Topic	Question	Done?
Connected Particles	3	
Connected Particles	4	
Connected Particles	8	
Connected Particles	21	
Dynamics	9	
Dynamics	13	
Dynamics	16	
Statics	15	
Statics	18	
Kinematics	7	
Kinematics	14	
Kinematics	7	
Kinematics	12	
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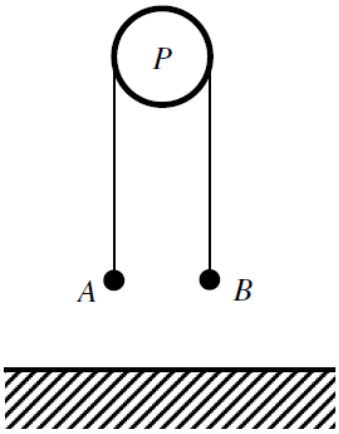
Moments	9	
Moments	14	
Moments	9	
Moments	15	
Moments	28	
Projectiles	8	
Projectiles	18	
Vectors	7	
Vectors	4	
Vectors	15	

Connected particles

Question 3 (***)



Two particles A and B of respective masses 3 kg and m kg are each attached to the two ends of a light inextensible string which passes over a smooth pulley P. The two particles are held at rest, both at a height of 1.28 m above a horizontal floor with the portions of the strings not in contact with the pulley vertical.

The system of the two particles is then released from rest with B accelerating towards the floor at 1.96 ms^{-2}, while A never reaches P.

a) For the period before B reaches the floor, calculate the tension in the string.

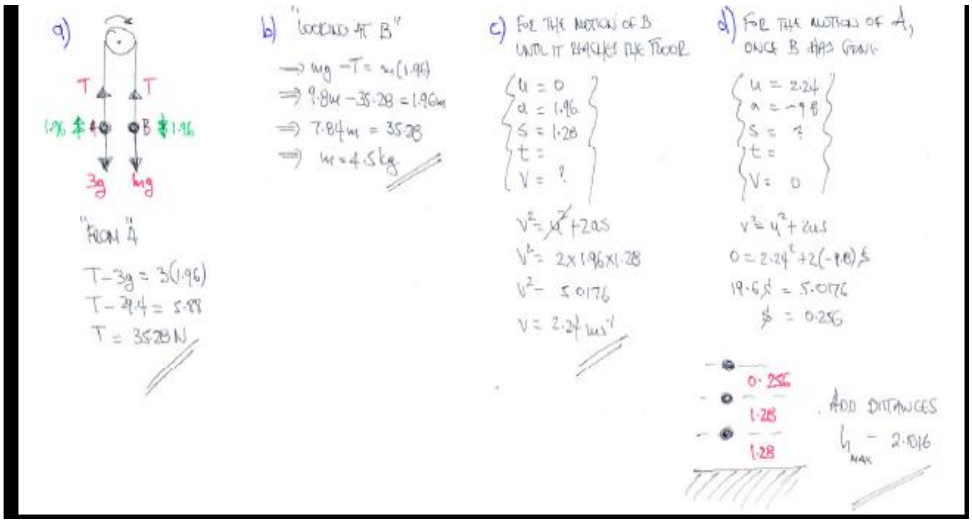
b) Determine the value of m.

c) Calculate the speed with which B strikes the floor.

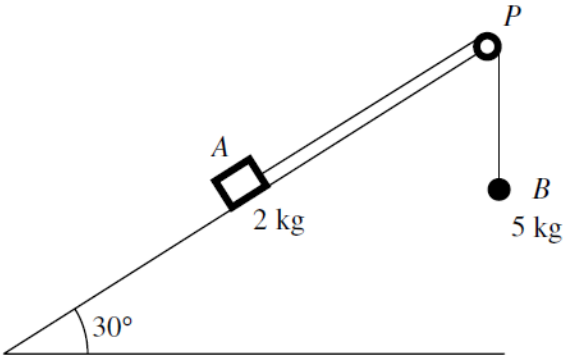
When B reaches the floor it remains at rest.

d) Determine the greatest height above the floor reached by A.

$\boxed{T = 35.28 \text{ N}}$, $\boxed{m = 4.5}$, $\boxed{v = 2.24 \text{ ms}^{-1}}$, $\boxed{h_{\max} = 2.816 \text{ m}}$



Question 4 (***)



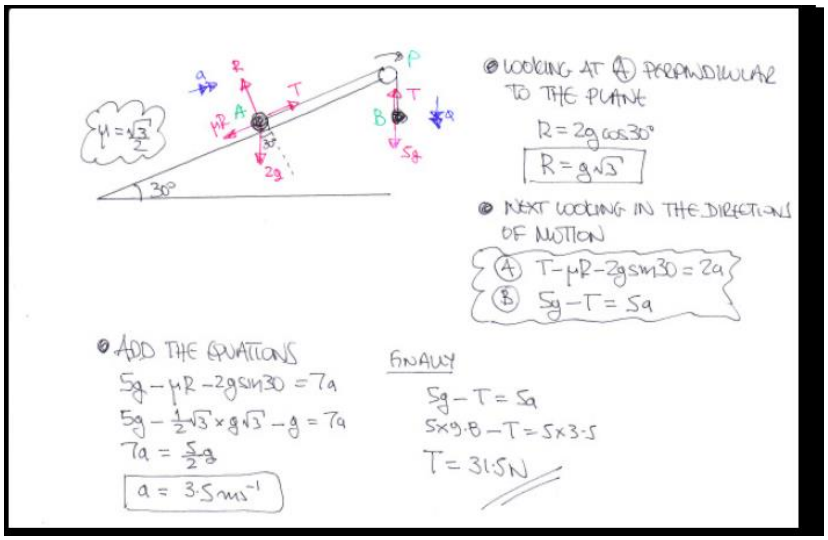
Two particles A and B, of mass 2 kg and 5 kg respectively, are attached to each of the ends of a light inextensible string. The string passes over a smooth pulley P, at the top of a fixed rough plane, inclined at $30°$ to the horizontal.

