



**Second Year**

**“A” level Maths**

**Survival Kit**

**Name.....**

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Date	Videos	Page
14 <sup>th</sup> September	Implicit Differentiation	4
17 <sup>th</sup> September	Inverse Trig – graphs and equations	5
19 <sup>th</sup> September	Compound and Double angles	6
21 <sup>st</sup> September	Consolidation	
24 <sup>th</sup> September	Continuing With Confidence (Tracking Test 1)	7
26 <sup>th</sup> September	Moments 1	8
28 <sup>th</sup> September	Moments 2	9
1 <sup>st</sup> October	Numerical Methods – locating roots	10
3 <sup>rd</sup> October	Numerical Methods – iterations; cobweb & staircase diagrams	11
5 <sup>th</sup> October	Numerical Methods – questions in context	12
8 <sup>th</sup> October	The Normal Distribution 1	13
10 <sup>th</sup> October	The Normal Distribution 2	14
12 <sup>th</sup> October	The Normal and Binomial Distributions	15
15 <sup>th</sup> October	Moments 3	16
17 <sup>th</sup> October	Moments 4	17
19 <sup>th</sup> October	Moments 5	18
29 <sup>th</sup> October	Further Trig Expressions	19
31 <sup>st</sup> October	Further Trig Equations	20
2 <sup>nd</sup> November	Integration by Substitution 1	21
5 <sup>th</sup> November	Integration by Substitution 2	22
7 <sup>th</sup> November	Integration by Parts 1	23
9 <sup>th</sup> November	Integration by Parts 2	24
12 <sup>th</sup> November	Consolidation	
19 <sup>th</sup> November	Connected Rates	25
21 <sup>st</sup> November	Sampling 1	26
23 <sup>rd</sup> November	Sampling 2	27
26 <sup>th</sup> November	Forming Differential Equations	28
28 <sup>th</sup> November	Solving Differential Equations 1	29
30 <sup>th</sup> November	Solving Differential Equations 2	30
3 <sup>rd</sup> December	Tracking Test 2	31
5 <sup>th</sup> December	Kinematics and variable acceleration 1	32
7 <sup>th</sup> December	Kinematics and variable acceleration 2	33
10 <sup>th</sup> December	Parametric Equations	34
12 <sup>th</sup> December	Modelling using parametric equations	35
14 <sup>th</sup> December	Consolidation	
17 <sup>th</sup> December	Consolidation	

4 <sup>th</sup> January	Scatter diagrams, Frequency Polygons, Histograms	36
7 <sup>th</sup> January	Regression lines	37
9 <sup>th</sup> January	Product Moment Correlation Coefficient 1	38
11 <sup>th</sup> January	Product Moment Correlation Coefficient 2	39
14 <sup>th</sup> January	Further Hypothesis Testing 1	40
16 <sup>th</sup> January	Further Hypothesis Testing 2	41
18 <sup>th</sup> January	Further Hypothesis Testing 3	42
21 <sup>st</sup> January	Tracking Test 3	43
23 <sup>rd</sup> January	Modulus graphs	44
25 <sup>th</sup> January	Consolidation	
28 <sup>th</sup> January	Arithmetic Sequences and Series	45
30 <sup>th</sup> January	Increasing, Decreasing and Periodic Sequences	46
1 <sup>st</sup> February	Consolidation	
4 <sup>th</sup> February	Graphical Inequalities	47
6 <sup>th</sup> February	Geometric Sequences and Series	48
8 <sup>th</sup> February	Use of Sequences and Series in Modelling	49
11 <sup>th</sup> February	Proof by deduction, proof by exhaustion, proof by counter example and proof by contradiction	50-53
13 <sup>th</sup> February	Consolidation	
15 <sup>th</sup> February	Consolidation	

### Pure: Implicit Differentiation

Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only. The finding of equations of tangents and normals to curves given parametrically or implicitly is required.

<https://youtu.be/am9WPDZL76M>



The equation of a curve is  $3x^3 + 2x^2y^3 + 4y^7 = 12$   
Find an expression for  $\frac{dy}{dx}$



### Pure: Inverse Trigonometric Functions

Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.

<https://youtu.be/hklOnHJx1t4>

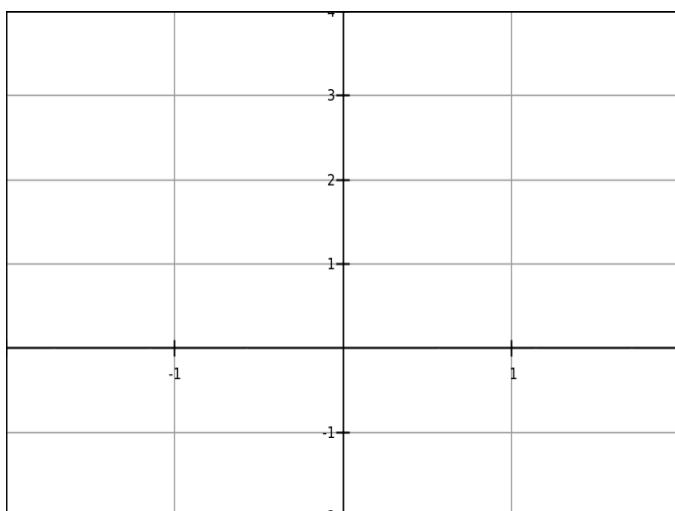


What is the difference between  $y = \sin^{-1}x$  and  $y = (\sin x)^{-1}$  ?

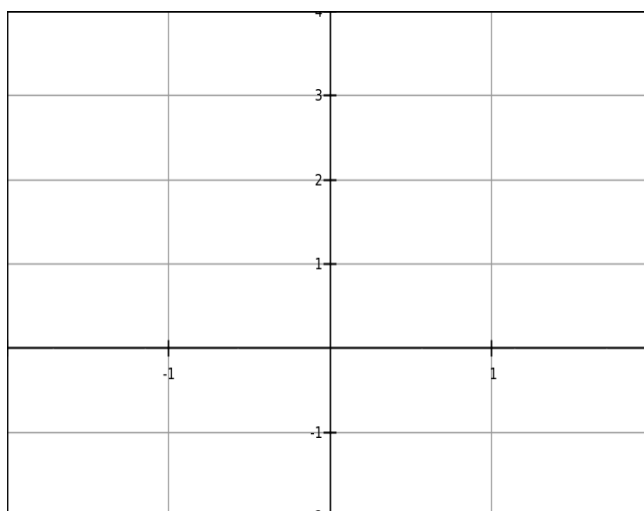
.....

