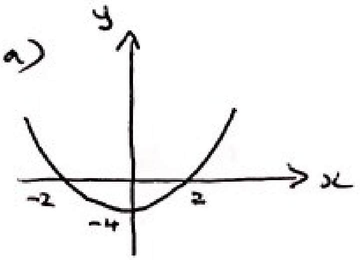
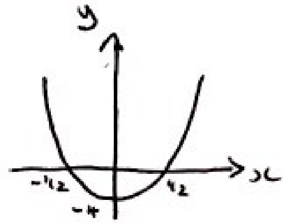
2nd Year Assignment 1

① a)



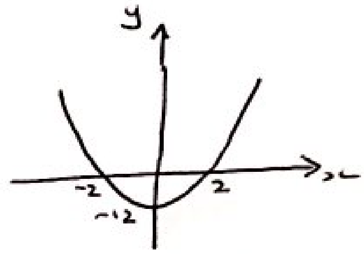
$y = f(x)$

b) i)



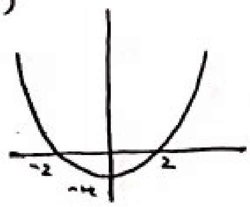
$y = f(4x)$

ii)



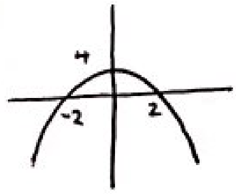
$y = 3f(x)$

iii)



$y = f(-x)$

iv)



$y = -f(x)$

② a) $\text{distance}^2 = (4a - -3a)^2 + (a - 2a)^2$

$= 49a^2 + a^2$

$= 50a^2$

$\therefore \text{distance} = a\sqrt{50} = 5a\sqrt{2}$ (take positive square root, since distance > 0)

b) i) $a = 1 \Rightarrow \text{distance} = 5\sqrt{2}$

ii) $a = 3 \Rightarrow \text{distance} = 15\sqrt{2}$

iii) $a = -5 \Rightarrow \text{distance} = -25 \times -\sqrt{2} = 25\sqrt{2}$ (distance > 0)

③ using my calculator

	mean	s.d.
Heathrow	12.7	1.88
Beijing	18.9	1.28

(3 s.f.)

Beijing is generally hotter and the temperatures in Heathrow are more variable

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(4) $s = h \quad u = 30 \quad v = \quad a = -9.8 \quad t = t$

use $s = ut + \frac{1}{2}at^2$

$\therefore h = 30t - 4.9t^2$

$\therefore 4.9t^2 - 30t + h = 0$

$\therefore t = \dfrac{30 \pm \sqrt{900 - 19.6h}}{9.8}$

$\therefore t_1 = \dfrac{30 + \sqrt{900 - 19.6h}}{9.8} \qquad t_2 = \dfrac{30 - \sqrt{900 - 19.6h}}{9.8}$

since $t_1 > t_2$

$t_1 - t_2 = 2.4$

$\therefore \dfrac{\left(30 + \sqrt{900 - 19.6h}\right) - \left(30 - \sqrt{900 - 19.6h}\right)}{9.8} = 2.4$

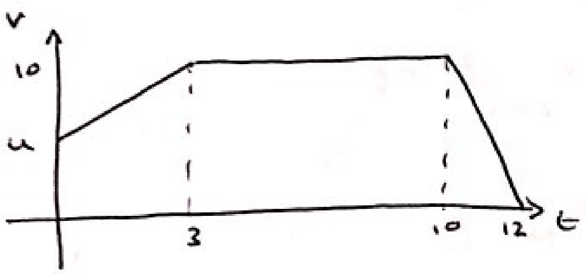
$\therefore \dfrac{2\sqrt{900 - 19.6h}}{9.8} = 2.4$

$\therefore 900 - 19.6h = \left(\dfrac{9.8 \times 2.4}{2}\right)^2$

$\therefore h = 38.8623$

$\approx 39\ m \quad (2\ sf)$

(5) a)



b) Area $= 100 \quad \therefore \dfrac{10+u}{2} \times 3 + 7 \times 10 + \frac{1}{2} \times 10 \times 2 = 100$

$\therefore 3\left(\dfrac{10+u}{2}\right) = 20$

$\therefore \dfrac{10+u}{2} = \dfrac{20}{3}$

$\therefore u = \dfrac{40}{3} - 10 = \dfrac{10}{3}\ ms^{-1}$

c) $v = u + at$

$\therefore 10 = \dfrac{10}{3} + 3a$

$\therefore a = \dfrac{20}{9}\ ms^{-2}$

6 a) $y = \frac{\arctan 2x}{x}$

$\therefore \frac{dy}{dx} = \frac{\frac{1}{1+(2x)^2} \times 2x - \arctan 2x}{x^2}$