Particle A is placed at rest on the incline plane while B is hanging freely at the end of the incline plane vertically below P, as shown in the figure above. The two particles, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

The particles are released from rest with the string taut. Particle A begins to move up the incline plane, where the coefficient between A and the plane is $\frac{1}{2}\sqrt{3}$.

Ignoring air resistance, calculate the tension in the string immediately after the particles are released.

$\boxed{T = 31.5 \text{ N}}$



Question 8 (***+)

A car of mass 1500 kg is towing a trailer of mass 1000 kg by means of a light inextensible rope. The car is experiencing a constant air resistance of 200 N, while the corresponding constant air resistance on the trailer is 300 N.

The car and trailer are modelled as particles, with the tow rope remaining taut and horizontal throughout the motion.

a) Given driving force of the car is 750 N, determine …

i. … the acceleration of the system.

ii. … the tension in the tow rope.

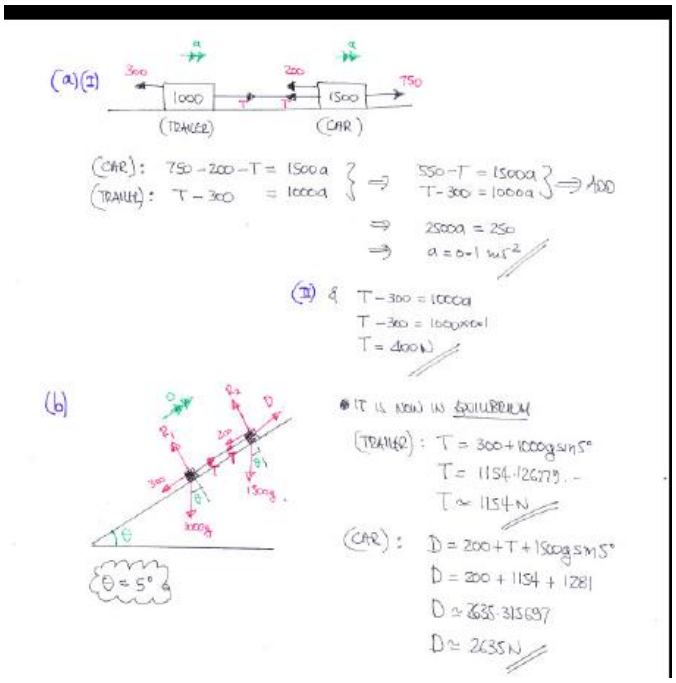
Later in the journey, the car and the trailer are ascending on a road which inclined at 5° to the horizontal. The air resistance on the car and trailer are unchanged.

b) Assuming that the system now moves with constant speed, calculate …

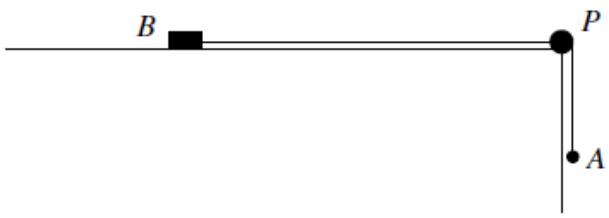
i. … a new figure for the tension in the tow rope.

ii. … a new figure for the driving force of the car..

$\boxed{a = 0.1\ \text{ms}^{-2}}$, $\boxed{T = 400\ \text{N}}$, $\boxed{T \approx 1154\ \text{N}}$, $\boxed{D \approx 2635\ \text{N}}$



Question 21 (****)



A particle A of mass 5 kg is connected to small box B of mass 7.5 kg by a light inextensible string. The string passes over a light smooth pulley P, which is located at the end of a rough horizontal house roof. The box is held on the roof with the particle hanging vertically at the end of the roof, as shown in the figure above.

The coefficient of friction between the box and the roof is 0.2.

The system is released from rest with the string taut.

a) Determine in any order…

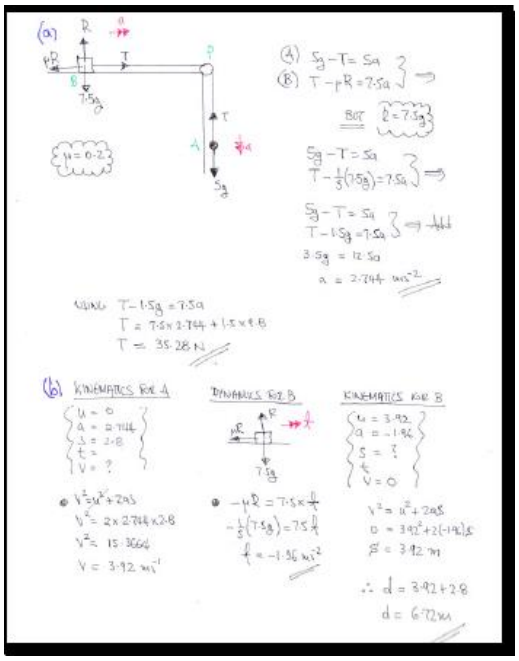
i. … the acceleration of the system.

ii. … the tension in the string.

On release B is at a distance d m from P. When A has moved a distance of 2.8 m the string breaks. In the subsequent motion B comes to rest as it reaches P.

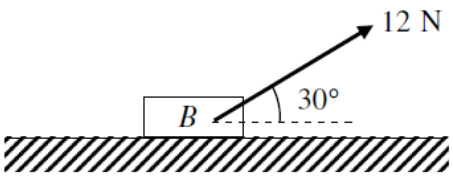
b) Calculate the value of d.