Sketch the graphs of

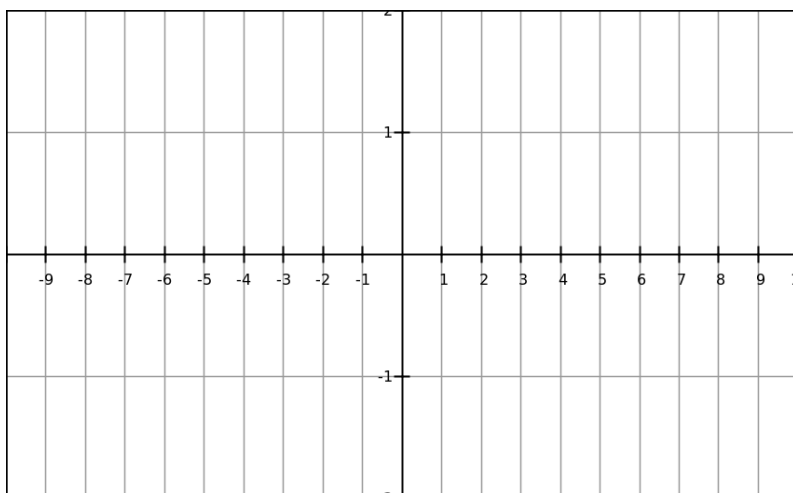
$$y = \arcsin x \quad (y = \sin^{-1}x)$$



$$y = \arccos x \quad (y = \cos^{-1}x)$$



$$y = \arctan x \quad (y = \tan^{-1}x)$$



### Pure: Compound and Double angles

Understand and use double angle formulae; use of formulae for  $\sin(A \pm B)$ ,  $\cos(A \pm B)$ , and  $\tan(A \pm B)$ , understand geometrical proofs of these formulae. To include application to half angles. Knowledge of the  $\tan \frac{1}{2}\theta$  formulae will *not* be required.

There's no new video to watch. Use your notes from June/July

$\sin(A \pm B) = \dots\dots\dots$

$\cos(A \pm B) = \dots\dots\dots$

$\tan(A \pm B) = \dots\dots\dots$

$\sin 2A = \dots\dots\dots$

$\tan 2A = \dots\dots\dots$

$\cos 2A = \dots\dots\dots$

$\cos 2A = \dots\dots\dots$

$\cos 2A = \dots\dots\dots$

Write these formulae out again but replace  $2A$  by  $\theta$

$\dots\dots\dots$

$\dots\dots\dots$

$\dots\dots\dots$

$\dots\dots\dots$

$\dots\dots\dots$

<b>Tracking Test 1</b>	
Use this space to write down what you have learned as a result of Tracking Test 1	

### **Mechanics: Moments 1**

Understand and use moments in simple static contexts. Equilibrium of rigid bodies.

<https://youtu.be/enYbO8kZ8Lo>



What does the moment of a force measure?

How do you calculate the moment of a force?

Complete this sentence: A body in equilibrium has equal

Copy the example in the video.



## Mechanics: Moments 2

There is no new video to watch

### Pure: Numerical Methods – Locating roots

Locate roots of  $f(x) = 0$  by considering changes of sign of  $f(x)$  in an interval of  $x$  on which  $f(x)$  is sufficiently well behaved. Understand how change of sign methods can fail.

Students should know that sign change is appropriate for continuous functions in a small interval. When the interval is too large sign may not change as there may be an even number of roots. If the function is not continuous, sign may change but there may be an asymptote (not a root).

<https://youtu.be/4JYb-IPtspU>



Show that there is a root of the equation  $\ln x = \frac{1}{x}$  between  $x = 1.7$  and  $x = 1.8$ .  
Underline the key sentence that you must write at the end of these questions.

Use iteration to find a solution to  $x^2 - 4x + 1 = 0$  correct to 2 d.p.

**Pure: Numerical Methods – iterations; cobweb and staircase diagrams**

Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams.  
Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy. Use an iteration of the form  $x_{n+1} = f(x_n)$  to find a root of the equation  $x = f(x)$  and show understanding of the convergence in geometrical terms by drawing cobweb and staircase diagrams.

<https://youtu.be/ulyZqj4w6WE>



Copy the four examples in the video

What is the difference between a cobweb and a staircase diagram?

What is the difference between convergence and divergence?

**Pure: Numerical Methods – questions in context including Newton-Raphson and the Trapezium Rule**

Use numerical methods to solve problems in context.

Iterations may be suggested for the solution of equations not soluble by analytic means.

Solve equations using the Newton-Raphson method and other recurrence relations of the form  $x_{n+1} = g(x_n)$ . Understand how such methods can fail.

For the Newton-Raphson method, students should understand its working in geometrical terms, so that they understand its failure near to points where the gradient is small.

Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.

For example, evaluate  $\int_0^1 \sqrt{2x+1} \, dx$  using the values of  $\sqrt{2x+1}$  at  $x = 0, 0.25, 0.5, 0.75$  and  $1$  and use a sketch on a given graph to determine whether the trapezium rule gives an over-estimate or an under-estimate.

No new video to watch. Use your notes from June/July

Use the trapezium rule with 4 strips to find an approximate value for  $\int_0^{\frac{\pi}{3}} \sec x \, dx$

Given that  $x^3 + 2x - 2 = 0$  has a root between 0 and 1, find the root to 2 decimal places using the Newton-Raphson method.

### Statistics: Normal Distribution 1

Understand and use the Normal distribution as a model; find probabilities using the Normal distribution. The notation  $X \sim N(\mu, \sigma^2)$  may be used. Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Questions may involve the solution of simultaneous equations. Students will be expected to use their calculator to find probabilities connected with the normal distribution.

<https://youtu.be/Wqw9cLRMPLO>



If  $X$  is the random variable “time taken in hours to complete a marathon by a group of males” and the mean is 3.8 hours and the variance is 0.5 hours, complete this notation for the distribution of  $X$

$$X \sim N( \quad , \quad )$$

What percentage of a normal distribution is within 1 standard deviation of the mean?.....

What percentage of a normal distribution is within 2 standard deviations of the mean?.....

What percentage of a normal distribution is within 3 standard deviations of the mean?.....

Describe how the  $Z$  distribution (the standard Normal distribution) works. (You will need to watch until the end of the video).

## Statistics: Normal Distribution 2

Link to histograms, mean, standard deviation, points of inflection

Students should know that the points of inflection on the normal curve are at  $x = \mu \pm \sigma$ . The derivation of this result is not expected.

<https://youtu.be/UpfdAe1Q78Y>



A high jumper knows from experience that she can clear a height of at least 1.65 m on 7 out of 10 attempts and can clear at least 1.78 metres once in 5 attempts. Assuming that the height she jumps, in cm, is given by the random variable  $X$  and is normally distributed, find to 1 decimal place, the mean and standard deviation of the heights that she can reach.

### Statistics: The Normal and binomial distributions 1

Links to the binomial distribution.

Students should know that when  $n$  is large and  $p$  is close to 0.5 the distribution  $B(n, p)$  can be approximated by  $N(np, np[1 - p])$ . The application of a continuity correction is expected.

Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate.

Students should know under what conditions a binomial distribution or a Normal distribution might be a suitable model.

