

Prove that there are infinitely many prime numbers

Assume that there is a finite number of prime numbers

List all the prime numbers: $p_1, p_2, p_3, \dots, p_n$

Consider the number $N = p_1 \times p_2 \times p_3 \times \dots \times p_n + 1$

When you divide N by any of the prime numbers,
you get a remainder of 1

So, none of $p_1, p_2, p_3, \dots, p_n$ is a factor of N

This means N must either be a prime number

or have a prime factor which is not

in the list $p_1, p_2, p_3, \dots, p_n$

Contradiction \times

\therefore There is an infinite number of prime numbers