$x = \frac{\sqrt{3}}{2} \Rightarrow \frac{dy}{dx} = \frac{\frac{1}{1+3} \times \sqrt{3} - \pi/3}{3/4}$

$= \frac{\frac{\sqrt{3}}{4} - \frac{\pi}{3}}{3/4}$

$= \left(\frac{\sqrt{3}}{4} - \frac{\pi}{3}\right)\frac{4}{3}$

$= \frac{\sqrt{3}}{3} - \frac{4\pi}{9}$

$= \frac{3\sqrt{3} - 4\pi}{9}$

$y = \arctan x$

$x = \tan y$

$\therefore \frac{dx}{dy} = \sec^2 y$

$= 1 + \tan^2 y$

$= 1 + x^2$

$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$

b) $x = \frac{\sqrt{3}}{2} \Rightarrow y = \frac{\arctan \sqrt{3}}{\sqrt{3}/2} = \frac{\pi}{3} \times \frac{2}{\sqrt{3}} = \frac{2\pi}{3\sqrt{3}} = \frac{2\pi\sqrt{3}}{9}$

Equation of normal $= y - y_1 = \frac{-1}{m}(x - x_1)$

$\therefore y - \frac{2\pi\sqrt{3}}{9} = \frac{9}{4\pi - 3\sqrt{3}}\left(x - \frac{\sqrt{3}}{2}\right)$

$\therefore y = \frac{9}{4\pi - 3\sqrt{3}}x - \frac{9\sqrt{3}}{8\pi - 6\sqrt{3}} + \frac{2\pi\sqrt{3}}{9}$

$= \frac{-9}{3\sqrt{3} - 4\pi}x + \frac{9\sqrt{3}}{6\sqrt{3} - 8\pi} + \frac{2\pi\sqrt{3}}{9}$

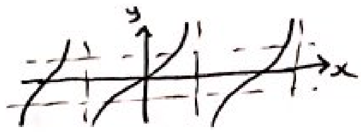
SECTION B

① a) $LHS \equiv \dfrac{1-\cos 2x}{1+\cos 2x} \equiv \dfrac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)} \equiv \dfrac{2\sin^2 x}{2\cos^2 x}$

$\equiv \tan^2 x \equiv RHS$

b) $\tan^2 x = 3$

$\therefore \tan x = \pm\sqrt{3}$



$\therefore x = \frac{\pi}{3}, -\frac{\pi}{3}$

② a) Let $A = 0$, $B = \frac{\pi}{6}$

$\therefore \sec(A+B) = \sec\frac{\pi}{6} = \dfrac{1}{\cos \pi/6} = \dfrac{2}{\sqrt{3}}$

$\sec A + \sec B = \dfrac{1}{\cos 0} + \dfrac{1}{\cos \pi/6} = 1 + \dfrac{2}{\sqrt{3}} \neq \dfrac{2}{\sqrt{3}}$

$\therefore \sec(A+B) \neq \sec A + \sec B \quad \forall A, B$

b) $LHS \equiv \tan\theta + \cot\theta \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$

$\equiv 2\operatorname{cosec} 2\theta \equiv RHS$

③ a) use $y = \dfrac{x-3}{7}$

$\therefore$ data becomes 7, 10, 4, 10, 5, 11, 2, 3

b) mean of coded data $= \dfrac{52}{8} = \dfrac{13}{2}$

c) mean of original data is M

$\therefore \dfrac{13}{2} = \dfrac{M-3}{7}$

$\therefore M = \dfrac{91}{2} + 3 = \dfrac{97}{2}$

4) a) Newton's first law of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.

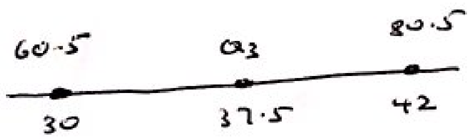
b) Newton's second law of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced. $F = ma$

c) Newton's third law states that for every action there is an equal and opposite reaction.