<https://youtu.be/g9OcfAyrENg>



Suppose that  $X \sim B(200, 0.75)$ . Find the approximate value of  $p(140 \leq X \leq 155)$

### Mechanics: Moments 3

Problems involving parallel and nonparallel coplanar forces, e.g. ladder problems.

<https://youtu.be/4XB-7EjmqTA>



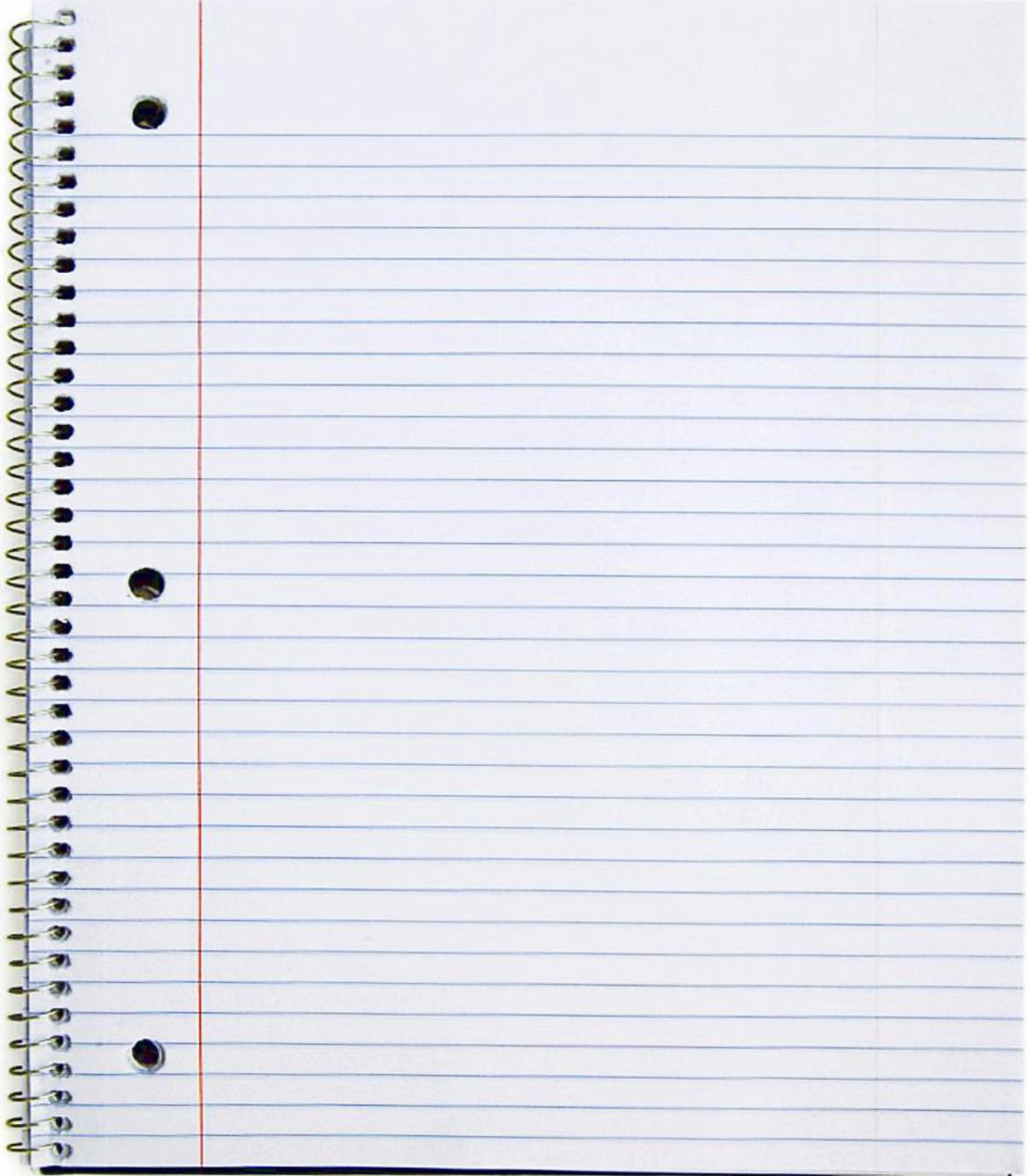
A uniform rod AB, of mass 6 kg and length 4 m is smoothly hinged at A. A light inextensible string is attached to the rod at C, where  $AC = 3$  m, and to the point D, vertically above A. If the string keeps the rod in equilibrium in a horizontal position and the angle between the string and the rod is  $40^\circ$ , calculate

- a) The tension in the string
- b) The magnitude and direction of the reaction at the hinge.



## Mechanics: Moments 4

There is no new video to watch



<b>Mechanics: Moments 5</b>	
There is no new video to watch	

### Pure: Further Trig Expressions

Understand and use expressions for  $a \cos \theta + b \sin \theta$  in the equivalent forms of  $r \cos (\theta \pm \alpha)$  or  $r \sin (\theta \pm \alpha)$

<https://youtu.be/dmrFYuNfkEg>



Solve the equation  $3 \cos x + 5 \sin x = 2$

$(0^\circ \leq x \leq 360^\circ)$

Show the full working.

### Pure: Further Trig Equations

Students should be able to solve equations such as  $a \cos \theta + b \sin \theta = c$  in a given interval.

There is no new video to watch

### Pure: Integration by Substitution 1

Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain (rule) and product rules respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.)

<https://youtu.be/WeAKe8uGQ1M>



Use integration by substitution to work out

$$\int x \sqrt{2x + 5} \, dx$$

Use integration by substitution to evaluate

$$\int_0^{\frac{\pi}{2}} \cos x \sqrt{1 + \sin x} \, dx$$

<b>Pure: Integration by Substitution 2</b>	
There is no new video to watch	

### Pure: Integration by Parts 1

Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules

respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method

but excludes reduction formulae.)