$\boxed{a = 2.744 \text{ ms}^{-2}}$, $\boxed{T = 35.28 \text{ N}}$, $\boxed{d = 6.72 \text{ m}}$



Dynamics

Question 9 (***+)



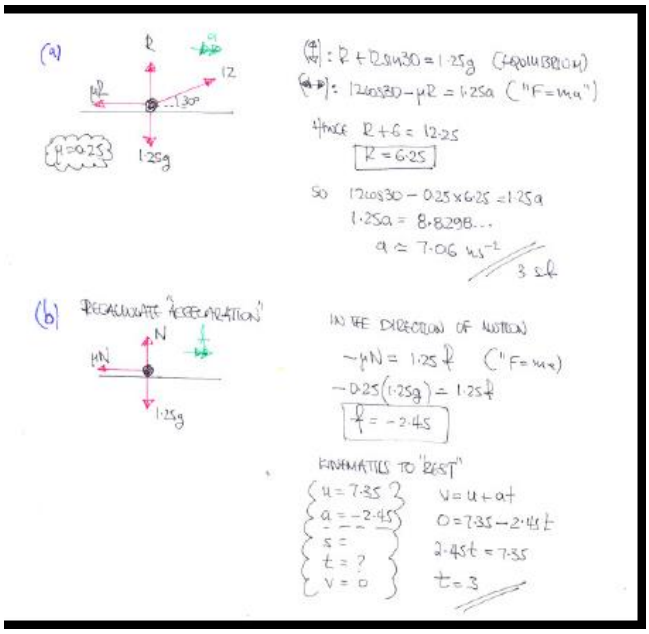
A box B of mass 1.25 kg is pulled along rough horizontal ground by a force of magnitude 12 N inclined at 30° to the horizontal, as shown in the figure above. The box is modelled as a particle moving on a rough horizontal plane where coefficient of friction between the particle and the plane is 0.25.

a) Determine the acceleration of the box.

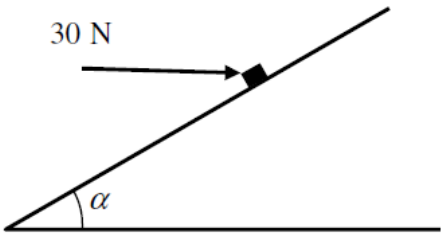
The pulling force is suddenly removed when the box has a speed of $7.35\ \text{ms}^{-1}$.

b) Find the time it takes the box to come to rest from the instant the pulling force was removed.

$\boxed{a \approx 7.06\ \text{ms}^{-2}}$, $\boxed{t = 3\ \text{s}}$



Question 13 (***+)



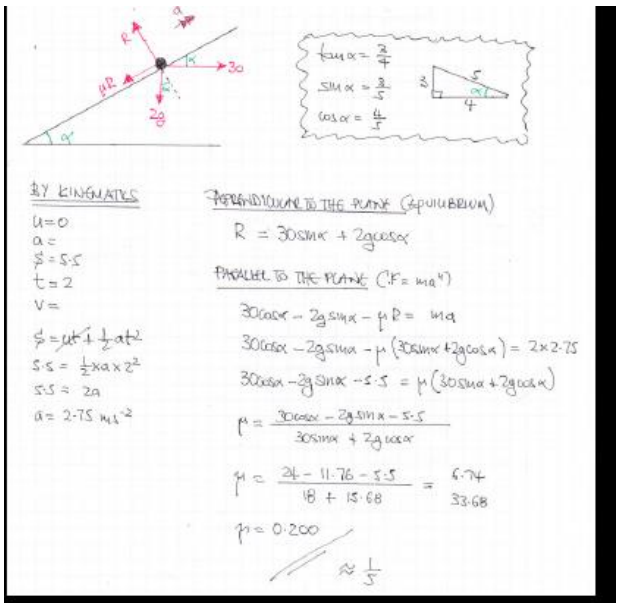
A particle of mass 2 kg, is pushed up a rough plane inclined at an angle θ to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a horizontal force 30 N, as shown in the above figure.

The force acts in a vertical plane, which contains the box and a line of greatest slope of the plane. The coefficient of friction between the box and the plane is $\frac{1}{2}$.

The box starts from rest and travels a distance of 5.5 m up the plane, in 2 seconds.

Determine the value of the coefficient of friction between the particle and the plane

$\boxed{\mu \approx 0.200}$



Question 16 (****)

A particle is **projected** down the line of greatest slope of a rough incline plane and moves along a straight line with constant acceleration $a\text{ ms}^{-2}$.

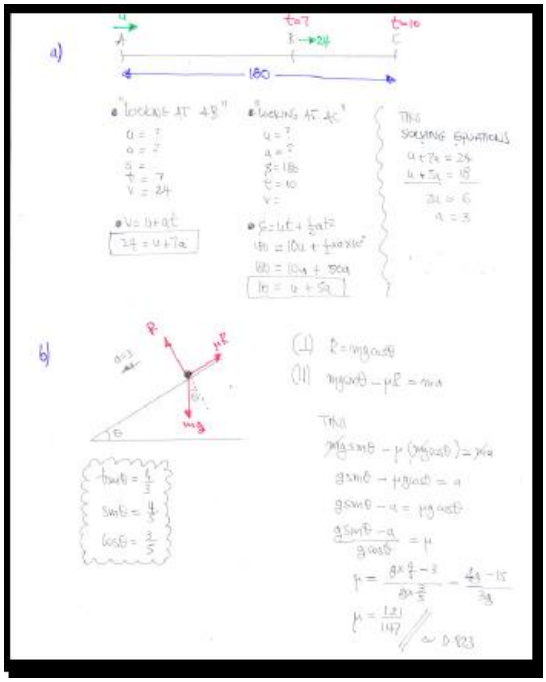
The particle achieves a speed of 24 ms^{-1}, 7 s after projection and covers 180 m in the first 10 seconds of its motion.

a) Assuming that the above described motion takes place entirely on the slope of the plane, determine the value of a.

