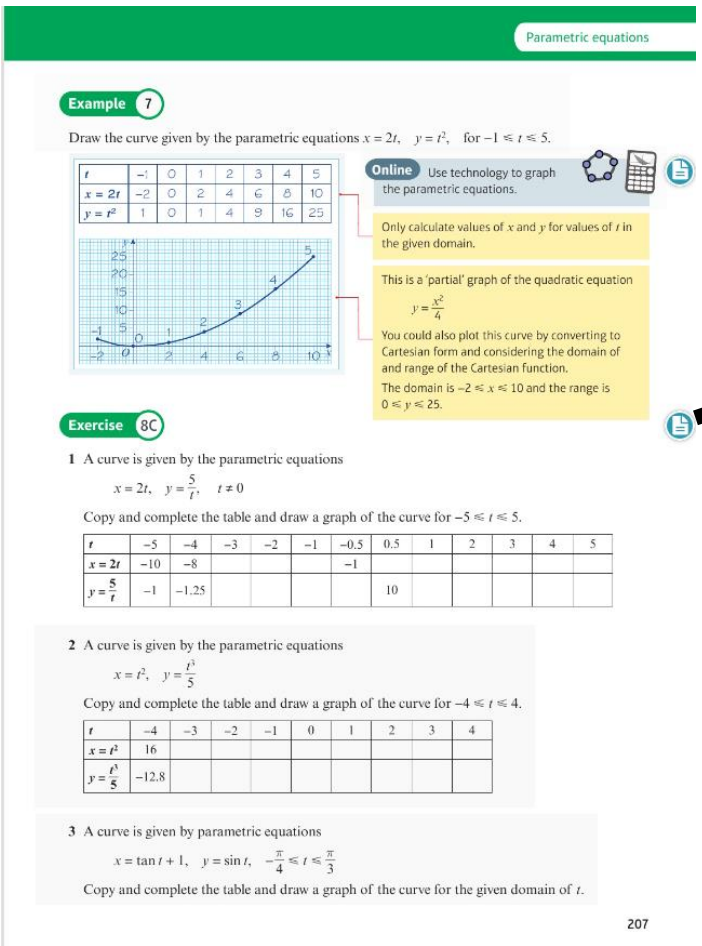
2. Pearson Active Learning

https://www.pearsonactivelearn.com/app/library

Username: All lower case. First initial followed by surname17. Double surnames have no space between names.
Password: Same as your username. All lower case.

(Some usernames / passwords are too short for the software to cope with, so if this does not work try 1718 instead of simply 17 after your surname & password.)

On line version of the textbooks. Worked examples, questions and solutions.



Parametric equations

Example 7

Draw the curve given by the parametric equations $x = 2t$, $y = t^2$, for $-1 \leqslant t \leqslant 5$.

t	−1	0	1	2	3	4	5
$x = 2t$	−2	0	2	4	6	8	10
$y = t^2$	1	0	1	4	9	16	25

Online Use technology to graph the parametric equations.

Only calculate values of x and y for values of t in the given domain.

This is a 'partial' graph of the quadratic equation $y = \frac{x^2}{4}$

You could also plot this curve by converting to Cartesian form and considering the domain of and range of the Cartesian function.
The domain is $-2 \leqslant x \leqslant 10$ and the range is $0 \leqslant y \leqslant 25$.

Exercise 8C

1 A curve is given by the parametric equations

$x = 2t$, $y = \frac{5}{t}$, $t \neq 0$

Copy and complete the table and draw a graph of the curve for $-5 \leqslant t \leqslant 5$.

t	−5	−4	−3	−2	−1	−0.5	0.5	1	2	3	4	5
$x = 2t$	−10	−8				−1						
$y = \frac{5}{t}$	−1	−1.25					10					

2 A curve is given by the parametric equations

$x = t^2$, $y = \frac{t^3}{5}$

Copy and complete the table and draw a graph of the curve for $-4 \leqslant t \leqslant 4$.

t	−4	−3	−2	−1	0	1	2	3	4
$x = t^2$	16								
$y = \frac{t^3}{5}$	−12.8								

3 A curve is given by parametric equations

$x = \tan t + 1$, $y = \sin t$, $-\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{3}$

Copy and complete the table and draw a graph of the curve for the given domain of t.