<https://youtu.be/3EITLQirl4E>



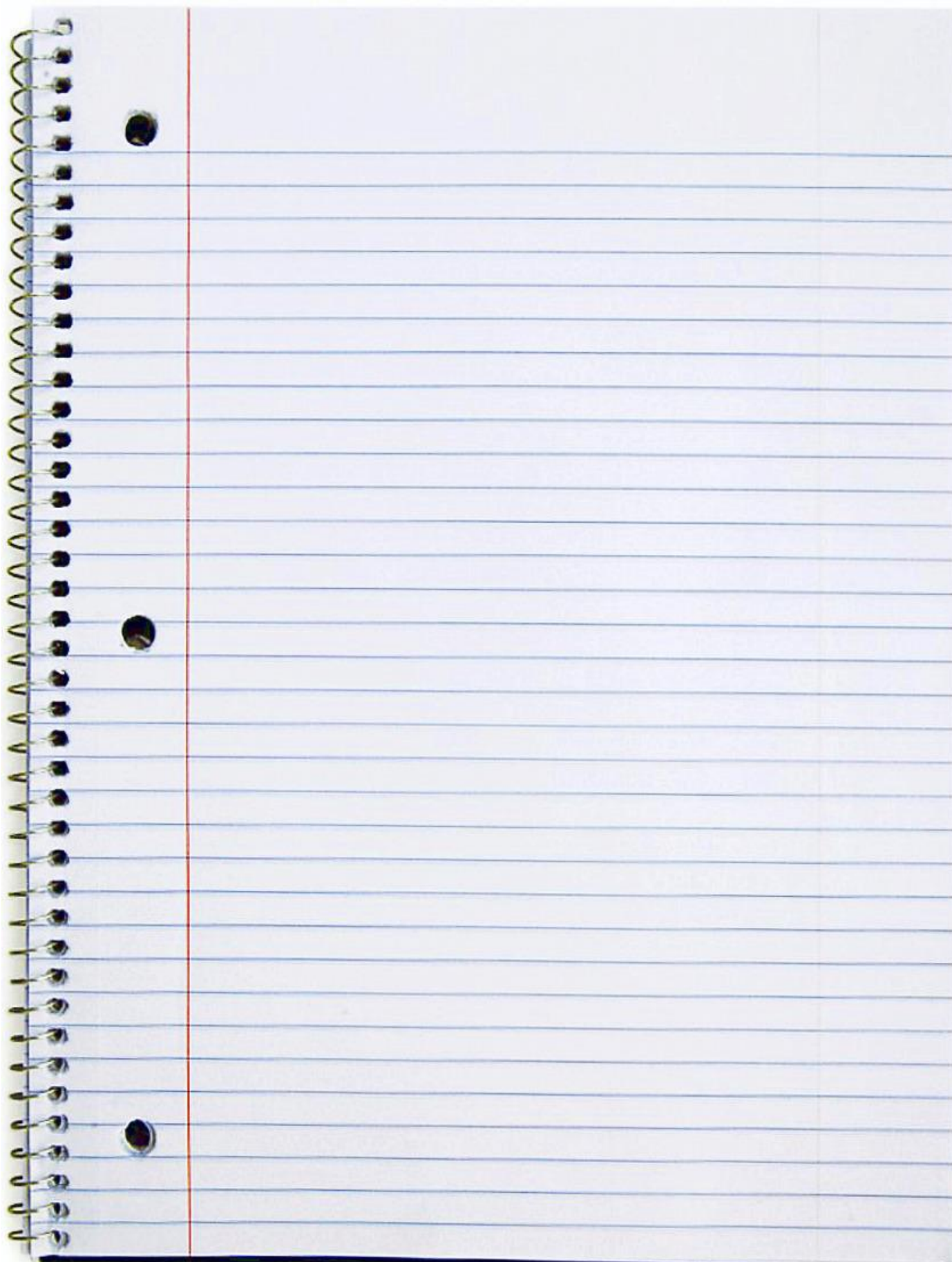
The formula for integration by parts is  $\int u \frac{dv}{dx} dx =$

Use integration by parts to work out  $\int x \cos x dx$



## Pure: Integration by Parts 2

There is no new video to watch





### Pure: Connected Rates

Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.

<https://youtu.be/OyeiYysYXZI>



The rate of change of the radius of a circle is  $5 \text{ cm s}^{-1}$ . Find the rate of change of the area of the circle when the radius is 3 cm.

### Statistics: Sampling 1

Understand and use the terms 'population' and 'sample'. Use samples to make informal inferences about the population. Understand and use sampling techniques, including simple random sampling and opportunity sampling. Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.

Students will be expected to comment on the advantages and disadvantages associated with a census and a sample. Students will be expected to be familiar with: simple random sampling, stratified sampling, systematic sampling, quota sampling and opportunity (or convenience) sampling.

<https://youtu.be/be9e-Q-jC-0>



Explain how the following sampling methods work:

Simple Random Sampling

Stratified Sampling

Systematic Sampling

Quota Sampling (refer to <https://prezi.com/0t0skvyb-kj2/difference-between-cluster-and-quota-sampling-/>)

Opportunity (or Convenience) Sampling

N.B. Cluster sampling is NOT in your syllabus



### Pure: Forming Differential Equations

Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand).

[https://youtu.be/dnWa5\\_3eNb8](https://youtu.be/dnWa5_3eNb8)



A population is growing at a rate which is proportional to the size of the population at a given time. Write down an equation for the rate of growth of the population.

Newton's Law of Cooling states that the rate of loss of temperature of a body is proportional to the excess temperature of the body over its surroundings. Write an equation that expresses this law.

### Pure: Solving Differential Equations 1

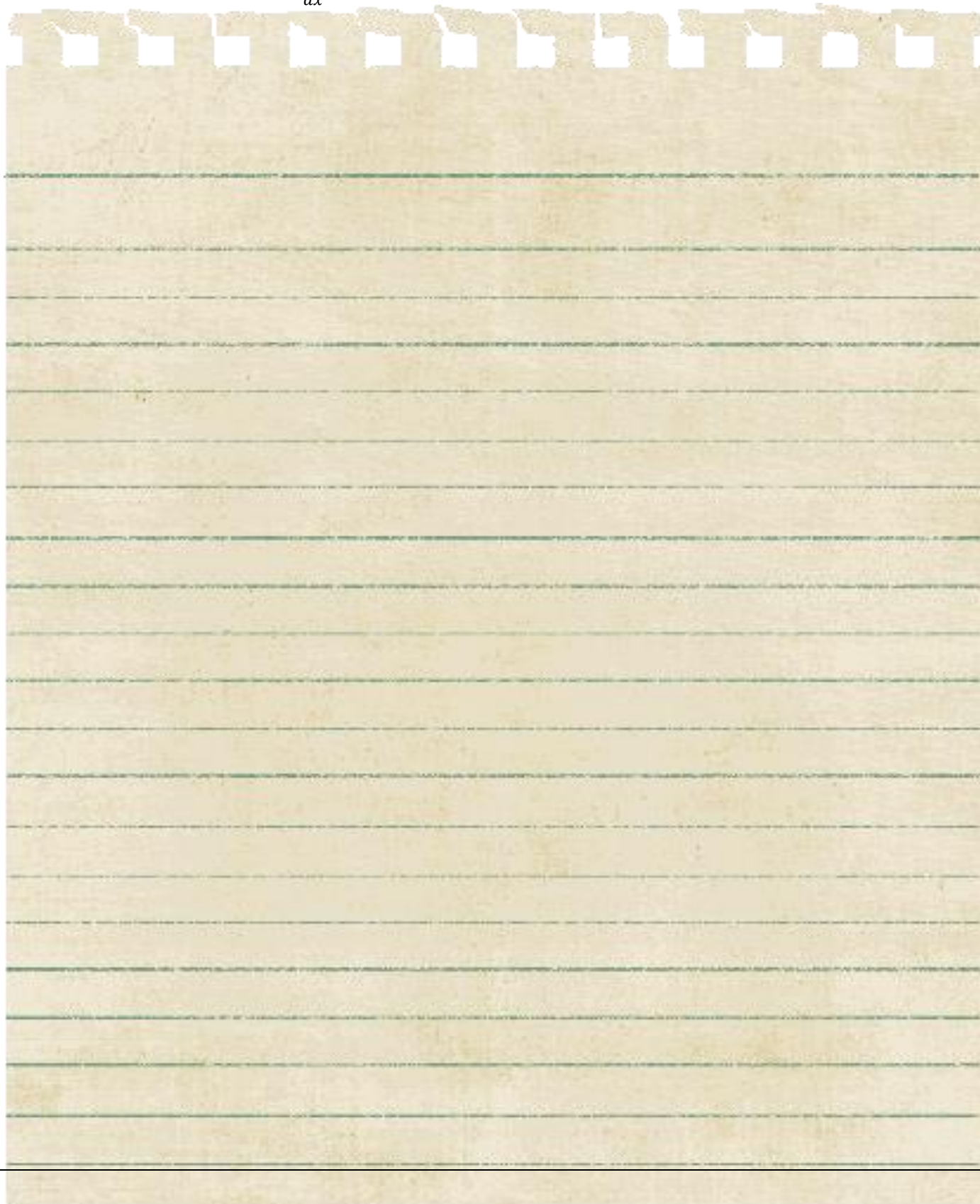
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions (Separation of variables may require factorisation involving a common factor.)

