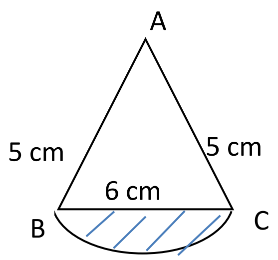
1) Sketch the graph of $y = 3(x - 2)^2 - 12$.
   a. Give the co-ordinates of the y-intercept
   b. Give the co-ordinates of the x-intercepts
   c. Give the co-ordinates of the minimum point
2) The equation $px^2 - 3px + 9 = 0$ has real roots. Find the range of possible values of p
3) On a co-ordinate diagram, sketch the points A(3,8) and B(-1,5)
   a. Find the equation of the straight line through A and B
   b. Find the equation of the normal to AB which passes through AB
   c. Give the co-ordinates of the midpoint of AB
   d. Find the length AB
   e. Give the equation of the circle which has AB as its diameter
4) Sketch the following graphs
   a. $y = \sin x$ $\quad 0 \le x \le 360°$
   b. $y = \cos 2x$ $\quad 0 \le x \le 360°$
   c. $y = 2\sin x$ $\quad 0 \le x \le 360°$
   d. $y = -\sin x$ $\quad 0 \le x \le 360°$
   e. $y = 1 + \tan x$ $\quad 0 \le x \le 360°$
   f. $y = 3\cos \frac{1}{2} x$ $\quad 0 \le x \le 360°$
5)



a) Calculate the angle BAC (use the cosine rule).
b) Calculate the arc length BC.
c) Calculate the area of the triangle ABC.
d) Calculate the area of the sector ABC.
e) Find the perimeter of the shaded area to 3 s.f.
f) Find the area of the shaded area to 3 s.f.

6) A curve has equation $y = \frac{1}{3}x^3 - 2x^2$
a) The gradient of the tangent when $x = -1$
b) The equation of the tangent when $x = 2$
c) Find the x coordinate where the gradient is -4
d) Given that the tangents to the curve at P and R are parallel to the line $y + 3x = 1$, find the x coordinate of P and R
e) Find the coordinates of the stationary points of the curve
f) Determine whether the stationery points are a minimum or maximum

7) A box of mass 5 kg lies on a smooth plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude P N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

(*a*) the magnitude of the normal reaction of the plane on the box,

(*b*) the value of P

8) A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point *A*. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point *B*.

(*a*) Sketch a speed-time graph to represent the motion of the car from *A* to *B*.

(*b*) Find the time for which the car is decelerating.

Given that the distance from *A* to *B* is 1960 m,

(*c*) find the time taken for the car to move from *A* to *B*.

9) Two particles *A* and *B* of masses *m* and *km* respectively are connected by a light inextensible string which passes over a smooth fixed pulley.

When the system is released from rest with both particles 0.5 m above the ground, particle *A* moves vertically upwards with acceleration $\frac{1}{4}g$ ms^{-2}.

(a) Find the value of *k*.

Given that *A* does not hit the pulley,

(b) calculate, correct to 3 significant figures, the speed with which *B* hits the ground.

