Prove $\sqrt{2}$ is irrational

Assume $\sqrt{2}$ is rational

$\therefore \sqrt{2} = \frac{a}{b}$ for some integers a and b

Also assume that a and b have no common factors

$\therefore 2 = \frac{a^2}{b^2}$

$\therefore a^2 = 2b^2$

$\therefore a^2$ is even which means a must be even

If a is even, it can be expressed in the form $a = 2n$ where n is an integer

$\therefore a^2 = 4n^2$

But $a^2 = 2b^2$

$\therefore 2b^2 = 4n^2$

$\therefore b^2 = 2n^2$

$\therefore b^2$ is even, which means b must be even

$\therefore$ Both a and b are even, which means they have a common factor

CONTRADICTION

$\therefore \sqrt{2}$ is an irrational number