1.

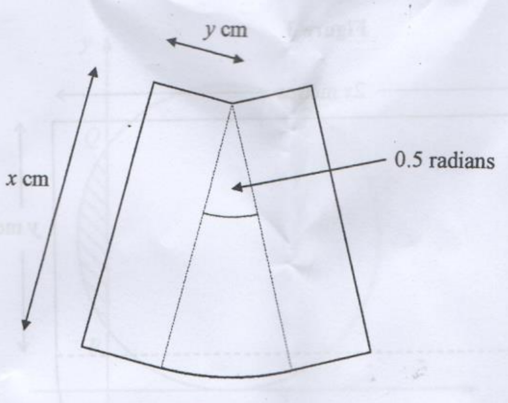

Figure 3

Figure 3 shows a design consisting of two rectangles measuring x cm by y cm joined to a circular sector of radius x cm and angle 0.5 radians.

Given that the area of the design is 50 cm^2,

(a) show that the perimeter, P cm, of the design is given by

$$P = 2x + \frac{100}{x}.$$

(5)

(b) Find the value of x for which P is a minimum. **(4)**

(c) Show that P is a minimum for this value of x. **(2)**

(d) Find the minimum value of P in the form $k\sqrt{2}$. **(2)**

2. A pencil holder is in the shape of an open circular cylinder of radius r cm and height h cm. The surface area of the cylinder (including the base) is 250 cm^2.

(a) Show that the volume, $V cm^3$, of the cylinder is given by $V = 125r - \frac{\pi r^3}{2}$.

(4)

(b) Use calculus to find the value of r for which V has a stationary value.

(3)

(c) Prove that the value of r you found in part (b) gives a maximum value for V.

(2)

(d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.

(2)

3.

Figure 3

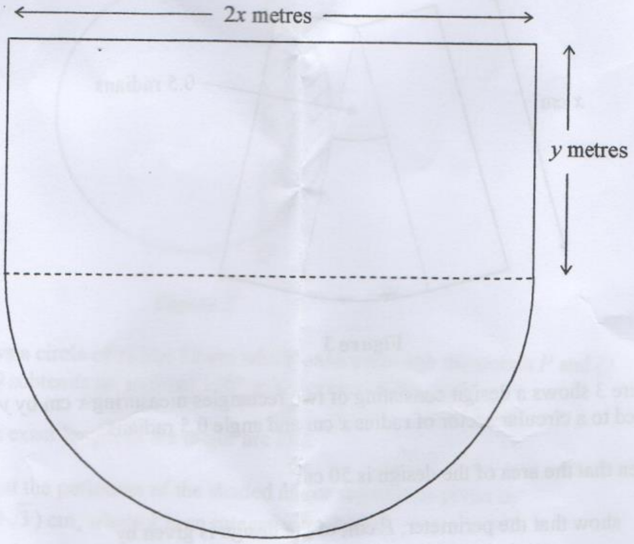

Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(*a*) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(*b*) Use calculus to find the value of x at which A has a stationary value. **(4)**

(*c*) Prove that the value of x you found in part (*b*) gives the maximum value of A. **(2)**

(*d*) Calculate, to the nearest m^2, the maximum area of the stage. **(2)**

4.

$$f(x) = 2 - x + 3x^{\frac{2}{3}}, \quad x > 0.$$

(*a*) Find $f'(x)$ and $f''(x)$. **(3)**

(*b*) Find the coordinates of the turning point of the curve $y = f(x)$. **(4)**

(*c*) Determine whether the turning point is a maximum or minimum point. **(2)**

5.

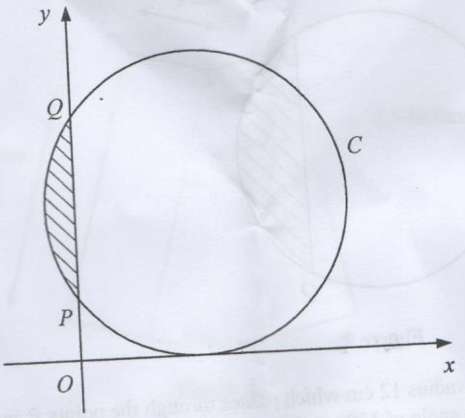

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 8x - 10y + 16 = 0.$$

(a) Find the coordinates of the centre and the radius of C. **(3)**

C crosses the y-axis at the points P and Q.

(b) Find the coordinates of P and Q. **(3)**

The chord PQ subtends an angle of θ at the centre of C.

(c) Using the cosine rule, show that $\cos\theta = \frac{7}{25}$. **(4)**

(d) Find the area of the shaded minor segment bounded by C and the chord PQ. **(4)**

Figure 1

6.

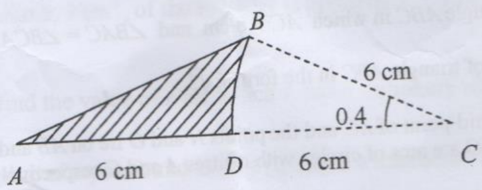

Figure 1 shows a logo ABD.

The logo is formed from triangle ABC. The mid-point of AC is D and $BC = AD = DC = 6$ cm. $\angle BCA = 0.4$ radians. The curve BD is an arc of a circle with centre C and radius 6 cm.

(a) Write down the length of the arc BD. **(1)**

(b) Find the length of AB. **(3)**

(c) Write down the perimeter of the logo ABD, giving your answer to 3 significant figures. **(1)**

7.

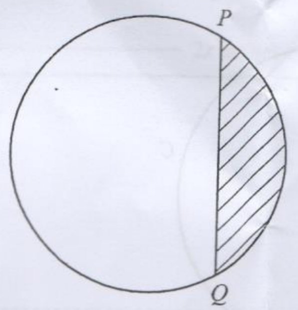

Figure 2

Figure 2 shows a circle of radius 12 cm which passes through the points P and Q. The chord PQ subtends an angle of 120° at the centre of the circle.

(a) Find the exact length of the major arc PQ. **(2)**

(b) Show that the perimeter of the shaded minor segment is given by $k(2\pi + 3\sqrt{3})$ cm, where k is an integer to be found. **(4)**

(c) Find, to 1 decimal place, the area of the shaded minor segment as a percentage of the area of the circle. **(4)**

8.

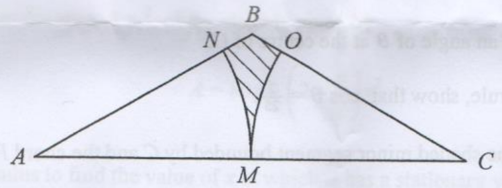

Figure 1

Figure 1 shows triangle ABC in which $AC = 8$ cm and $\angle BAC = \angle BCA = 30°$.

(a) Find the area of triangle ABC in the form $k\sqrt{3}$. **(5)**

The point M is the mid-point of AC and the points N and O lie on AB and BC such that MN and MO are arcs of circles with centres A and C respectively.

(b) Show that the area of the shaded region $BNMO$ is $\frac{8}{3}(2\sqrt{3} - \pi)$ cm^2. **(4)**