The plane is inclined at $\arctan\frac{4}{3}$ to the horizontal.

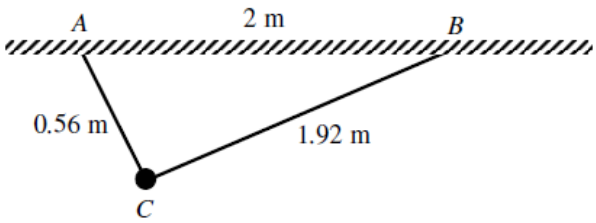
b) Calculate the coefficient of friction between the particle and the plane.

$\boxed{a = 3}$, $\boxed{\mu = \frac{121}{147} \approx 0.823}$



Statics

Question 15 (****)



The points A and B are 2 m apart and lie on a fixed horizontal ceiling. A particle C, of weight 100 N, is suspended by two strings from A and B, so that $AC = 0.56$ m and $BC = 1.92$ m as shown in the figure above. The particle hangs in equilibrium.

a) Show clearly that $\measuredangle ACB = 90°$.

b) Calculate the tension in each of the two strings.

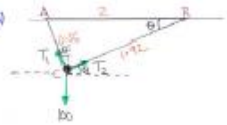
The particle is next suspended by two different strings from A and B, so that $AC = 0.8$ m and $BC = 2.5$ m.

The particle still hangs in equilibrium.

c) Show further that the tension in the string AC is 100 N.

$\boxed{}$, $\boxed{T_{BC} = 28 \text{ N}}$, $\boxed{T_{AC} = 96 \text{ N}}$

(a)



$|AC|^2 + |BC|^2 = 0.56^2 + 1.92^2$
$= 0.3136 + 3.6864$
$= 4$
$= |AB|^2$

AS THE THREE LENGTHS OF $\triangle ABC$, SATISFY THE PYTHAGOREAN RELATIONSHIP $A\hat{C}B = 90$

(b) FIRSTLY $\sin\theta = \frac{0.56}{2} = 0.28 = \frac{7}{25}$

$\cos\theta = \frac{1.92}{2} = 0.96 = \frac{24}{25}$

$\left.\begin{array}{l} (\updownarrow)\ T_1\cos\theta + T_2\sin\theta = 100 \\ (\leftrightarrow)\ T_1\sin\theta = T_2\cos\theta \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{24}{25}T_1 + \frac{7}{25}T_2 = 100 \\ \frac{7}{25}T_1 = \frac{24}{25}T_2 \end{array}\right\} \Rightarrow \boxed{T_1 = \frac{24}{7}T_2}$

THUS $\frac{24}{25}\times\left(\frac{24}{7}T_2\right) + \frac{7}{25}T_2 = 100$

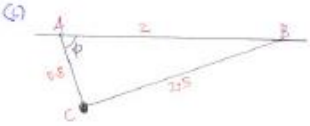
$\frac{25}{7}T_2 = 100$

$T_2 = 28\text{N}$

& $T_1 = \frac{24}{7}\times 28$

$T_1 = 96\text{N}$

(c)



- IF $\phi \geqslant 90$, $\triangle C$ WILL BE VERTICAL AND THE STRING BC SLACKS
- BY THE COSINE RULE

$2.5^2 = 2^2 + 0.5^2 - 2\times2\times0.5\cos\phi$

$6.25 = 4 + 0.64 - 2\cos\phi$

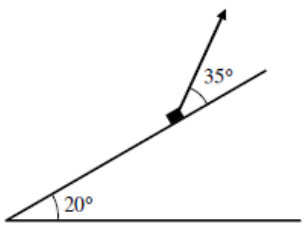
$2\cos\phi = -1.61$

$\cos\phi = -0.805$

$\phi \simeq 142.6°$

$\therefore$ AC IS VERTICAL, SO THE TENSION IN AC EQUALS THE WEIGHT, $\therefore$ 100N

Question 18 (****)



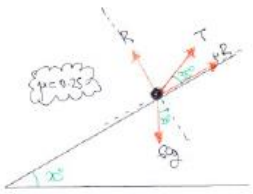
A box of mass 60 kg is held in limiting equilibrium, on a fixed rough inclined plane, by a rope. The plane is at an angle of 20° to the horizontal, as shown in the figure above.

The rope lies in a vertical plane containing a line of greatest slope of the incline plane and is inclined to the plane at an angle 35°.

The rope is modelled as a light inextensible string and the box is modelled as a particle. The coefficient of friction between the box and the plane is $\frac{1}{4}$.

Determine the **least** possible tension in the rope.

$\boxed{T \simeq 93.1887\ldots \text{ N}}$



LEAST TENSION ⟹ LIMITING FRICTION UP THE PLANE

MAX TENSION ⟹ LIMITING FRICTION DOWN THE PLANE

(I) — (⊥): $R + T\sin 35° = 60g\cos 20°$

(II) — (∥): $T\cos 35° + \mu R = 60g\sin 20$

BY SUBSTITUTION (I): $\boxed{R = 60g\cos 20 - T\sin 35°}$

(II): $T\cos 35 + \mu(60g\cos 20 - T\sin 35) = 60g\sin 20$

$T\cos 35 + 60\mu g\cos 20 - \mu T\sin 35 = 60g\sin 20$

$T\cos 35 - \mu T\sin 35 = 60g\sin 20 - 60\mu g\cos 20$

$T(\cos 35 - \mu\sin 35) = 60g(\sin 20 - \mu\cos 20)$

$T = \dfrac{60g(\sin 20 - \mu\cos 20)}{\cos 35 - \mu\sin 35}$

$T = 93.18873777\ldots$

$T \simeq 93.2\text{N}$

Kinematics

Question 7 (***)

A car is travelling along a straight horizontal road. It starts from rest at point A and accelerates uniformly at a ms^{-2}, reaching a speed of 18 ms^{-1}.