Students may be asked to sketch members of the family of solution curves.

<https://youtu.be/q9a52OxY3Ww>



Solve the differential equation  $\frac{dy}{dx} = x \cos y$ . Use the boundary condition that  $y = 0$  when  $x = 1$



### Pure: Solving Differential Equations 2

Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.

The validity of the solution for large values should be considered.

<https://youtu.be/L8zgpJ7ISNg>



A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at  $3^{\circ}\text{C}$  and  $t$  minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is  $\theta^{\circ}\text{C}$

The rate of change of the water in the bottle is modelled by the differential equation ,

$$\frac{d\theta}{dt} = \frac{3 - \theta}{125}$$

By solving the differential equation, show that  $\theta = Ae^{-0.008t} + 3$ , where A is a constant

<b>Tracking Test 2</b>	
Use this space to write down what you have learned as a result of Tracking Test 2	



### Mechanics: Kinematics and Variable Acceleration 1

Use calculus in kinematics for motion in a straight line:  $v = \frac{dr}{dt}$ ,  $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$

$$r = \int v \, dt, v = \int a \, dt$$

<https://youtu.be/xvVkAG7o1T8>



A particle P is moving on the x-axis. At time  $t$  seconds, the displacement  $x$  metres from O is given by

$$x = t^4 - 32t + 12$$

Find

- a) the speed of P when  $t=3$ ,
- b) the value of  $t$  for which P is instantaneously at rest,
- c) the magnitude of the acceleration of P when  $t=1.5$ .



## Mechanics: Kinematics and Variable Acceleration 2

Extend to 2 dimensions using vectors.

Differentiation and integration of a vector with respect to time.

e.g. Given  $\mathbf{r} = t^2\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$ , find  $\dot{\mathbf{r}}$  and  $\ddot{\mathbf{r}}$  at a given time.

No video to watch  
but try this  
question

The displacement of a particle is given by  $\mathbf{r} = t^2\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$ ,

a) Find  $\dot{\mathbf{r}}$  when  $t=3$

N.B.  $\dot{\mathbf{r}}$  is an expression for velocity

b) Find  $\ddot{\mathbf{r}}$  when  $t=6$

and  $\ddot{\mathbf{r}}$  is an expression for acceleration

### Pure: Parametric Equations

Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.

For example:  $x = 3 \cos t$ ,  $y = 3 \sin t$  describes a circle centre  $O$  radius 3

$x = 2 + 5 \cos t$ ,  $y = -4 + 5 \sin t$  describes a circle centre  $(2, -4)$  with radius 5

$x = 5t$ ,  $y = \frac{5}{t}$  describes the curve  $xy = 25$  (or  $y = \frac{25}{x}$ )

$x = 5t$ ,  $y = 3t^2$  describes the quadratic curve  $25y = 3x^2$  and other familiar curves covered in the specification.

Students should pay particular attention to the domain of the parameter  $t$ , as a specific section of a curve may be described.

Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only. The finding of equations of tangents and normals to curves given parametrically or implicitly is required.

Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines, or between two curves. This includes curves defined parametrically.

<https://youtu.be/nFClKAxutOc>



A curve is defined by the parametric equations  $x=2t$  and  $y = t^2$ .

What is the Cartesian equation of this curve?

A curve is defined by the parametric equations  $x = t^2$  and  $y = 2t(3-t)$ . Find the area between this curve and the  $x$  axis which is bounded by the  $y$ -axis and the line  $x = 3$

**Pure: Modelling using parametric equations**

Use parametric equations in modelling in a variety of contexts. A shape may be modelled using parametric equations or students may be asked to find parametric equations for a motion. For example, an object moves with constant velocity from (1, 8) at  $t = 0$  to (6, 20) at  $t = 5$ . This may also be tested in kinematics in the mechanics paper.

No video to watch but try this question

An object moves with constant velocity from (1, 8) at  $t = 0$  to (6, 20) at  $t = 5$ .

a) What is the x displacement in the first 5 seconds

b) What is the y displacement in the first 5 seconds

c) What is the x velocity after 5 seconds (use  $s = ut + \frac{1}{2}at^2$ , with  $a = 0$ , since the velocity is constant)

d) What is the y velocity after 5 seconds

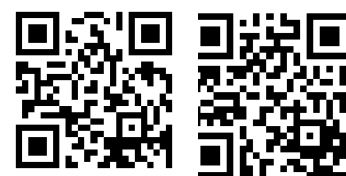
e) Write down the positions of x and y after t seconds using "Final position = Initial Position + Displacement" and Displacement = Velocity x Time, since the velocity is constant.

### Statistics: Scatter Diagrams, Frequency Polygons, Histograms

Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency. Connect to probability distributions. Students should be familiar with histograms, frequency polygons, box and whisker plots (including outliers) and cumulative frequency diagrams.