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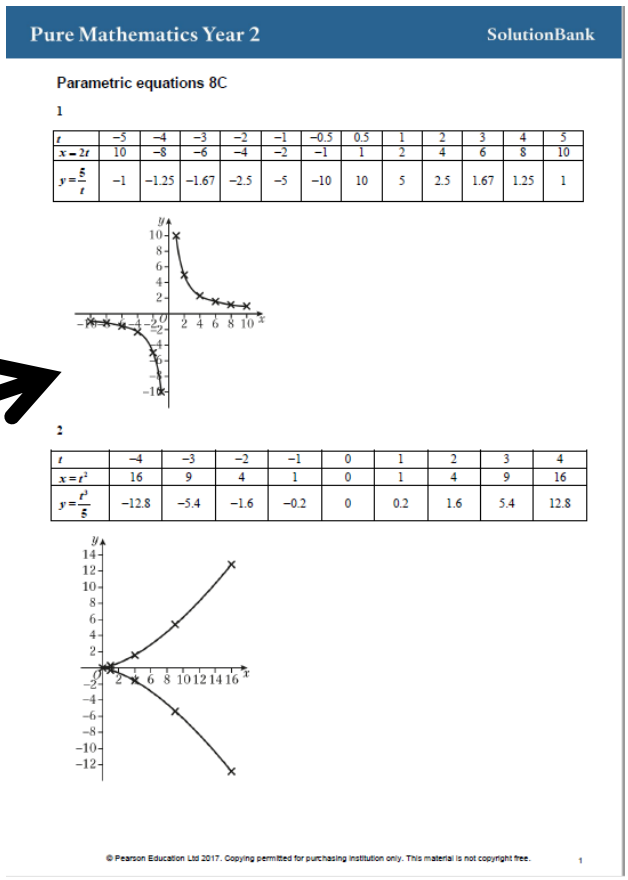


Pure Mathematics Year 2 SolutionBank

Parametric equations 8C

1

t	−5	−4	−3	−2	−1	−0.5	0.5	1	2	3	4	5
$x = 2t$	10	−8	−6	−4	−2	−1	1	2	4	6	8	10
$y = \frac{5}{t}$	−1	−1.25	−1.67	−2.5	−5	−10	10	5	2.5	1.67	1.25	1

2

t	−4	−3	−2	−1	0	1	2	3	4
$x = t^2$	16	9	4	1	0	1	4	9	16
$y = \frac{t^3}{5}$	−12.8	−5.4	−1.6	−0.2	0	0.2	1.6	5.4	12.8

1

3. Maths Genie

https://mathsgenie.co.uk/newalevel.html

Videos, Questions and solutions to the whole syllabus

A Level Mechanics and Statistics

Topic	Videos	Exam Questions	Solutions
Probability	Videos	Probability	Solutions
Statistical Distributions	Videos	The Normal Distribution Using the Normal Distribution to approximate the Binomial	Solutions Solutions
Statistical Hypothesis Testing	Videos	Correlation Hypothesis Testing Mean of Normal Distribution Hypothesis Testing Non Linear Regression	Solutions Solutions Solutions
Forces	Videos	Resolving Forces Resolving Forces 2 Connected Particles	Solutions Solutions Solutions
Kinematics	Videos	Kinematics with Vectors Kinematics with Calculus Projectiles	Solutions Solutions Solutions
Moments	Videos	Moments Statics of Rigid Bodies	Solutions Solutions

4. Assignment 15

Good revision of the Large Data Set

https://www.mickmacve.com/uploads/2/9/5/2/29527671/2nd_year_15.pdf

2. Copy and complete this table

Attribute	Units	Discrete or Continuous?	Min	Max	Qualitative or Quantitative	Possible values
Daily Mean Temperature						
Daily Total Rainfall						
Daily Total Sunshine						
Daily Maximum Relative Humidity						
Daily Mean Windspeed						
Daily Maximum Gust						
Daily Mean Wind Direction						
Cardinal Wind Direction						
Daily Maximum Gust Direction						
Cardinal Gust Direction						
Cloud Cover						
Visibility						
Daily Mean Pressure						

3. Lauren wants to find the average daily mean windspeed in Hurn in 1987.
She only has access to the large data set. She uses it to obtain a random sample of the daily mean windspeeds, t knots, on n days in Hurn in 1987.
The data collected by Lauren are summarised as follows

$$\sum(t-5)=55, \quad \bar{t}=10$$

(a) Find n.

Lauren uses the same sampling method to estimate that the average daily mean windspeed in Hurn in 2015 was 11 mph.

(b) Convert 11 mph into knots.

(c) Hence, compare the average daily mean windspeed in Hurn in 1987 and 2015.