The car then travels at constant speed for T s. Finally the car begins to decelerate uniformly at 0.75 ms^{-2} coming to rest at point B.

a) Sketch a speed time graph to show the motion of the car from A to B.

b) Determine the time for which the car decelerates.

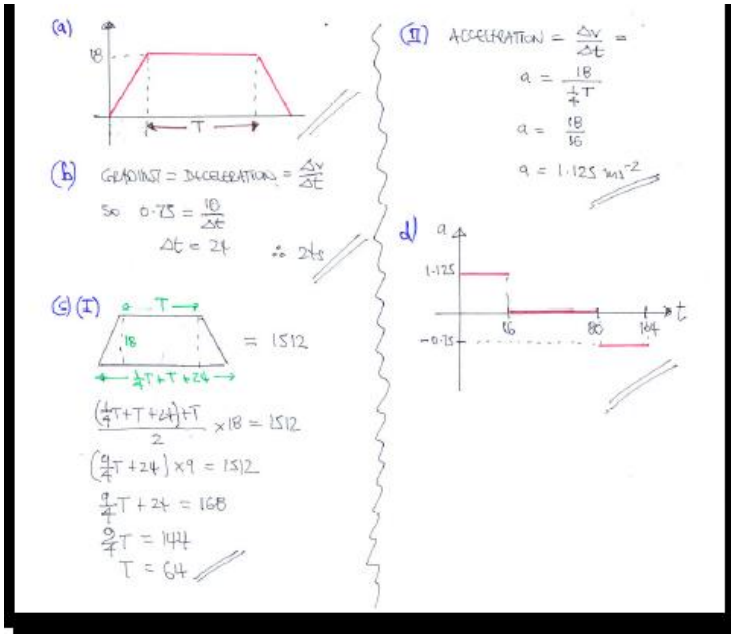
It is further given that the car accelerates for $\frac{1}{4}T$ s and the distance AB is 1512 m.

c) Calculate …

i. … the value of T.

ii. … the value of a.

d) Sketch an acceleration time graph to show the motion of the car from A to B.



Question 14 (****)

Two trains, T_1 and T_2, start together from rest, at time $t = 0$, at a station A and move along parallel straight horizontal tracks.

Both trains come to rest at the next station B after 240 s.

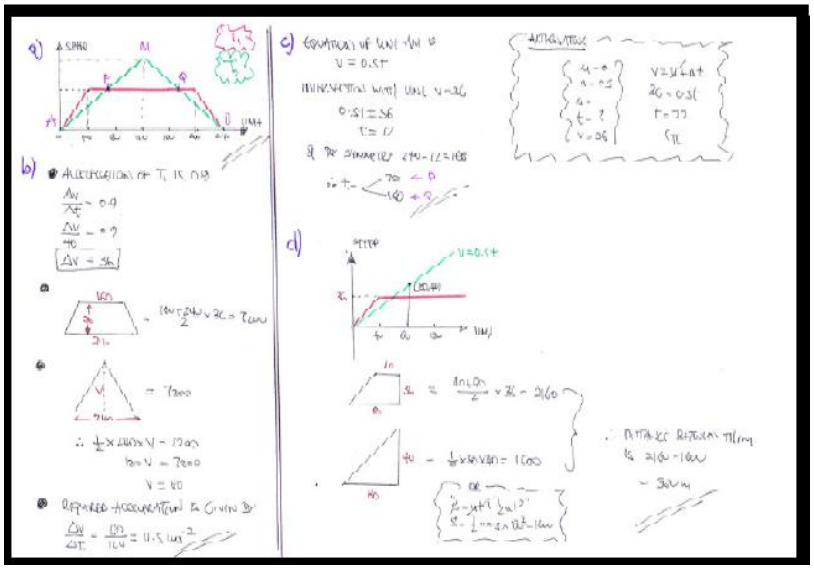
- T_1 moves with constant acceleration 0.9 ms^{-2} for 40 s, then moves at constant speed for 160 s, and then moves with constant deceleration for the last 40 s.
- T_2 moves with constant acceleration for 120 s, and then moves with constant deceleration for the last 120 s.

a) Sketch, on the same axes, the speed–time graphs for the motion of the two trains between the two stations.

b) Find the acceleration of T_2 for the first 120 s of its journey.

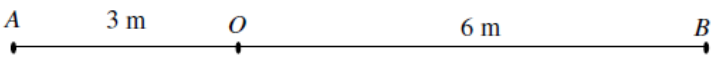
c) Determine the times when T_1 and T_2 are moving with the same speed.

d) Calculate the distance between T_1 and T_2, 80 s after they start.



Question 7 (***+)

A particle is travelling along a straight line with constant acceleration a ms^{-2}.



The points A, O and B lie in that order on this straight line, as shown in the figure above. The distance AO is 3 m and the distance OB is 6 m.

The particle is initially observed passing through O with speed u ms^{-1} and 4 s later is observed to be passing through B with speed 7 ms^{-1}, in the direction OB.

a) Find in any order the value of a and the value of u.

b) Prove that the particle never passes through A.

