SECTION A

1. A person throws a ball in a Sports Hall. The height of the ball, h m, can be modelled in relation to the horizontal distance from the point it was thrown from by the quadratic equation

$$h = -\frac{3}{10}x^2 + \frac{5}{2}x + \frac{3}{2}$$

The Hall has a sloping ceiling which can be modelled with equation $h = \frac{15}{2} - \frac{1}{5}x$

Determine whether the model predicts that the ball will hit the ceiling.

2. a) Find an equation of the straight line passing through the points with co-ordinates (-1,5) and (4,-2), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.
   b) The line crosses the x-axis at the point A and the y-axis at the point B and O is the origin. Find the area of ΔAOB

3. The table shows the daily maximum relative humidity, h (%), and the daily mean visibility, v (dm) in Heathrow for the first two weeks in September 2015.

h	94	95	92	80	97	94	93	90	87	95	93	92	91	98
V	2600	2900	3900	4300	2800	2400	2700	3500	3000	2200	2200	3300	2800	2200

The equation of the regression line (the line of best fit) of v on h is $v = 12700 - 106h$

a) Give an interpretation of the value of the gradient on the regression line.
b) Use your knowledge of the large data set to explain whether there is likely to be a causal relationship between humidity and visibility (i.e. will an increase in humidity mean there could be a decrease in visibility?)
c) Give reasons why it would not be reliable to use this regression equation to predict
   i) the mean visibility on a day with 100% humidity (think: "extrapolation")
   ii) the humidity on a day with visibility of 3000 dm. (i.e. is it OK to calculate h, given v)
d) State two ways in which better use could be made of the large data set to produce a model describing the relationship between humidity and visibility.

(Don't be reluctant to look at the answers if you are not sure what to write but make sure you UNDERSTAND and LEARN from the answers.)

4. Two cars A and B are moving in the same direction along a straight horizontal road. At time t=0, they are side by side, passing a point O on the road. Car A travels at a constant speed of $30\ ms^{-1}$. Car B passes O with a speed of $20\ ms^{-1}$, and has a constant acceleration of $4\ ms^{-1}$ Find

a) the speed of B when it has travelled 78 m from O
b) the distance from O of A when B is 78 m from O
c) the time when B overtakes A

5. A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates from rest at a rate of $3\ ms^{-1}$ for 8 s and then maintains the velocity she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to $30\ ms^{-1}$. C passes M as they both pass a pedestrian.

a) On the same diagram, sketch velocity-time graphs to illustrate the motion of M and C
b) Find the distance of the pedestrian from the road junction.

6. Find the following integrals

$a) \int \sin^5 3x \cos 3x \, dx$

$b) \int \cos x \, e^{sinx} \, dx$

$c) \int \frac{e^{2x}}{e^{2x} + 3} dx$

SECTION B

1. Find an expression in terms of x and y for $\frac{dy}{dx}$, given that

a) $x^2 + y^3 = 2$ b) $y^3 + 3x^2y - 4x = 0$

c) $e^x y = xe^y$ d) $\sqrt{xy} + x + y^2 = 0$

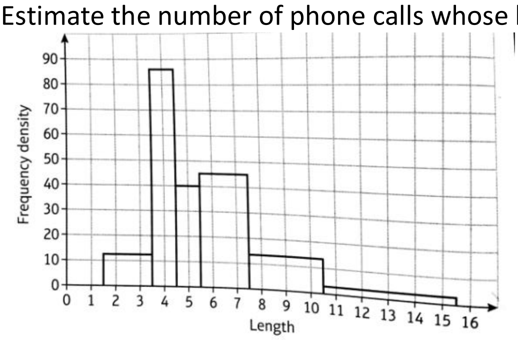
2. $f(x) = \frac{4}{(2x+1)(1-2x)}$, $x \neq \pm\frac{1}{2}$

a) Given that $f(x) = \frac{A}{2x+1} + \frac{B}{1-2x}$, find the values of A and B

b) Hence find $\int f(x)dx$, writing your answer as a single logarithm

c) Find $\int_1^2 f(x)dx$, giving your answer in the form $ln\, k$ where k is a rational constant.

3. A histogram looks like a bar chart but the frequency is NOT given by the height.
The frequency in a histogram is proportional to the AREA.
In this histogram the constant of proportionality is equal to 1 so the frequency = area.
BEWARE: This is not always the case. It's very unusual for the constant of proportionality to be equal to 1
Estimate the number of phone calls whose length was between 5½ and 7½ minutes.



4. A girl cycles from Appledore to Benfield and then from Benfield to Charlesville. The displacement from Appledore to Benfield is 10**i** + 3**j** km. The displacement from Benfield to Charlesville is -7**i** + 12**j** km

a) Find the magnitude of the displacement from Appledore to Charlesville.
b) Find the total distance the girl has cycled in getting from Appledore to Charlesville.
c) Work out the angle that the vector from Appledore to Charlesville makes with the unit vector **i**.

5. A sample of the daily maximum relative humidity is taken from the large data set for Leuchars and for Camborne during 2015. The data is given in the table.