<https://youtu.be/9rfToo6MMsg>

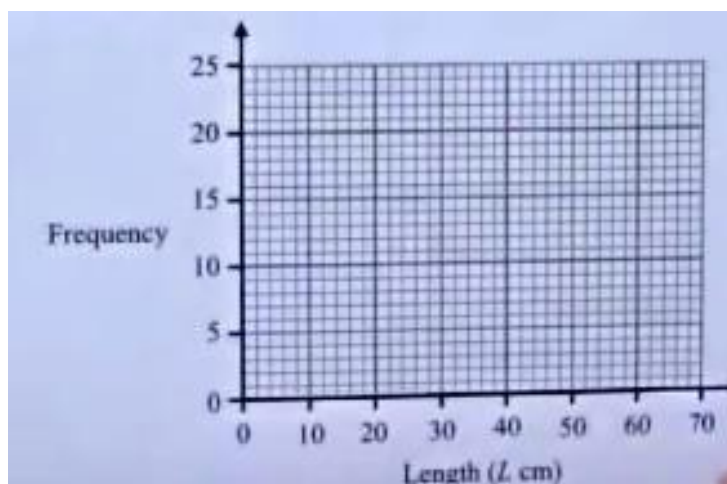
[https://youtu.be/1nW8t1QV5\\_A](https://youtu.be/1nW8t1QV5_A)



This table gives information about the length of branches on a bush

Length $L$ (cm)	$0 \leq L < 10$	$10 \leq L < 20$	$20 \leq L < 30$	$30 \leq L < 40$	$40 \leq L < 50$	$50 \leq L < 60$
Frequency	20	12	10	8	6	0

Draw a frequency polygon to show this information



The lifetime of a bulb in hours is given in this table.

The width of the 95-105 class is 2 cm and the height is 9 cm

Find the width and height of the 105-130 class.

Lifetime in hours	Frequency
90-95	5
95-105	12
105-130	16

### Statistics: Regression lines

Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded).  
Students should be familiar with the terms explanatory (independent) and response (dependent) variables. Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than  $x$  and  $y$  may be used. Use of interpolation and the dangers of extrapolation. Variables other than  $x$  and  $y$  may be used. Change of variable may be required, e.g. using knowledge of logarithms to reduce a relationship of the form  $y = ax^n$  or  $y = kb^x$  into linear form to estimate  $a$  and  $n$  or  $k$  and  $b$ .

Understand informal interpretation of correlation.

Use of terms such as positive, negative, zero, strong and weak are expected.

Understand that correlation does not imply causation.

<https://youtu.be/1e0p0rYD36E>



Explain the terms dependent (or response) variable and independent (or explanatory) variable.

In the example,  $y = 13.01 + 0.49x$

What is the gradient of the line?

What is the y-intercept of the line?

What are the physical meanings of 13.01 and 0.49 in the example

In the second example,  $y = 19.8 - 2.1x$

What is the gradient of the line?

What is the y-intercept of the line?

What are the physical meanings of 19.8 and 2.1 in this example

## Statistics: Product Moment Correlation Coefficient 1

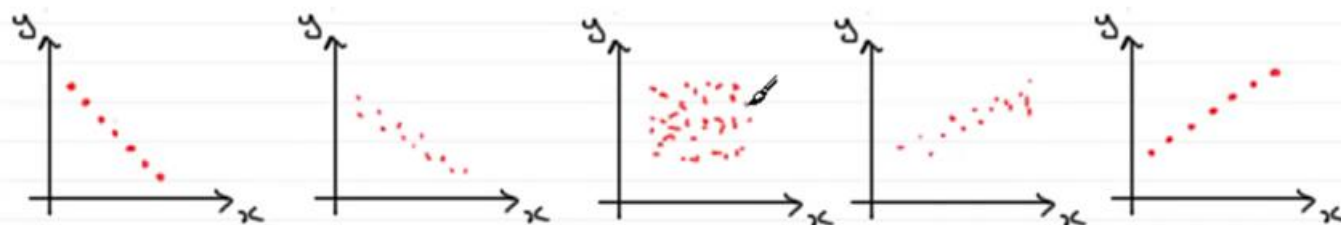
Extend Hypothesis testing to correlation coefficients as measures of how close data points lie to a straight line.

Students should know that the product moment correlation coefficient  $r$  satisfies  $|r| \leq 1$  and that a value of  $r = \pm 1$  means the data points all lie on a straight line.

<https://youtu.be/BXXtkYOqAfM>



What are the approximate values of  $r$  in each of these diagrams



What are the minimum and maximum possible values of  $r$ ?

It can be shown that  $r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$

What is  $S_{xy}$ ?

What is  $S_{xx}$ ?

What is  $S_{yy}$ ?

Calculate  $r$  for this data

### Marks in two tests

maths	$x$	4	5	5	6	7	7	8	9	10	10
physics	$y$	6	6	7	7	7	8	8	9	9	10

### Statistics: Product Moment Correlation Coefficient 2

Be able to interpret a given correlation coefficient using a given  $p$ -value or critical value (calculation of correlation coefficients is excluded).

Students will be expected to calculate a value of  $r$  using their calculator but use of the formula is not required. Hypotheses should be stated in terms of  $\rho$  with a null hypothesis of  $\rho = 0$  where  $\rho$  represents the population correlation coefficient.

Tables of critical values or a  $p$ -value will be given.

[https://youtu.be/EVXwJgDGF\\_U](https://youtu.be/EVXwJgDGF_U)