$\boxed{u = -4}$, $\boxed{a = 2.75}$

a) LOOKING AT "OB"

$u = ?$, $a = ?$, $s = 6\text{ m}$, $t = 4\text{ s}$, $v = 7\text{ ms}^{-1}$

$s = \frac{u+v}{2} \times t$

$6 = \frac{u+7}{2} \times 4$

$6 = 2(u+7)$

$3 = u+7$

$u = -4\text{ ms}^{-1}$

(MOVING UP)

$v = u + at$

$7 = -4 + a \times 4$

$11 = 4a$

$a = \frac{11}{4}$

$a = 2.75\text{ ms}^{-2}$

b) LOOKING AT THE JOURNEY FROM O TOWARDS A

EITHER

$u = -4$, $a = 2.75$, $s = -3$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$-3 = -4t + \frac{1}{2}(2.75)t^2$

$-3 = -4t + \frac{11}{8}t^2$

$-24 = -32t + 11t^2$

$0 = 11t^2 - 32t + 24$

CHECK DISCRIMINANT

$b^2 - 4ac = (-32)^2 - 4 \times 11 \times 24$

$= 1024 - 1056 < 0$

PARTICLE NEVER REACHES DISPLACEMENT OF -3, IE POINT A

OR

$u = -4$, $a = 2.75$, $s = ?$, $v = 0$

$v^2 = u^2 + 2as$

$0 = (-4)^2 + 2(2.75)s$

$-16 = 5.5s$

$s = -2.909\ldots > -3$

$\therefore$ NEVER REACHES A

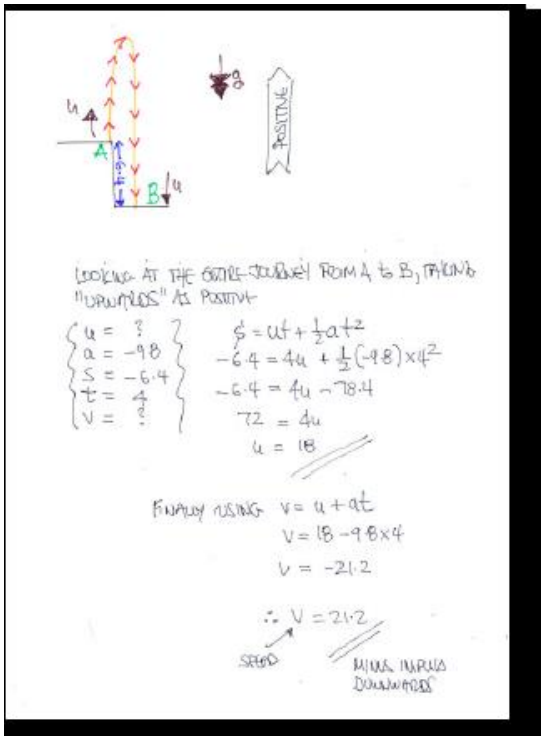
Question 12 (****)

A particle is projected vertically upwards with speed u ms^{-1}, from a balcony which lies 6.4 m above level horizontal ground.

The particle is moving freely under gravity and strikes the ground 4 s later with speed v ms^{-1}.

Calculate in any order the value of u and the value of v.

$\boxed{u = 18}$, $\boxed{v = 21.2}$



Variable acceleration

Question 8 (***+)

A particle P is moving on the x axis and its velocity v ms^{-1} in the positive x direction, t seconds after a given instant, is given by

$$v = t^2 - 2t - 24\,,\ t \geq 0\,.$$

When $t = 3$, P is observed passing through the origin.

a) Find the acceleration of P when $t = 3$.

b) Determine the distance of P from O when it is instantaneously at rest.

c) Find the time at which P is passing through O again.

$\boxed{a = 4\,\text{ms}^{-2}}$, $\boxed{d = 36\,\text{m}}$, $\boxed{t = \sqrt{72} \approx 8.49}$

$V = t^2 - 2t - 24 \qquad t=3,\ x=0$

(a) $a = \frac{dv}{dt} = 2t - 2$ when $t=3$, $a = 2\times3 - 2$

$a = 4\ \text{ms}^{-2}$

(b) $V=0$, $t^2 - 2t - 24 = 0$

$(t+4)(t-6) = 0$

$\boxed{t=6}$

$x = \int t^2 - 2t - 24\ dt$

$\boxed{x = \frac{1}{3}t^3 - t^2 - 24t + C}$

when $t=3$ $x=0$

$0 = 9 - 9 - 72 + C$

$C = 72$

$x = \frac{1}{3}t^3 - t^2 - 24t + 72$

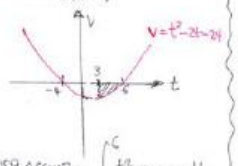
when $t=6$, $x = 72 - 36 - 144 + 72 = -36$

$\therefore$ A DISTANCE OF 36m

ALTERNATIVE

$V = t^2 - 2t - 24$

$V = (t+4)(t-6)$



DISPLACEMENT $= \int_3^6 t^2 - 2t - 24\ dt$

$= \left[\frac{1}{3}t^3 - t^2 - 24t\right]_3^6$ etc

(c) $x=0$ $\quad \frac{1}{3}t^3 - t^2 - 24t + 72 = 0$

$t^3 - 3t^2 - 72t + 216 = 0$

$(t-3)(t^2 + 0t - 72)$

$(t-3)(t^2 - 72) = 0$

$t = 3$ or $t = \sqrt{72}$ $(t>0)$

(≈ 8.49)

Question 10 (****)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = 4t - 9,\ t \geq 0.$$

When $t = 1$, P is moving with a velocity of -3 ms^{-1}.

a) Find the minimum velocity of P.

b) Determine the times when P is instantaneously at rest.

c) Find the distance travelled by P in the first $4\frac{1}{2}$ seconds of its motion.