Leuchars	100	98	100	100	100	100	100	100	94	100	91	100	100	89	100
Camborne	92	95	99	96	100	100	90	98	81	99	100	99	91	98	100

a) Find the median and quartiles for both samples
b) Compare the two samples

6. Find $\frac{dy}{dx}$, if a) $y = \frac{(e^x+3)^3}{cosx}$, b) $y = \sin(5x)\ln(cosx)$

SECTION C

1. Given that $y = \arctan\left(\frac{1-x}{1+x}\right)$, find $\frac{dy}{dx}$ in terms of x only

2. a) When θ is small, show that the equation $\frac{4cos3\theta-2+5\sin\theta}{1-sin2\theta}$ can be written as 9θ+2

   b) Hence write down the value of $\frac{4cos3\theta-2+5\sin\theta}{1-sin2\theta}$ when θ is small

3. Two particles P and Q are moving along the same straight horizontal line with constant acceleration $2\ ms^{-2}$ and $3.6\ ms^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $4ms^{-1}$. One second later Q passes through A with speed $3ms^{-1}$, moving in the same direction as P.

   a) Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A.
   b) Find the value of t where the particles meet.
   c) Find the distance of A from the point where the particles meet.

4. A particle of mass 0.3 kg is on a rough plane which is inclined at an angle of $30°$ to the horizontal. The particle is held at rest on the plane by a force of magnitude 3 N acting up the plane, in a direction parallel to a line of greatest slope of the plane. The particle is on the point of slipping up the plane. Find the coefficient of friction between the particle and the plane.

5. Anna and Bella are sometimes late for school. The events A and B are defined as follows:
A is the event that Anna is late for school
B is the event that Bella is late for school
p(A) = 0.3, p(B) = 0.7 and p(A'∩B') = 0.1
On a randomly selected day, find the probability that

a) both Anna and Bella are late to school
b) Anna is late to school given that Bella is late to school.

Their teacher suspects that Anna and Bella being late for school is linked in some way.
c) Comment on her suspicion, showing your working.

6. Find the value of $\frac{dy}{dx}$ at the point $(\frac{5}{2}, 4)$ on the curve with equation $y^{\frac{1}{2}} + y^{-\frac{1}{2}} = x$

ANSWERS

SECTION A

1 Yes.

2 a) 7x + 5y – 18 = 0 b) $\frac{162}{35}$

3. a) For each percent increase in daily maximum relative humidity there is a decrease of 106 m in daily mean visibility.

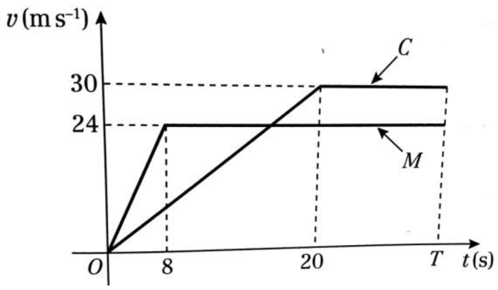
b) High levels of relative humidity cause mist or fog which will decrease visibility. Hence there is likely to be a causal relationship

c) i) The prediction for 100% is outside the range of the data (extrapolation) so is less likely to be accurate
ii) The regression equation should only be used to predict a value for v given h

d) Data is only useful for analysing the first two weeks of September. Random values throughout September should be used and analysis made of the whole month. The sample size could also be increased across multiple months as data between May and October is available.

4. a) 32 ms^{-1} $b)$ 90 m c) 5 s

5. a)



b) 720 m

6. a) $\frac{1}{18}\sin^6 3x + c$ b) $e^{sinx} + c$ c) $\frac{1}{2}\ln|e^{2x} + 3| + c$

SECTION B

1) a) $-\frac{2x}{3y^2}$ b) $\frac{4-6xy}{3x^2+3y^2}$ c) $\frac{e^x y-e^y}{xe^y-e^x}$ d) $\frac{-2\sqrt{xy}-y}{4y\sqrt{xy}+x}$

2) a) A=2, B=2 b) $\ln\left|\frac{2x+1}{1-2x}\right|+c$ c) $\ln\frac{5}{9}$

3) 90

4 a) 15.3 m b) 24.3 m c) 78.7°

5a) Leuchars: median = 100, lower quartile = 98, upper quartile = 100

Camborne: median = 98, lower quartile = 92, upper quartile = 100

b) The median humidity in Leuchars is higher than the median humidity in Camborne. So Leuchars is, on average more humid than Camborne.

The spread of the humidities for Camborne is greater than the spread of humidities for Leuchars. So the humidity in Camborne is more variable than the humidity in Camborne.

6a) $(e^x+3)^2(\frac{(e^x+3)\sin x+3e^x\cos x)}{\cos^2 x}$ b) $5\cos 5x\ln(\cos x)\ -\tan x\sin 5x$

SECTION C

1. $-\frac{1}{1+x^2}$
2. 2
3. a) P: $4t+t^2$, Q:$3(t-1)+1.8(t-1)^2$ b) 6 c) 60 m
4. 0.601 (accept 0.6)
5. a) 0.1

   b) 0.143 (3 s.f.)

   c) p(A) x p(B) $\neq$ p(A) ∩ p(B) so the events are not independent.
6. $\frac{16}{3}$