(d) With reference to the large data set, state **one** limitation of your conclusion in part (c).

(e) Explain how Lauren can
(i) improve her data collection method
(ii) improve her data processing
to allow for a more reliable comparison in part (c).

4. The table shows the mean daily temperatures at each of the eight weather stations for August 2015

	Camborne	Heathrow	Hurn	Leeming	Leuchars	Beijing	Jacksonville	Perth
Mean daily mean temp °C	15.4	18.1	16.2	15.6	14.7	26.6	26.4	13.6

a) Give a geographical reason why the temperature in August might be lower in Perth than in Jacksonville

b) Comment on whether this data supports the conclusion that coastal locations experience lower average temperatures than inland locations.

Examination Technique

Multiple attempts and crossing out

"Crossed-out work should be marked **unless** the candidate has replaced it with an alternative response."

"Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete."

Circle or highlight key phrases in questions.

Tick off question parts as you go so you don't leave any parts out

Key phrases might include

3 d.p.	Hence	Exact Answer	Write down
Simplified Fraction	Nearest Integer	x is an integer, x>1	
Express the probability as a %		Show - Prove - Verify	

Read the key phrases carefully

- "**Give an exact answer**" means leave your answer with a fraction or π or √ or ln or e in it. Don't give a rounded decimal.
- "**Hence**" means use what you have just found out.
- "**Write down**" means that there is no working – this shouldn't take you very long
- "**Interpret**" means that you need to write a sentence in the context of the question

Prove, Show, Verify

- ***Prove*** means LHS≡ ⋯ ≡ ⋯ ≡ ⋯ ≡ RHS

  e.g. prove that $1 + \tan^2\theta \equiv \sec^2\theta$……. ∴Proof complete or QED

- ***Show*** means use the information to get an answer

  e.g. Show that $x = 1.41$ is a solution to the equation $x^2 = 2$ correct to 3 s.f.

- ***Verify*** means substitute a value

  e.g. verify that $x = 7$ is a solution to $x^3 - 2x - 329 = 0$

  When $x = 7$, $x^3 - 2x - 329 = 7^3 - 2\text{x}7 - 329 \ = 343 - 14 - 329 = 0$

  ∴ $x = 7$ is a solution to $x^3 - 2x - 329 = 0$

Prove or Show

- If you're asked to "Prove" or "Show" something, the last line in your working should state the answer. It's not just enough to write "As required" or "Q.E.D." – you must write out the statement.
- e.g. f(x) = $2x^3 - 7x^2 + 4x - 4$.

  Use the factor theorem to show that($x - 2$) is a factor of f(x).

  f(2) = 2 x 8 – 7 x 4 + 4 x 4 - 4 = 0

  $\therefore (x - 2)$ is a factor of f(x)

Scan the paper.

- You can do the questions in any order. Start with all the familiar questions first. There may be questions that look unfamiliar. Do them last.

Read The Question

- Read the question after you've finished it, to check you've done what it asks you to.

(*c*) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (*a*) and hence find an approximate value for $\sqrt{2}$. Give your answer in the form $\frac{p}{q}$, where p and q are integers. **(2)**

Write on the exam paper

You can write on the exam paper. Draw diagrams to help you (e.g. area under a graph)

Do a sketch for those questions that involve tangents and normal. Don't try to hold the information in your head. If in doubt, sketch it out.

Write in black ink only. Don't use tippex or highlighters as your answer paper is scanned.

Attempt every part of every question

If you can't do part (a) don't give up. Make up a value for your answer to part (a) and then use it in subsequent parts of the question to earn method marks.

Improve Levels of Accuracy

Always write the full calculator display down first.

Then check the level of accuracy required in the question.

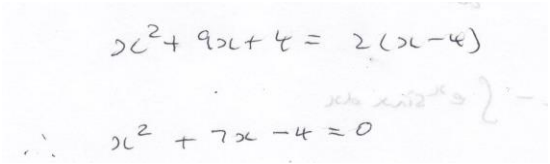
Assume 3 significant figures if no accuracy stated.

Always use unrounded answers in any subsequent calculations.

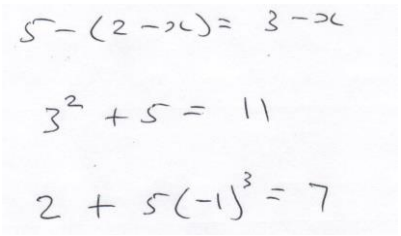
Remember g = 9.8

Beware of taking shortcuts with your working.