$\boxed{v_{\min} = -6.125\,\text{ms}^{-1}}$, $\boxed{t = \frac{1}{2},\ 4}$, $\boxed{d = \frac{389}{24} \approx 16.21\,\text{m}}$

(a) $a = 4t - 9$

$\frac{dv}{dt} = 4t - 9$

$v = \int 4t - 9 \, dt$

$\boxed{v = 2t^2 - 9t + C}$

$t = 1 \quad v = -3$

$-3 = 2 - 9 + C$

$C = 4$

$\therefore \boxed{v = 2t^2 - 9t + 4}$

THIS HAS A MINIMUM

$\frac{dv}{dt} = 4t - 9$ (CONTD)

SOLVE FOR ZERO, GIVES $t = \frac{9}{4}$

$V_{MIN} = 2\left(\frac{9}{4}\right)^2 - 9\left(\frac{9}{4}\right) + 4$

$V_{MIN} = -6.125 \text{ ms}^{-1}$

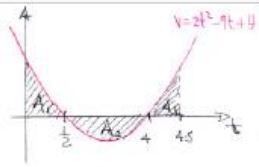
(b) PARTICLE AT REST, $v = 0$

$2t^2 - 9t + 4 = 0$

$(2t - 1)(t - 4) = 0$

$t = 4, \frac{1}{2}$

(c)



- $A_1 = A_2$ (SYMMETRY OF QUADRATIC)
- $A_1 = \int_0^{\frac{1}{2}} 2t^2 - 9t + 4 \, dt$

$= \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_0^{\frac{1}{2}}$

$= \left(\frac{1}{12} - \frac{9}{8} + 2\right) - (0)$

$= \frac{23}{24}$

- $A_2 = \int_{\frac{1}{2}}^{4} 2t^2 - 9t + 4 \, dt$

$= \left[\frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t\right]_{\frac{1}{2}}^{4}$

$= \left(\frac{128}{3} - 72 + 16\right) - \left(\frac{1}{12} - \frac{9}{8} + 2\right)$

$= -\frac{40}{3} - \frac{23}{24}$

$= -\frac{343}{24}$

$\therefore$ DISTANCE $= 2 \times \frac{23}{24} + \frac{343}{24}$

$= \frac{389}{24}$

$\approx 16.21 \text{ m}$

Question 7 (***+)

The acceleration $\mathbf{a}$ ms^{-2} of a particle P of mass 0.2 kg, t s after a given instant is given by

$$\mathbf{a} = (2t - 4)\mathbf{i} + 3\mathbf{j}, \; t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing along the positive x axis and along the positive y axis, respectively.

a) Find the magnitude of the resultant force acting on P, when $t = 4$.

It is further given that when $t = 0$, P is at the point A with position vector $(-18\mathbf{i} - 24\mathbf{j})$ m and has velocity $(3\mathbf{i} - 9\mathbf{j})$ ms^{-1}.

b) Find the value of t when the particle is at rest.

c) Show that when $t = 6$, P is on the y axis and state its distance from A.

d) Determine the value of t when the particle is on the x axis.

$\boxed{F = 1\text{N}}$, $\boxed{t = 3}$, $\boxed{18 \text{ m}}$, $\boxed{t = 8}$

(a) $\underline{a} = (2t-4)\underline{i} + 3\underline{j}$

• WHEN $t=4$

$\Rightarrow \underline{a} = 4\underline{i} + 3\underline{j}$

$\Rightarrow |\underline{a}| = \sqrt{4^2+3^2} = 5$

• "$F = ma$"

$\Rightarrow F = 0.2 \times 5$

$\Rightarrow F = 1\text{N}$

(b) $\underline{v} = \int \underline{a}\, dt$

$\Rightarrow \underline{v} = \int (2t-4)\underline{i} + 3\underline{j}\, dt$

$\Rightarrow \underline{v} = (t^2-4t+C)\underline{i} + (3t+D)\underline{j}$

• WHEN $t=0$ $\underline{v}_A = 3\underline{i} - 9\underline{j}$

$3\underline{i} - 9\underline{j} = C\underline{i} + D\underline{j}$

$C = 3$
$D = -9$

$\boxed{\underline{v} = (t^2-4t+3)\underline{i} + (3t-9)\underline{j}}$

TO BE AT REST BOTH COMPONENTS MUST BE ZERO

$\underline{j}$: $3t-9=0$, $t=3$

CHECK THE $\underline{i}$: $3^2-4\times3+3=0$

$\therefore t=3$

(c) $\underline{r} = \int \underline{v}\, dt$

$\Rightarrow \underline{r} = \int (t^2-4t+3)\underline{i} + (3t-9)\underline{j}\, dt$

$\Rightarrow \underline{r} = (\frac{1}{3}t^3-2t^2+3t+E)\underline{i} + (\frac{3}{2}t^2-9t+F)\underline{j}$

• WHEN $t=0$, $\underline{r}_A = -18\underline{i} + 24\underline{j}$

$\therefore -18\underline{i} - 24\underline{j} = E\underline{i} + F\underline{j}$

$E = -18$
$F = -24$

$\boxed{\underline{r} = (\frac{1}{3}t^3-2t^2+3t-18)\underline{i} + (\frac{3}{2}t^2-9t-24)\underline{j}}$

WHEN $t=6$

$\underline{r} = (72-72+18-18)\underline{i} + (54-54-24)\underline{j}$

$\underline{r} = 0\underline{i} - 24\underline{j}$

IE CO-ORDINATES $(0,-24)$, SO IT LIES ON THE y AXIS

DISTANCE BETWEEN $(0,-24)$ & $(-18,-24)$ IS 18 UNITS

(BY INSPECTION)

(d) ON THE x AXIS, "$y=0$"

$\frac{3}{2}t^2 - 9t - 24 = 0$

$3t^2 - 18t - 48 = 0$

$t^2 - 6t - 16 = 0$

$(t-8)(t+2) = 0$

$t = 8$

Question 8 (****)

The position vector, velocity and acceleration of a particle P, t s after a given instant are denoted by $\mathbf{r}$ m, $\mathbf{v}$ ms^{-1} and $\mathbf{a}$ ms^{-2}.