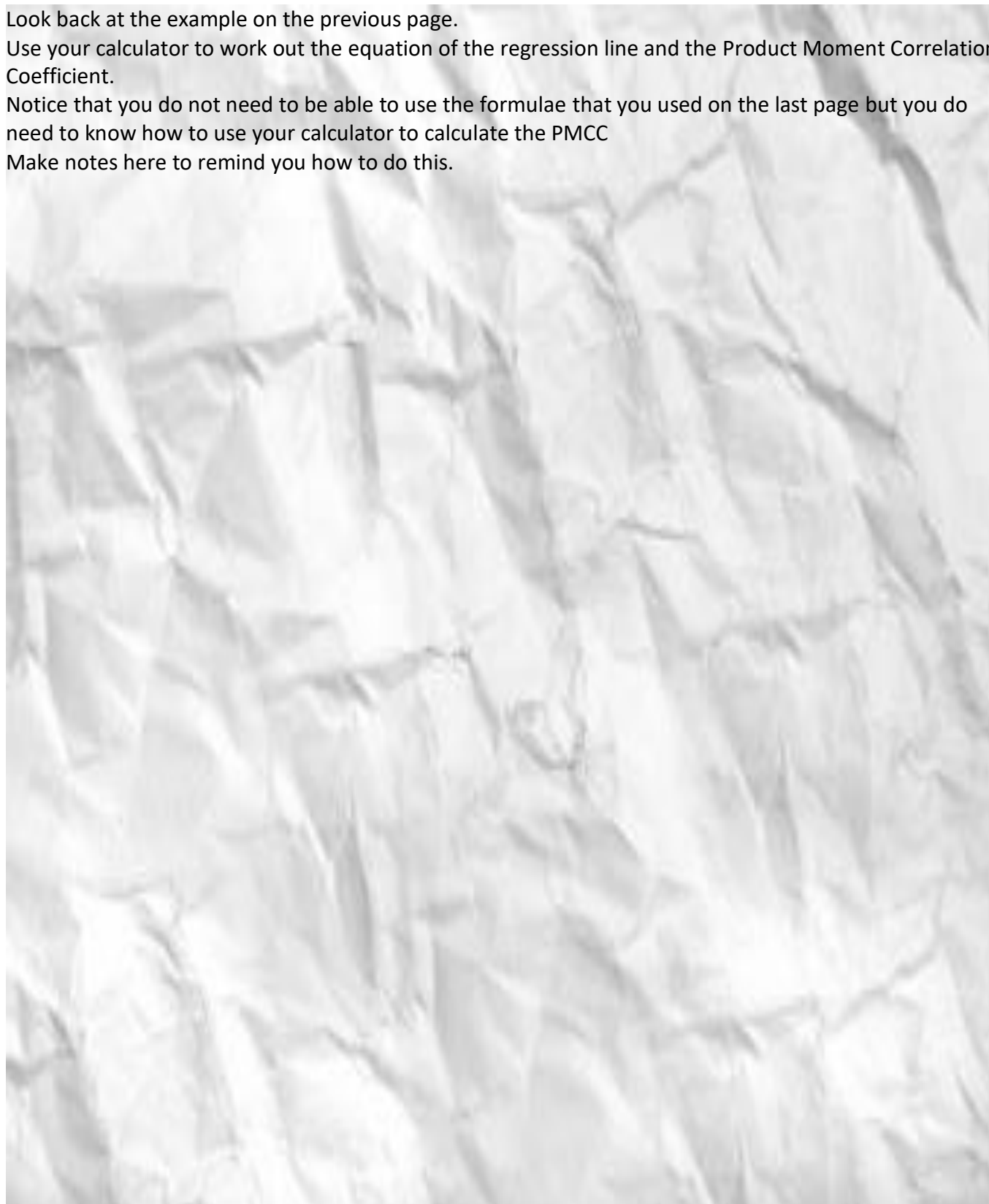


Look back at the example on the previous page.

Use your calculator to work out the equation of the regression line and the Product Moment Correlation Coefficient.

Notice that you do not need to be able to use the formulae that you used on the last page but you do need to know how to use your calculator to calculate the PMCC

Make notes here to remind you how to do this.



### Statistics: Further Hypothesis Testing 1

Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context.

Students should know that: If  $X \sim N(\mu, \sigma^2)$  then  $X \sim N(\mu, \frac{\sigma^2}{n})$  and that a test for  $\mu$  can be carried out using:  $\frac{X - \mu}{\sigma/\sqrt{n}} \sim N(0, 1^2)$

No proofs required. Hypotheses should be stated in terms of the population mean  $\mu$ . Knowledge of the Central Limit Theorem or other large sample approximations is not required.

<https://youtu.be/9PkYISeHHI4>



Heights of Japanese Sixth Form students are Normally distributed.

In 1975 the population height was 168 cm with a standard deviation of 8 cm.

In 2015 it is claimed that due to a Western diet, students are now taller.

A random sample of 50 students had a mean height of 169 cm

Test at the 5% significance level whether the students are now taller.







<b>Tracking Test 3</b>	
Use this space to write down what you have learned as a result of Tracking Test 3	

## Pure: Modulus Graphs

The modulus of a linear function.

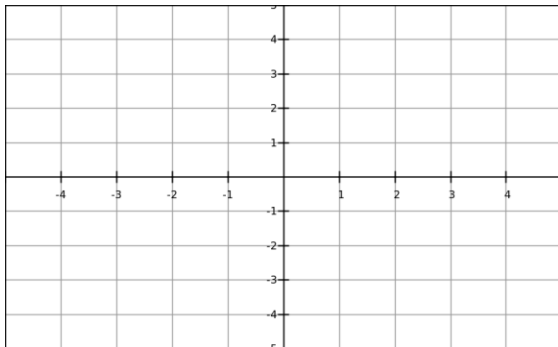
Students should be able to sketch the graphs of  $y = |ax + b|$ . They should be able to use their graph. For example, sketch the graph with equation  $y = |2x - 1|$  and use the graph to solve the equation  $|2x - 1| = x$  or the inequality  $|2x - 1| > x$

<https://youtu.be/dxzM3Yl0X5s>

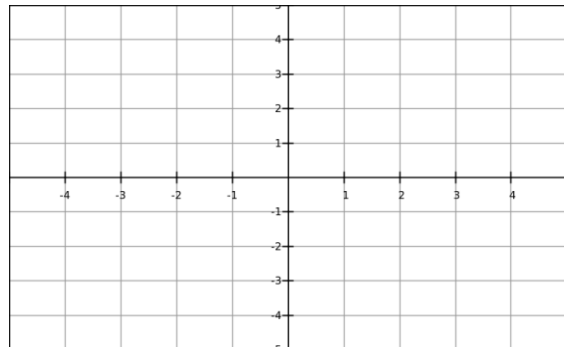


Sketch the following functions

$$f(x) = |3x + 2|$$



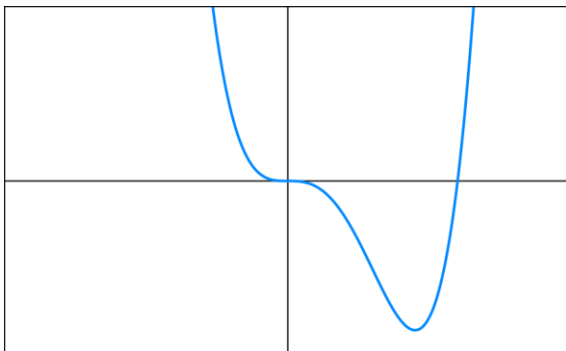
$$f(x) = |x| - 2$$



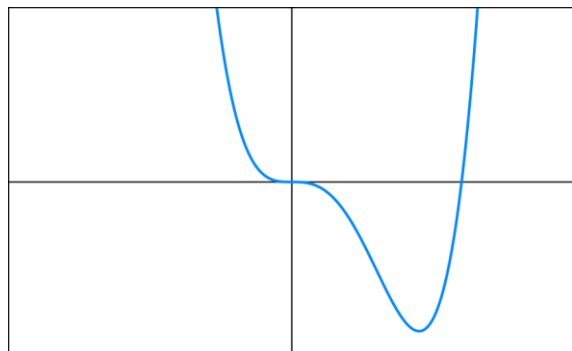
The graph shows the function  $f(x)$

Sketch the graphs of

$$y = |f(x)|$$



$$y = f(|x|)$$



### Pure: Arithmetic Sequences and Series

Understand and work with arithmetic sequences and series, including the formulae  
for  $n$ th term and the sum to  $n$  terms  
The proof of the sum formula for an arithmetic sequence should be known  
including the formula for the sum of the first  $n$  natural numbers.

