Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 1 or 2: Pure Mathematics

Practice Paper C **Time: 2 hours**	**Paper Reference(s)** **9MA0/01 or 9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are xx questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1. The functions f and g are defined by $f(x) = e^{2x} + 4$, $x \in \mathbb{R}$ and $g(x) = \ln(x+1)$, $x \in \mathbb{R}, x > -1$

a Find fg(x) and state its range. **(4 marks)**

b Solve fg(x) = 85 **(3 marks)**

2. A curve C has equation $4^x = 2xy$ for $x > 0$

Find the exact value of $\frac{dy}{dx}$ at the point C with coordinates (2, 4).

(5 marks)

3. (a) Show that $\cos 7x + \cos 3x = 2\cos 5x \cos 2x$ by expanding $\cos(5x + 2x)$ and $\cos(5x - 2x)$ using the compound-angle formulae.

(3 marks)

(b) Hence find $\int (\cos 5x \cos 2x)\, dx$.

(3 marks)

4. The temperature of a mug of coffee at time t can be modelled by the equation

$$\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t},$$

where $\mathrm{T}(t)$ is the temperature, in °C, of the coffee at time t minutes after the coffee was poured into the mug and T_R is the room temperature in °C.

Using the equation for this model,

(a) explain why the initial temperature of the coffee is independent of the initial room temperature.

(2 marks)

(b) Calculate the temperature of the coffee after 10 minutes if the room temperature is 20 °C.

(2 marks)

(Total for Question 1 is 7 marks)

5. Prove by contradiction that if n is odd, $n^3 + 1$ is even.

(5 marks)

6. A curve C has parametric equations $x = \sec^2 t + 1, y = 2\sin t, -\frac{\pi}{4} \leq t \leq \frac{\pi}{4}$.

(a) Show that a cartesian equation of C is $y = \sqrt{\frac{8-4x}{1-x}}$ for a suitable domain which should be stated.

(4 marks)

(b) Find the equation of the normal where the curve C crosses the x axis

(3 marks)

7. An infinite geometric series has first four terms $1-4x+16x^2-64x^3+...$ The series is convergent.

(a) Find the set of possible values of x for which the series converges.

(2 marks)

Given that $\sum_{r=1}^{\infty}(-4x)^{r-1}=4$,

(b) calculate the value of x.

(3 marks)

8. $f(x) = 2 - 3\sin^3 x - \cos x$, where x is in radians.

(a) Show that $f(x) = 0$ has a root α between $x = 1.9$ and $x = 2.0$.

(2 marks)

Using $x_0 = 1.95$ as a first approximation,

(b) apply the Newton–Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(5 marks)

9. Given that $(b-a)\mathbf{i}-2abc\mathbf{j}+2\mathbf{k}=10\mathbf{i}-96\mathbf{j}+(7a+5b)\mathbf{k}$, find the values of *a*, *b* and *c*.

(6 marks)

10. Use proof by contradiction to show that there are no positive integer solutions to the statement $x^2 - y^2 = 1$.

(5 marks)

11. The function g(x) is defined by $g(x) = x^2 - 8x + 7$, $x \in \mathbb{R}$, $x > 4$.

Find $g^{-1}(x)$ and state its domain and range.

(6 marks)

12. Given that $x = \sec 4y$, find:

a $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y **(2 marks)**

b Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x\sqrt{x^2 - 1}}$

where k is a constant which should be found. **(3 marks)**

13 The curve C has equation $y = x^3 + 6x^2 - 12x + 6$.

(a) Show that C is concave on the interval $[-5, -3]$.

(3 marks)

(b) Find the coordinates of the point of inflection.

(3 marks)

14 Find $\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \sin 4x\left(1-\cos 4x\right)^3 \, \mathrm{d}x$.

(4 marks)

15 $$\frac{4x^2 - 4x - 9}{(2x+1)(x-1)} \equiv A + \frac{B}{2x+1} + \frac{C}{x-1}.$$

(a) Find the values of the constants A, B and C.

(6 marks)

(b) Hence, or otherwise, expand $\frac{4x^2 - 4x - 9}{(2x+1)(x-1)}$ in ascending powers of x, as far as the x^2 term.

(6 marks)

(c) Explain why the expansion is not valid for $x = \frac{3}{4}$.

(1 mark)

16 A large cylindrical tank has radius 40 m. Water flows into the cylinder from a pipe at a rate of $4000\pi \text{ m}^3 \text{ min}^{-1}$. At time t, the depth of water in the tank is h m. Water leaves the bottom of the tank through another pipe at a rate of $50\pi h \text{ m}^3 \text{ min}^{-1}$.

(a) Show that t minutes after water begins to flow out of the bottom of the cylinder, $160\frac{\text{d}h}{\text{d}t} = 400 - 5h$.

(6 marks)

When $t = 0$ min, $h = 50$ m.

(b) Find the exact value of t when $h = 60$ m.

(6 marks)