2nd Year Assignment 2

SECTION A

(1) If the ball hits the ceiling, there is a point of intersection between the ball and the ceiling.

i.e. $-\frac{3}{10}x^2 + \frac{5}{2}x + \frac{3}{2} = \frac{15}{2} - \frac{1}{5}x$

$\times 10 \Rightarrow -3x^2 + 25x + 15 = 75 - 2x$

$\therefore \quad 0 = 3x^2 - 27x + 60$

For a point of intersection $b^2 - 4ac \geq 0$

$b^2 - 4ac = 27^2 - 4 \times 3 \times 60$

$= 729 - 720$

$= 9 > 0$

$\therefore$ The ball hits the wall.

(2) a) $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\therefore \frac{y - 5}{-2 - 5} = \frac{x - -1}{4 - -1}$

$\therefore 5y - 25 = -7x - 7$

$\therefore 7x + 5y - 18 = 0$

b) A is where $y = 0 \Rightarrow x = \frac{18}{7}$

B is where $x = 0 \Rightarrow y = \frac{18}{5}$

ΔAOB has area $= \frac{1}{2} \times \frac{18}{7} \times \frac{18}{5} = \frac{324}{70} = \frac{162}{35}$

(3) see answers

(4)

	A	B
s		78
u	30	20
v	30	v
a	0	4
t		

a) for B $v^2 = u^2 + 2as$

$= 20^2 + 2 \times 78 \times 4$

$\therefore v = 32\ ms^{-1}$

b) for B $v = u + at$

$\therefore 32 = 20 + 4t$

$\therefore t = 3$

for A $s = ut + \frac{1}{2}at^2$

$\therefore s = 30 \times 3 = 90\ m$

c)

	A	B
s	x	x
u	30	20
v	30	
a	0	4
t	t	t

For A $s = ut + \frac{1}{2}at^2 \Rightarrow x = 30t \Rightarrow t = \frac{x}{30}$

for B $s = ut + \frac{1}{2}at^2 \Rightarrow x = \frac{20x}{30} + \frac{1}{2} \times 4 \times \frac{x^2}{900}$

$\therefore x = \frac{2}{3}x + \frac{x^2}{450}$

$\therefore \frac{1}{3}x = \frac{x^2}{450}$

$\therefore x = 0$ or $\frac{1}{3} = \frac{x}{450}$

$\therefore x = 150$

$\therefore t = \frac{150}{30} = 5\ s$

better to sub $x = 30t$

$30t = 20t + 2t^2$

$\therefore 10t = 2t^2$

$\therefore t = 0$ or $10 = 2t$

$\therefore t = 5\ s$

(5) a) see answers

b) The areas must be equal if they have travelled the same distance

for C $\quad A = \frac{1}{2} \times 20 \times 30 + (T-20)30$

$= 300 + 30T - 600$

$= 30T - 300$

for M $\quad A = \frac{1}{2} \times 24 \times 8 + (T-8)24$

$= 96 + 24T - 192$

$= 24T - 96$

$\therefore 24T - 96 = 30T - 300$

$204 = 6T$

$\therefore T = 34$

$\therefore A = 30 \times 34 - 300 = 720$ m

(6) a) $\frac{d}{dx}(\sin^6 3x) = 6\sin^5 3x \cos 3x \times 3 = 18\sin^5 3x \cos 3x$

$\therefore \int \sin^5 3x \cos 3x \, dx = \frac{1}{18}\sin^6 3x + c$

b) $\frac{d}{dx}(e^{\sin x}) = e^{\sin x}\cos x$

$\therefore \int e^{\sin x}\cos x \, dx = e^{\sin x} + c$

c) $\frac{d}{dx}(e^{2x}+3) = 2e^{2x}$

$\therefore \int \frac{e^{2x}}{e^{2x}+3} \, dx = \frac{1}{2}\ln|e^{2x}+3| + c$

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SECTION B

1) a) $\frac{d}{dx}(x^2+y^3) = \frac{d}{dx}(2)$

$\therefore 2x + 3y^2\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{-2x}{3y^2}$

b) $\frac{d}{dx}(y^3 + 3x^2y - 4x) = \frac{d}{dx}(0)$

$\therefore 3y^2\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} - 4 = 0$

$\therefore \frac{dy}{dx} = \frac{4-6xy}{3y^2+3x^2}$

c) $\frac{d}{dx}(e^x y) = \frac{d}{dx}(xe^y)$

$\therefore e^x y + e^x\frac{dy}{dx} = e^y + xe^y\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{e^y - e^x y}{e^x - xe^y}$

d) $\frac{d}{dx}(\sqrt{xy} + x + y^2) = \frac{d}{dx}(0)$

$\therefore \frac{1}{2}(xy)^{-\frac{1}{2}}\left(y + x\frac{dy}{dx}\right) + 1 + 2y\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{-1-\frac{1}{2}y(xy)^{-1/2}}{\frac{1}{2}x(xy)^{-1/2}+2y} = \frac{-(xy)^{1/2}-\frac{1}{2}y}{\frac{1}{2}x+2y(xy)^{1/2}}$ *