(1) a) $x=0 \Rightarrow y = 3(-2)^2-12$

$= 3\times4-12$

$=0$ $\quad\therefore (0,0)$

b) $y=0 \Rightarrow 3(x-2)^2-12=0$

$\therefore 3(x-2)^2 = 12$

$\therefore (x-2)^2 = 4$

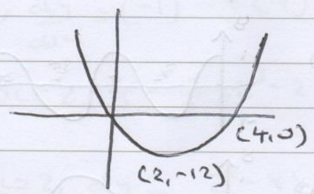
$\therefore x-2 = \pm2$

$\therefore x=2+2$ or $-2+2$

$\therefore x=4$ or $x=0$ $\quad\therefore (4,0)$ and $(0,0)$

c) minimum occurs when $(x-2)^2=0$

i.e. $x=2$ and $y=3\times0-12=-12$ $\quad\therefore (2,-12)$



(2) For real roots $b^2-4ac \geq 0$

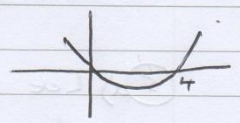
$\therefore (-3p)^2-4(p)(9)\geq0$

$\therefore 9p^2-36p\geq0$

$\therefore p^2-4p\geq0$

$\therefore p(p-4)\geq0$

$\therefore p\geq4$ or $p\leq0$



(3) a) $x_1=3 \quad y_1=8 \qquad x_2=-1 \quad y_2=5$

equation is $\dfrac{y-8}{5-8} = \dfrac{x-3}{-1-3}$

i.e. $\dfrac{y-8}{-3} = \dfrac{x-3}{-4}$

$\therefore -4y+32=-3x+9$

$\therefore 3x-4y+23=0$

b) gradient of AB $= 3/4$

$\therefore$ gradient of normal $= -4/3$

$\therefore$ equation is $y - 8 = -\frac{4}{3}(x-3)$

i.e. $3y - 24 = -4x + 12$

$\therefore 4x + 3y - 36 = 0$

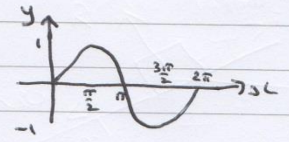
c) $(1, 6\frac{1}{2})$

d) length $= \sqrt{4^2+3^2} = 5$

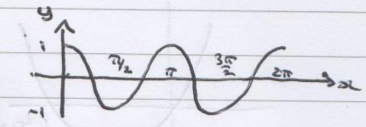
e) $(x-1)^2 + (y - 6\frac{1}{2})^2 = \left(\frac{5}{2}\right)^2$

i.e. $(x-1)^2 + (y-6\frac{1}{2})^2 = \frac{25}{4}$

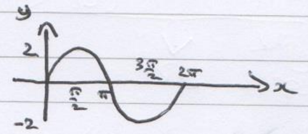
4 a)



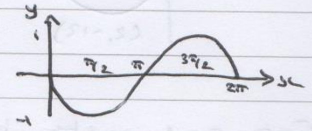
b)



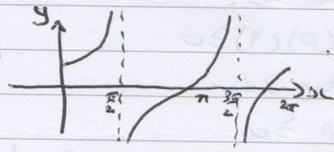
c)



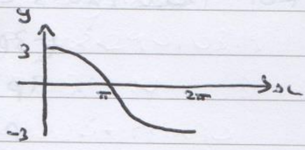
d)



e)



f)



5 a) Let $B\hat{A}C = x°$

$\therefore 6^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \cos x$

$\therefore 36 = 25 + 25 - 50 \cos x$

$\therefore 36 = 50 - 50 \cos x$

$\therefore 50 \cos x = 50 - 36$

$\therefore 50 \cos x = 14$

$\therefore \cos x = 14/50 = 0.28$

$\therefore x = 73.739795° = 73.7°$ (3 s.f.)

$= 1.287002218 = 1.29$ (3 s.f.)

b) arc $BC = 5 \times 1.29 = 6.43501109 = 6.44$ cm (3 s.f.)

5 ctd c) area $ABC = \frac{1}{2} \times 5 \times 5 \times \sin x = 12 \text{ cm}^2$

d) area sector $ABC = \frac{1}{2} \times 5^2 \times 1.29 = 16.08752773$

$= 16.1 \text{ cm}^2$ (3 s.f.)

e) perimeter $= 6 + 6.44 = 12.4$ cm (3 sf)

f) segment $= 16.08752773 - 12 = 4.09 \text{ cm}^2$ (3 sf)

