Prove that the first n terms of an arithmetic series have a sum equal to $\frac{n}{2}(2a+(n-1)d)$

Let $S_n = a + (a+d) + (a+2d) + \dots + (a+(n-2)d) + (a+(n-1)d)$ — ①

$\therefore S_n = (a+(n-1)d) + (a+(n-2)d) + \dots + (a+2d) + (a+d) + a$ — ②

① + ② $\Rightarrow$ $2S_n = (a+a+(n-1)d) + (a+d+a+(n-2)d) + \dots$

$\dots + (a+(n-2)d+a+d) + (a+(n-1)d+a)$

$= (2a+(n-1)d) + (2a+(n-1)d) + \dots + (2a+(n-1)d) + (2a+(n-1)d)$

$= n(2a+(n-1)d)$

$\therefore S_n = \frac{n}{2}(2a+(n-1)d)$

Prove that the sum of the first n natural numbers is given by $\frac{n}{2}(n+1)$

Let $S_n = 1+2+3+ \dots + (n-2)+(n-1)+n$ — ①

$\therefore S_n = n+(n-1)+(n-2)+\dots+3+2+1$ — ②

① + ② $\Rightarrow$ $2S_n = (1+n) + (2+n-1) + (3+n-2) + \dots + (n-2+3) + (n-1+2) + (n+1)$

$= (n+1)+(n+1)+(n+1)+\dots+(n+1)+(n+1)+(n+1)$

$= n(n+1)$

$\therefore S_n = \frac{n}{2}(n+1)$

Prove that the sum of a geometric sequence is given by

$$S = \frac{a(r^n - 1)}{r-1}$$

Let $S_n = a + ar + ar^2 + \dots\dots + ar^{n-2} + ar^{n-1}$ —— ①

$\therefore rS_n = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n$ — ②

② − ① $\Rightarrow$ $rS_n - S_n = ar^n - a$

$\therefore S_n \cdot (r-1) = a(r^n - 1)$

$\therefore S_n = \frac{a(r^n-1)}{r-1}$