5)

	1-20	21-40	41-60	61-80	81-100
f	5	10	15	12	8
cumulative frequency	5	15	30	42	50

upper quartile $= \frac{3}{4} \times 50 = 37.5^{th}$ value

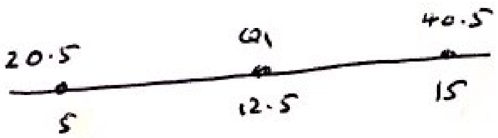


$$\frac{Q_3 - 60.5}{80.5 - 60.5} = \frac{37.5 - 30}{42 - 30}$$

$$\therefore Q_3 - 60.5 = 20 \times \frac{7.5}{12}$$

$$\therefore Q_3 = 73$$

lower quartile $= \frac{1}{4} \times 50 = 12.5^{th}$ value



$$\frac{Q_1 - 20.5}{40.5 - 20.5} = \frac{12.5 - 5}{15 - 5}$$

$$\therefore Q_1 - 20.5 = 20 \times \frac{7.5}{10}$$

$$\therefore Q_1 = 35.5$$

$\therefore$ Interquartile range $= 73 - 35.5 = 37.5$

⑥ a) $(2+5\sqrt{x})(2+5\sqrt{x}) = 4 + 25x + 10\sqrt{x} + 10\sqrt{x}$

$= 4 + 20\sqrt{x} + 25x$

i.e. $k = 20$

b) $\int (2+5\sqrt{x})^2\,dx = \int 4 + 20\sqrt{x} + 25x\,dx$

$= 4x + 20 \times \frac{2}{3} x^{3/2} + \frac{25}{2} x^2 + c$

$= 4x + \frac{40}{3} x^{3/2} + \frac{25}{2} x^2 + c$

SECTION C

1 a) $(\sec\theta - \cos\theta)^2 = \tan\theta - \sin^2\theta$

$\therefore \sec^2\theta - 2 + \cos^2\theta = \tan\theta - \sin^2\theta$

$\therefore \sin^2\theta + \cos^2\theta - 2 = \tan\theta - \sec^2\theta$

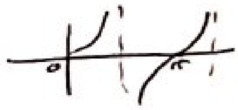
$\therefore -1 = \tan\theta - (1 + \tan^2\theta)$

$\therefore \tan^2\theta - \tan\theta = 0$

$\therefore \tan\theta(\tan\theta - 1) = 0$

$\therefore \tan\theta = 0$ or $\tan\theta = 1$

$\therefore \theta = 0, \pi/4, \pi$



b) $3\sec\frac{1}{2}\theta = 2\tan^2\frac{1}{2}\theta$

$\therefore 3\sec\frac{1}{2}\theta = 2(\sec^2\frac{1}{2}\theta - 1)$

$\therefore 2\sec^2\frac{1}{2}\theta - 3\sec\frac{1}{2}\theta + 2 = 0$

$\therefore (2\sec\frac{1}{2}\theta + 1)(\sec\frac{1}{2}\theta - 2) = 0$

$\therefore \sec\frac{1}{2}\theta = -\frac{1}{2}$ or $\sec\frac{1}{2}\theta = 2$

$\therefore \cos\frac{1}{2}\theta = -2$ No solution

or $\cos\frac{1}{2}\theta = \frac{1}{2} \Rightarrow \frac{1}{2}\theta = 60°, 300° \ldots$

$\therefore \theta = 120°$ since $0 \leq \theta \leq 360°$

c) $\tan^2 2\theta = \sec 2\theta - 1$

$\therefore \sec^2 2\theta - 1 = \sec 2\theta - 1$

$\therefore \sec 2\theta(\sec 2\theta - 1) = 0$

$\therefore \sec 2\theta = 0$ or $\sec 2\theta = 1$

$\therefore \cos 2\theta \to \infty$ no solution

or $\cos 2\theta = 1 \Rightarrow 2\theta = 0, 360, \ldots$

$\therefore \theta = 0, 180°$

1 ctd

d) $\sec^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 1$

$\therefore 1 + \tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} - 1 = 0$

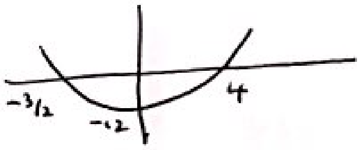
$\therefore \tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$

$\therefore (\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$

$\therefore \tan\theta = 1$ or $\tan\theta = \sqrt{3}$

$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{3}, \frac{4\pi}{3}$

2 a)



b) $A(-\frac{3}{2}, 0)$ $B(4, 0)$ $C(0, -12)$

c) $A = \int_{-3/2}^{4} (x-4)(2x+3)\,dx$

$= \int_{-3/2}^{4} 2x^2 - 5x - 12\,dx$

$= \left[\frac{2}{3}x^3 - \frac{5}{2}x^2 - 12x\right]_{-3/2}^{4}$

$= \left(\frac{2}{3}\times 64 - \frac{5}{2}\times 16 - 12\times 4\right) - \left(\frac{2}{3}\times -\frac{27}{8} - \frac{5}{2}\times\frac{9}{4} - 12\times -\frac{3}{2}\right)$