6 a) $y = \frac{1}{3}x^3 - 2x^2$

$\therefore \frac{dy}{dx} = x^2 - 4x$

$x = -1, \frac{dy}{dx} = (-1)^2 - 4(-1) = 1 + 4 = 5$

b) $x = 2, y = \frac{1}{3} \times 2^3 - 2 \times 2^2 = \frac{8}{3} - 8 = -16/3$

$x = 2, \frac{dy}{dx} = 2^2 - 4 \times 2 = -4$

$\therefore$ equation of the tangent is

$y - -\frac{16}{3} = -4(x-2)$

$\therefore 3y + 16 = -12x + 24.$

$\therefore 12x + 3y - 8 = 0$

c) gradient $= -4 \Rightarrow x^2 - 4x = -4$

$\therefore x^2 - 4x + 4 = 0$

$\therefore (x-2)^2 = 0$

$\therefore x = 2$

d) Gradient of $y + 3x = 1$ is -3

$\therefore \frac{dy}{dx} = -3$ i.e. $x^2 - 4x = -3$

$\therefore x^2 - 4x + 3 = 0$

$\therefore (x-1)(x-3) = 0$

$\therefore x = 1$ or $x = 3$

(6) e) $\frac{dy}{dx} = 0 \Rightarrow x^2 - 4x = 0$

ctd

$\therefore x(x-4) = 0$

$\therefore x = 0$ or $x = 4$

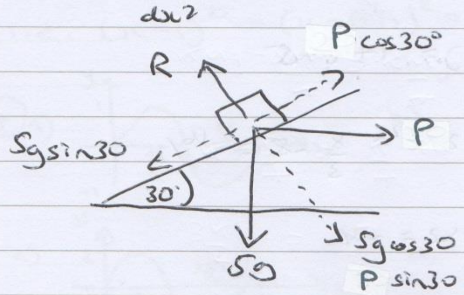
$\therefore$ co-ordinates are $(0,0)$ and $(4, -^{32}/_3)$

f) $\frac{d^2y}{dx^2} = 2x - 4$

$x = 0$, $\frac{d^2y}{dx^2} = -4 < 0 \therefore (0,0)$ is a maximum

$x = 4$, $\frac{d^2y}{dx^2} = 8 - 4 > 0 \therefore (4, -^{32}/_3)$ is a minimum

(7)



Resolve // to slope, $P\cos 30 = 5g\sin 30$

$\therefore P = 28.29016319$

$= 28.3\,N$ (3 sf)

Resolve ⊥ to slope $R = 5g\cos 30 + P\sin 30$

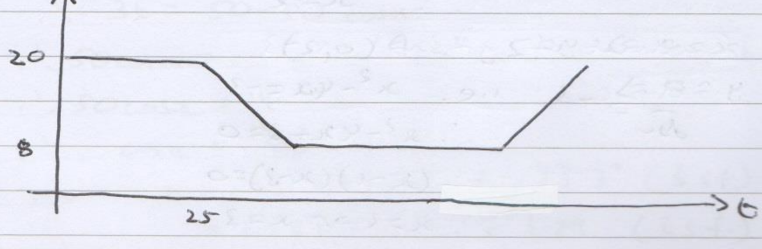
$= 5g\cos 30 + 28.3\sin 30$

$= 56.58032638$

$= 56.6\,N$ (3 sf)

(8)

a)



(8) ctd

b) In the second phase of the journey

$s =$

$u = 20$ $\qquad v = u + at$

$v = 8$ $\qquad \therefore 8 = 20 - 0.4t$

$a = -0.4$ $\qquad \therefore 0.4t = 12$

$t = ?$ $\qquad \therefore t = 30$

c) $s = \frac{u+v}{2} t$ $\qquad \therefore s = \frac{20+8}{2} \times 30$

$= 420m$

In the first phase of the journey

$s =$

$u = 20$ $\qquad s = \frac{u+v}{2} t$

$v = 20$

$a = 0$ $\qquad = 20 \times 25$

$t = 25$ $\qquad = 500m$

In the third phase of the journey

$s =$

$u = 8$ $\qquad s = \frac{u+v}{2} t$

$v = 8$

$a =$ $\qquad = 8 \times 60$

$t = 60$ $\qquad = 480m$

In the fourth phase of the journey

$s =$

$u = 8$ $\qquad s = 1960 - 500 - 420 - 480 = 560$

$v = 20$

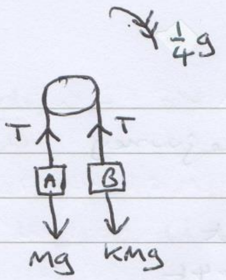
$a =$ $\qquad s = \frac{u+v}{2} t$

$t =$

$\therefore 560 = \frac{20+8}{2} t$

$\therefore t = 40s$ $\qquad \therefore$ total time $= 25 + 30 + 60 + 40$
$= 155s$

9 a)



For A $\quad T - Mg = \frac{M}{4}g$

$\therefore T = \frac{5Mg}{4}$

For B $\quad kMg - T = \frac{kMg}{4}$

$\therefore kMg - \frac{5Mg}{4} = \frac{kMg}{4}$

$\therefore k - \frac{5}{4} = \frac{k}{4}$

$\therefore \frac{3k}{4} = \frac{5}{4}$

$\therefore k = \frac{5}{3}$

b) $s = 0.5$
$u = 0$ $\qquad v^2 = u^2 + 2as$
$v =$ $\qquad \therefore v^2 = 2 \times \frac{1}{4}g \times 0.5$
$a = \frac{1}{4}g$
$t =$ $\qquad \therefore v = 1.57 \text{ ms}^{-1}$ (3sf)