It doesn't take that long to write out an intermediate step.



Avoid basic arithmetic and algebraic errors.



Check your work

In many questions, it's possible to take your answer and substitute values back in to the question

Know your Calculator!

Angle units

Pure – radians unless you see the degrees sign °

Mechanics – normally degrees

Use brackets to tell the calculator the order of operations.

Use all the functions

- Statistics (mean, standard deviation)
- Distributions (Normal, Binomial)
- Solving quadratic equations
- Solving simultaneous equations
- Integration & Differentiation

Write in the correct space

- Write within the space given for each question. Don't do question 5 in question 6's space as the papers are scanned in question by question. If you need additional paper ask for some. Label parts of questions clearly (a), (b), (c) etc.

Timing

- Don't spend too long on one question.
- Keep a close eye on the time.
- Put a watch on your desk rather than keep looking at the clock on the wall.
- Make sure you know how many marks for each question and aim for a minute per mark.

Use the correct notation

Integration

$\int \sin^2 x \ dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ NOT $\int \sin^2 x = \frac{1}{2}x - \frac{1}{4}\sin 2x$

Differentiation

$y = 3x^2 \quad \therefore \frac{dy}{dx} = 6x$ NOT $\frac{d}{dx} = 6x$

Formulae

- Learn the formulae.
- Quote the general form first then substitute in the correct values.

  e.g. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

  $\frac{d}{dx}\left(\frac{e^{2x}}{\sin x}\right) = \frac{2e^{2x}\sin x - e^{2x}\cos x}{\sin^2 x}$

Learn The Formulae

Appendix 1: Formulae

Formulae that students are expected to know for A Level Mathematics are given below and will not appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation $y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$u_n = a + (n-1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \frac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and area of circle, radius r and diameter d:

$C = 2\pi r = \pi d \quad A = \pi r^2$

Pythagoras' theorem:

In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse, $c^2 = a^2 + b^2$

Area of a trapezium = $\frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:

$s = r\theta \quad A = \frac{1}{2}r^2\theta$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c,\ x\neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $= \int_a^b y\,dx\ (y\geqslant 0)$

Vectors

$$\left|x\mathbf{i}+y\mathbf{j}+z\mathbf{k}\right|=\sqrt{\left(x^2+y^2+z^2\right)}$$

Statistics

The mean of a set of data: $\bar{x}=\frac{\sum x}{n}=\frac{\sum \mathrm{f}x}{\sum \mathrm{f}}$

The standard Normal variable: $Z=\frac{X-\mu}{\sigma}$ where $X\sim \mathrm{N}\left(\mu,\sigma^2\right)$

Mechanics

Forces and Equilibrium

Weight = mass × g

Friction: $F\leqslant \mu R$

Newton's second law in the form: $F=ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v=\frac{\mathrm{d}r}{\mathrm{d}t}\qquad a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2r}{\mathrm{d}t^2}$$

$$r=\int v\,\mathrm{d}t\qquad v=\int a\,\mathrm{d}t$$

Make sure you know your way around the formula book

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x\ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

Small angle approximations

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan \theta \approx \theta$$

where θ is measured in radians

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec kx$	$k \sec kx \tan kx$
$\cot kx$	$-k \operatorname{cosec}^2 kx$
$\operatorname{cosec} kx$	$-k \operatorname{cosec} kx \cot kx$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

$f(x)$	$\int f(x)\,dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\quad\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\quad\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Numerical Methods

The trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Statistics

Probability

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B \mid A)$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')}$$

For independent events A and B,

$$P(B \mid A) = P(B)$$
$$P(A \mid B) = P(A)$$
$$P(A \cap B) = P(A)\,P(B)$$

Standard deviation

Standard deviation = √(Variance)

Interquartile range = $\mathrm{IQR} = \mathrm{Q}_3 - \mathrm{Q}_1$

For a set of n values $x_1, x_2, \ldots x_i, \ldots x_n$

$$\mathrm{S}_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Standard deviation = $\sqrt{\frac{\mathrm{S}_{xx}}{n}}$ or $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$

Discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

Sampling distributions

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$$

Statistical tables

The following statistical tables are required for A Level Mathematics:

Binomial Cumulative Distribution Function (see page 29)

Percentage Points of The Normal Distribution (see page 34)

Critical Values for Correlation Coefficients: Product Moment Coefficient (see page 37)

Random Numbers (see page 38)

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$