$= \frac{-2\sqrt{xy}-y}{x+4y\sqrt{xy}}$

* Multiply numerator and denominator by $\sqrt{xy}$

(2)

a) $$\frac{4}{(2x+1)(1-2x)} = \frac{A}{2x+1} + \frac{B}{1-2x}$$

$$\therefore 4 = A(1-2x) + B(2x+1)$$

$$x = \tfrac{1}{2} \Rightarrow 4 = 2B \Rightarrow B = 2$$

$$x = -\tfrac{1}{2} \quad 4 = 2A \Rightarrow A = 2$$

b) $$\therefore \int f(x)\,dx = \int \frac{2}{2x+1}\,dx + \int \frac{2}{1-2x}\,dx$$

$$= \ln|2x+1| - \ln|1-2x| + c$$

$$= \ln\left|\frac{2x+1}{1-2x}\right| + c$$

$$= \ln\left|\frac{2x+1}{1-2x}\right| + \ln A$$

$$= \ln\left|A\frac{(2x+1)}{1-2x}\right|$$

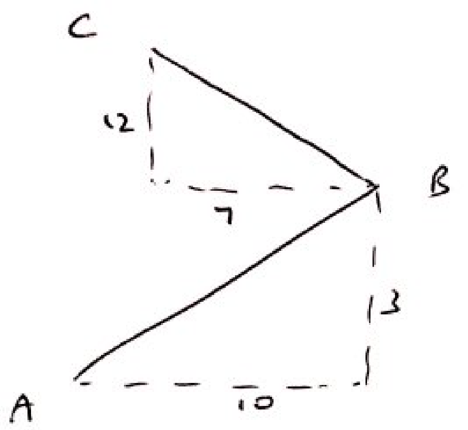
c) $$\int_1^2 f(x)\,dx = \left[\ln\left|\frac{2x+1}{1-2x}\right|\right]_1^2$$

$$= \ln\left|\frac{5}{-3}\right| - \ln\left|\frac{3}{-1}\right|$$

$$= \ln\frac{5}{3} - \ln 3 = \ln 5/9$$

(3) Area $= 2 \times 45 = 90$

(4)



$\vec{AC} = 3\underline{i} + 15\underline{j}$

$AB = \sqrt{109}$ $\quad BC = \sqrt{193}$ $\quad AC = \sqrt{234}$

a) $\sqrt{234} = 15.3$ KM

b) $AB + BC = 24.3$ KM

c) $\tan^{-1} \frac{15}{3} = 78.7°$

(5) a) L 89, 91, 94, 98, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100

C 81, 99 91, 92, 95, 96, 98, 98, 99, 99, 99, 100, 100, 100, 100

Q_1 $\quad Q_2$ $\quad Q_3$

b) see answers

(6) a) $y = \dfrac{(e^{x}+3)^3}{\cos x}$ $\quad \therefore \dfrac{dy}{dx} = \dfrac{3(e^{x}+3)^2 e^{x} \cos x + \sin x (e^{x}+3)^3}{\cos^2 x}$

$$= \frac{(e^{x}+3)^2}{\cos^2 x}\left(3e^{x}\cos x + (e^{x}+3)\sin x\right)$$

b) $y = \sin 5x \ln(\cos x)$

$$\therefore \frac{dy}{dx} = 5\cos 5x \ln(\cos x) - \frac{\sin 5x \sin x}{\cos x}$$

$$= 5\cos 5x \ln(\cos x) - \sin 5x \tan x$$

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SECTION C

① $\tan y = \dfrac{1-x}{1+x}$

$\therefore \dfrac{d}{dx}(\tan y) = \dfrac{d}{dx}\left(\dfrac{1-x}{1+x}\right)$

$\therefore \sec^2 y \dfrac{dy}{dx} = \dfrac{-(1+x)-(1-x)}{(1+x)^2}$

$\therefore (1+\tan^2 y)\dfrac{dy}{dx} = \left[\dfrac{-2}{(1+x)^2}\right]$

$\therefore \left(1 + \dfrac{(1-x)^2}{(1+x)^2}\right)\dfrac{dy}{dx} = \dfrac{-2}{(1+x)^2}$

$\therefore \dfrac{(1+x)^2+(1-x)^2}{(1+x)^2}\dfrac{dy}{dx} = \dfrac{-2}{(1+x)^2}$

$\therefore \dfrac{1+2x+x^2+1-2x+x^2}{(1+x)^2}\dfrac{dy}{dx} = \dfrac{-2}{(1+x)^2}$

$\therefore 2(1+x^2)\dfrac{dy}{dx} = -2$

$\therefore \dfrac{dy}{dx} = \dfrac{-1}{(1+x)^2}$

② a) θ is small $\Rightarrow \dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1-\sin 2\theta} \simeq \dfrac{4\left(1-\frac{(3\theta)^2}{2}\right)-2+5\theta}{1-2\theta}$