$= \left(\frac{128}{3} - 40 - 48\right) - \left(-\frac{9}{4} - \frac{45}{8} + 18\right)$

$= -\frac{136}{3} - \frac{81}{8}$

$= -\frac{1331}{24}$

$\therefore$ The area is $\frac{1331}{24}$ below the x axis

(3) $\underline{F}_1 + \underline{F}_2 = m\underline{a}$

$$\therefore \underline{a} = \frac{\underline{F}_1 + \underline{F}_2}{m}$$

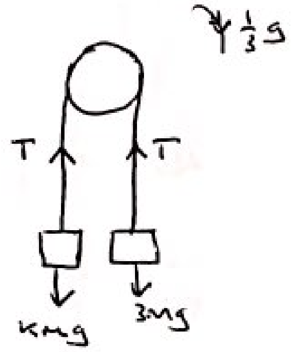
a) $\underline{a} = \dfrac{-\underline{i} + 8\underline{j}}{0.25} = -4\underline{i} + 32\underline{j}$

b) $\underline{a} = \dfrac{5\underline{i} - \underline{j}}{6} = \frac{5}{6}\underline{i} - \frac{1}{6}\underline{j}$

c) $\underline{a} = \dfrac{-15\underline{i} - 10\underline{j}}{15} = -\underline{i} - \frac{2}{3}\underline{j}$

d) $\underline{a} = \dfrac{-2\underline{i} + 9\underline{j}}{1.5} = -\frac{4}{3}\underline{i} + 6\underline{j}$

(4)



b) $3mg - T = 3m \times \frac{1}{3}g$ —— (1)

$T - kmg = km \times \frac{1}{3}g$ —— (2)

(1) + (2) $\Rightarrow 3mg - kmg = mg + \frac{kmg}{3}$

$\therefore 2g = k\left(\frac{4g}{3}\right)$

$\therefore k = 1.5$

a) use equation (1)

$T = 3mg - mg = 2mg$

c) Mag of acceleration is the same

d) $s =$ $u = 0$ $v =$ $a = \frac{1}{3}g$ $t = 1.8$

$s = ut + \frac{1}{2}at^2 \Rightarrow s = 0 + \frac{1}{2} \times \frac{1}{3}g \times 1.8^2 \Rightarrow s = 0.54g$

$v = u + at \Rightarrow v = 0.6g$

once Q reaches the plane, P is moving with acceleration $= -g$

$s =$ $u = 0.6g$ $v = 0$ $a = -g$ $t =$

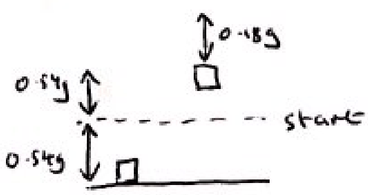
$v^2 = u^2 + 2as$

$\therefore 0 = 0.36g^2 - 2gs$

$\therefore 0 = g(0.36g - 2s)$

$\therefore 2s = 0.36g$

$\therefore s = 0.18g$



$0.54g$
$0.54g$
$0.18g$
$\overline{1.26g}$

⑤ Put in order

13, 17, 19, 20, 21, 21, 22, 23, 24, 25, 25, 25, 26, 26, 26, 27, 29,
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
Q_1 (7) Q_2 (13)

30, 30, 31, 33, 35, 38, 46, 78
18 19 20 21 22 23 24 25
Q_3 (19)

25 numbers

$Q_1 = \frac{25}{4} = 6.25 = 7^{th}$ number $= 22$

$Q_2 = \frac{25+1}{2} = 13^{th}$ number $= 26$

$Q_3 = \frac{3}{4} \times 25 = 18.75 = 19^{th}$ number $= 30$

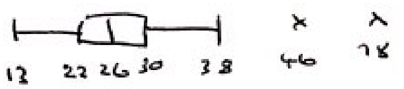
$Q_3 - Q_1 = 30 - 22 = 8 = IQR$

$1.5 \times 8 = 12$

$Q_1 - 1.5 \times 8 = 10$

$Q_3 + 1.5 \times 18 = 42$

$\therefore$ 46 and 78 are outliers



⑥ $f(x) = 9x^4 - 18x^3 - x^2 + 2x$

$f(2) = 9 \times 16 - 18 \times 8 - 4 + 4$
$= 0$

$\therefore x - 2$ is a factor.

clearly x is a factor

$\therefore f(x) = x(9x^3 - 18x^2 - x + 2)$
$= x(x-2)(9x^2 - 1)$
$= x(x-2)(3x+1)(3x-1)$

$\therefore f(x) = 0 \Rightarrow x = 0, 2, -1/3, 1/3$

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