[https://youtu.be/x\\_D3HXHNnv4](https://youtu.be/x_D3HXHNnv4)



The formulae for Arithmetic Sequences and Series use the following symbols. What do they stand for?

$a =$

$n =$

$d =$

$S =$

$l =$

a) What is the formula for  $U_n$  (the  $n^{\text{th}}$  term of a sequence)? .....

b) What is the formula for  $S_n$ ? .....

c) What is the alternative formula for  $S_n$ ? .....

d) In the space below, write out the proof for the formula in b)

### Pure: Increasing, Decreasing and Periodic Sequences

Work with sequences including those given by a formula for the  $n^{\text{th}}$  term and those generated by a simple relation of the form  $x_{n+1} = f(x_n)$ ; increasing sequences; decreasing sequences; periodic sequences.

For example  $u_n = \frac{1}{3n+1}$  describes a decreasing sequence as  $u_{n+1} < u_n$  for all integers  $n$ ;  $u_n = 2^n$  is an increasing sequence as  $u_{n+1} > u_n$  for all integers  $n$ ;  $u_{n+1} = \frac{1}{u_n}$  for  $n > 1$  and  $u_1 = 3$  describes a periodic sequence of order 2

There is no video to watch



### Pure: Graphical Inequalities

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.  
e.g. solving  $ax + b > cx + d$ ,  $px^2 + qx + r \geq 0$ ,  $px^2 + qx + r < ax + b$  and interpreting the third inequality as the range of  $x$  for which the curve  $y = px^2 + qx + r$  is below the line with equation  $y = ax + b$ . These would be reducible to linear or quadratic inequalities.

Express solutions through correct use of 'and' and 'or', or through set notation. So, e.g.  $x < a$  or  $x > b$  is equivalent to  $\{x : x < a\} \cup \{x : x > b\}$  and  $\{x : c < x\} \cap \{x : x < d\}$  is equivalent to  $x > c$  and  $x < d$ .

Represent linear and quadratic inequalities such as  $y > x + 1$  and  $y > ax^2 + bx + c$  graphically. Shading and use of dotted and solid line convention is required.

<https://youtu.be/qZrM1X2TeEA>



Find the set of values for which  $12 + 4x > x^2$

### Pure: Geometric Sequences and Series

Understand and work with geometric sequences and series, including the formulae for the  $n$ th term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of  $|r| < 1$ ; modulus notation. The proof of the sum formula should be known. Given the sum of a series students should be able to use logs to find the value of  $n$ . The sum to infinity may be expressed as  $S_{\infty}$

<http://youtu.be/ZBvja3B0Jco>



What is the formula for the  $n$ th term of a geometric sequence?

In this formula,  
What is  $a$ ?

What is  $r$ ?

What is  $n$ ?

What is the formula for the sum of the first  $n$  terms of a Geometric Series?

Write out the proof of this formula.

What is the formula for the sum to infinity of a Geometric Series?

For what values of  $r$  is this  
formula valid?



### Pure: Use of Sequences and Series in Modelling

Use sequences and series in modelling. Examples could include amounts paid into saving schemes, increasing by the same amount (arithmetic) or by the same percentage (geometric) or could include other series defined by a formula or a relation.

[https://youtu.be/a7rzOtpZzeY?list=PLjnEnKniBbCTYvLTELT\\_aZd-t6y65F3M5](https://youtu.be/a7rzOtpZzeY?list=PLjnEnKniBbCTYvLTELT_aZd-t6y65F3M5)



\$500 is invested at 3.5% interest each year. Each year a further \$500 is invested.

After how many years is the investment worth \$20 000

### Pure: Proof by deduction

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion

Examples of proofs:

- Proof by deduction e.g. using completion of the square, prove that  $n^2 - 6n + 10$  is positive for all values of  $n$  or, for example, differentiation from first principles for small positive integer powers of  $x$  or proving results for arithmetic and geometric series. This is the most commonly used method of proof throughout this specification.

<https://youtu.be/SBvurpUG81w>



Prove that for any four consecutive integers, the difference between the product of the last two and the product of the first two of these numbers is equal to their sum

### Pure: Proof by exhaustion

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion

Examples of proofs:

- Proof by exhaustion This involves trying all the options. Suppose  $x$  and  $y$  are odd integers less than 7. Prove that their sum is divisible by 2.

<https://youtu.be/0PgXAzJ2QNo>



Prove that 53 is a prime number

### Pure: Proof by counter example

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion

Examples of proofs:

- Disproof by counter example e.g. show that the statement “ $n^2 - n + 1$  is a prime number for all values of  $n$ ” is untrue

<https://youtu.be/mxcGpNji4ik>



Is  $p - q \leq p^2 - q^2$  for all values of  $p$  and  $q$ ?

Prove your answer

### Pure: Proof by contradiction

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion

Examples of proofs:

- Proof by contradiction (including proof of the irrationality of  $\sqrt{2}$  and the infinity of primes, and application to unfamiliar proofs).

<https://youtu.be/VNZoB0qao1U>



Prove that  $\sqrt{2}$  is irrational

## Critical Values for Correlation Coefficients

These tables concern tests of the hypothesis that a population correlation coefficient  $\rho$  is 0. The values in the tables are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

Product Moment Coefficient					Sample size, $n$
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10
0.4187	0.5214	0.6021	0.6851	0.7348	11
0.3981	0.4973	0.5760	0.6581	0.7079	12
0.3802	0.4762	0.5529	0.6339	0.6835	13
0.3646	0.4575	0.5324	0.6120	0.6614	14
0.3507	0.4409	0.5140	0.5923	0.6411	15
0.3383	0.4259	0.4973	0.5742	0.6226	16
0.3271	0.4124	0.4821	0.5577	0.6055	17
0.3170	0.4000	0.4683	0.5425	0.5897	18
0.3077	0.3887	0.4555	0.5285	0.5751	19
0.2992	0.3783	0.4438	0.5155	0.5614	20
0.2914	0.3687	0.4329	0.5034	0.5487	21
0.2841	0.3598	0.4227	0.4921	0.5368	22
0.2774	0.3515	0.4133	0.4815	0.5256	23
0.2711	0.3438	0.4044	0.4716	0.5151	24
0.2653	0.3365	0.3961	0.4622	0.5052	25
0.2598	0.3297	0.3882	0.4534	0.4958	26
0.2546	0.3233	0.3809	0.4451	0.4869	27
0.2497	0.3172	0.3739	0.4372	0.4785	28
0.2451	0.3115	0.3673	0.4297	0.4705	29
0.2407	0.3061	0.3610	0.4226	0.4629	30
0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60
0.1550	0.1982	0.2352	0.2776	0.3060	70
0.1448	0.1852	0.2199	0.2597	0.2864	80
0.1364	0.1745	0.2072	0.2449	0.2702	90
0.1292	0.1654	0.1966	0.2324	0.2565	100