$= \dfrac{4\left(1-\frac{9\theta^2}{2}\right)-2+5\theta}{1-2\theta} = \dfrac{4-18\theta^2-2+5\theta}{1-2\theta}$

$= \dfrac{2+5\theta-18\theta^2}{1-2\theta} = \dfrac{18\theta^2-5\theta-2}{2\theta-1} = \dfrac{(9\theta+2)(2\theta-1)}{2\theta-1} = 9\theta+2$

b) θ is small $\Rightarrow 9\theta+2 \to 2$

3

		P	Q
a)	s	S_p	S_Q
	u	4	3
	v		
	a	2	3.6
	t	t	t−1

P $\quad s = ut + \frac{1}{2}at^2 \Rightarrow S_p = 4t + \frac{2t^2}{2} = 4t + t^2$

Q $\quad s = ut + \frac{1}{2}at^2 \Rightarrow S_Q = 3(t-1) + 1.8(t-1)^2$

b) meet where $S_p = S_Q$

$\therefore 4t + t^2 = 3(t-1) + 1.8(t-1)^2$

$\therefore 4t + t^2 = 3t - 3 + 1.8t^2 - 3.6t + 1.8$

$\therefore \quad 0 = 0.8t^2 - 4.6t - 1.2$

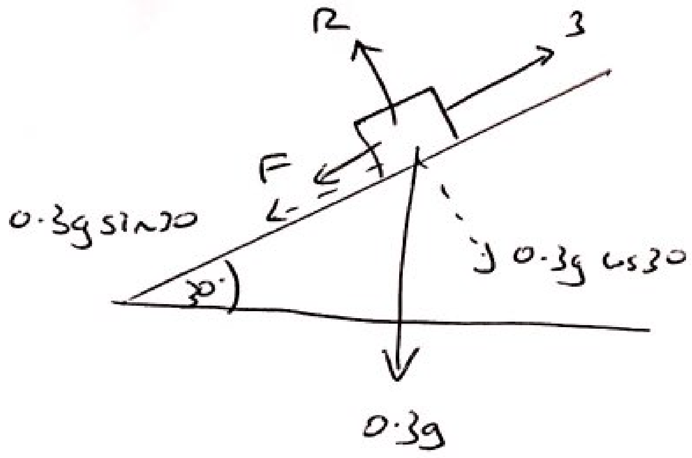
$\therefore t = \dfrac{4.6 \pm \sqrt{4.6^2 + 4 \times 1.2 \times 0.8}}{1.6}$

$= \dfrac{4.6 \pm 5}{1.6} = 6 \text{ or } -\frac{1}{2}$

$\therefore t = 6 \quad (t \geq 0)$

c) $S_p = 4 \times 6 + 6^2 = 60\text{m}$

(4)



$\nwarrow\!\searrow$ $R = 0.3g\cos 30 = 2.546$

$\swarrow\!\nearrow$ $F + 0.3g\sin 30 = 3$

$\therefore F = 1.53$

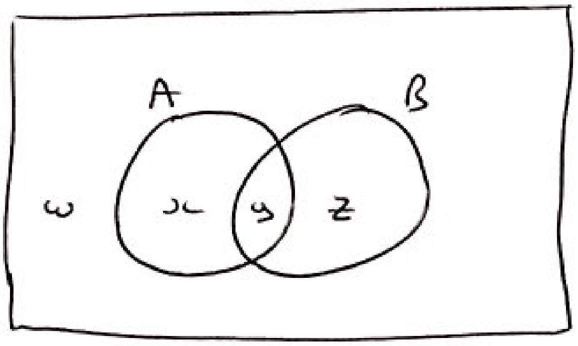
$F = \mu R \Rightarrow \mu = \dfrac{1.53}{2.546} = 0.601$

(6) $\dfrac{d}{dx}\left(y^{1/2} + y^{-1/2}\right) = \dfrac{d}{dx}(x)$

$\therefore \left(\frac{1}{2}y^{-1/2} - \frac{1}{2}y^{-3/2}\right)\dfrac{dy}{dx} = 1$

$y = 4 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\frac{1}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{1}{8}} = \dfrac{1}{\frac{1}{4} - \frac{1}{16}} = \dfrac{16}{3}$

(5)



$P(A) = 0.3 \Rightarrow x + y = 0.3$

$P(B) = 0.7 \Rightarrow y + z = 0.7$

$P(A' \cap B') = 0.1 \Rightarrow w = 0.1$

$\Rightarrow x + y + z = 0.9$

$\therefore z = 0.6$

$\therefore x = 0.2$

$\therefore y = 0.1$

$\therefore$ a) 0.1

b) $\dfrac{y}{y+z} = \dfrac{0.1}{0.7} = 0.143$

c) $P(A \cap B) = y = 0.1$

$P(A) \times P(B) = 0.3 \times 0.7 = 0.21$

$\therefore$ events are not independent.