When $t=1$, $\mathbf{r} = 9\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 13\mathbf{i} + \mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

It is further given that P has a constant acceleration of $6\mathbf{i}$ ms^{-2}.

a) Determine the distance of P from the origin O, when $t=3$.

b) Show that P is moving on the curve with equation

$$x = 3y^2 + y - 5.$$

$\boxed{\approx 47.17\text{ m}}$

$\underline{a} = 6\underline{i}$ SUBJECT TO $t=1$ $\underline{r} = 9\underline{i} + 2\underline{j}$

$\underline{v} = 13\underline{i} + \underline{j}$

(a) $\underline{a} = 6\underline{i} + 0\underline{j}$

$\underline{v} = \int 6\underline{i} + 0\underline{j} \, dt$

$\boxed{\underline{v} = (6t + A)\underline{i} + B\underline{j}}$

WHEN $t=1$, $\underline{v} = 13\underline{i} + \underline{j}$

$13\underline{i} + \underline{j} = (6+A)\underline{i} + B\underline{j}$

$A = 7$

$B = 1$

$\boxed{\underline{v} = (6t+7)\underline{i} + \underline{j}}$

$\underline{r} = \int (6t+7)\underline{i} + 1\underline{j} \, dt$

$\underline{r} = (3t^2 + 7t + C)\underline{i} + (t + D)\underline{j}$

WHEN $t=1$ $\underline{r} = 9\underline{i} + 2\underline{j}$

$9\underline{i} + 2\underline{j} = (10 + C)\underline{i} + (1 + D)\underline{j}$

$C = -1$

$D = 1$

$\boxed{\underline{r} = (3t^2 + 7t - 1)\underline{i} + (t+1)\underline{j}}$

WHEN $t=3$ $\underline{r} = 47\underline{i} + 4\underline{j}$

$\therefore$ DISTANCE FROM O IS $\sqrt{47^2 + 4^2}$

$\simeq 47.17$ m (2dp)

(b) $\underline{r} = (3t^2 + 7t - 1)\underline{i} + (t+1)\underline{j}$

$x = 3t^2 + 7t - 1$, $y = t + 1$ $\Rightarrow t = y - 1$

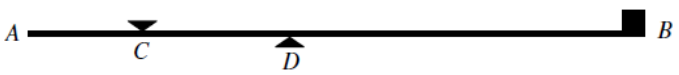
$x = 3(y-1)^2 + 7(y-1) - 1$

$x = 3y^2 - 6y + 3 + 7y - 7 - 1$

$x = 3y^2 + y - 5$

Moments

Question 9 (***)

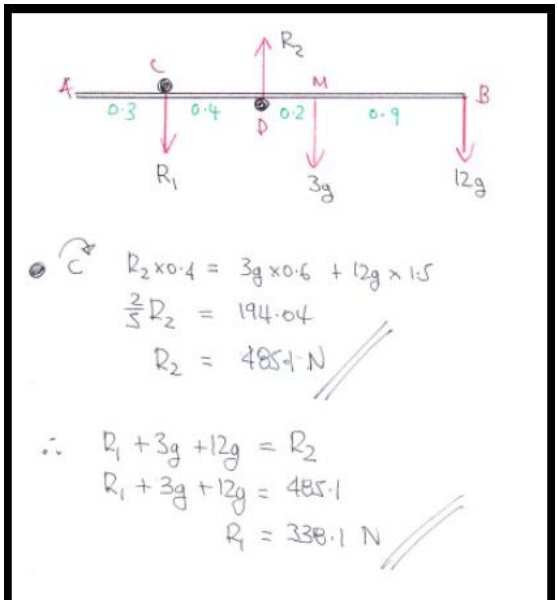


The figure above shows a uniform rod AB of length 1.8 m and mass 3 kg, held in a horizontal position by two small smooth pegs C and D.

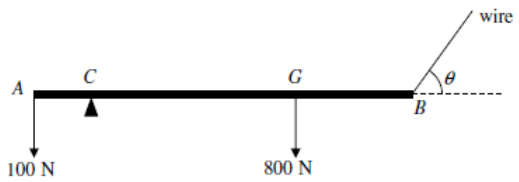
A particle of mass 12 kg, is placed at B.

Given that $|AC| = 0.3$ m and $|CD| = 0.4$ m, determine the magnitude of each of the forces exerted on the rod by the pegs.

$\boxed{R_C = 338.1 \text{ N}}$, $\boxed{R_D = 485.1 \text{ N}}$



Question 14 (***+)



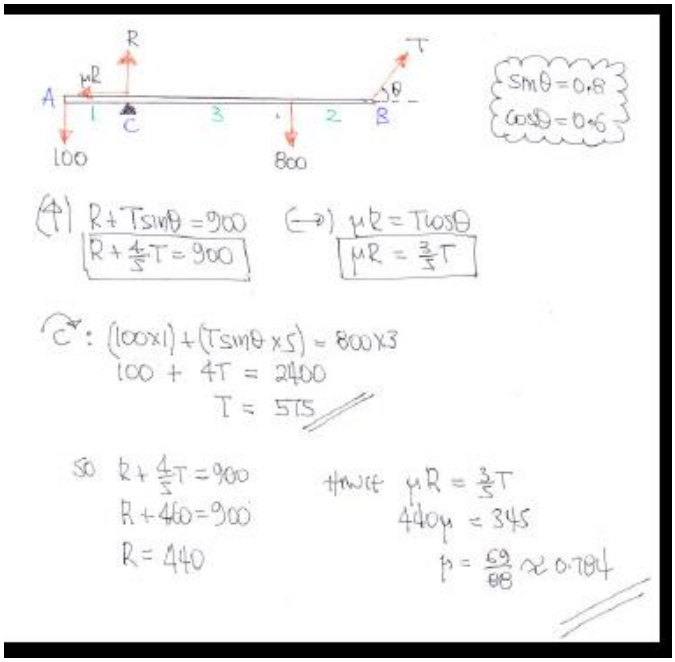
A thin rigid non uniform beam AB of length 6 m and weight 800 N has its centre of mass at G, where $AG = 4$ m. An additional weight of 100 N is fixed at A.

The beam lies in a horizontal position supported by a rough peg at C, where $AC = 1$ m, and a light inextensible wire attached at B.

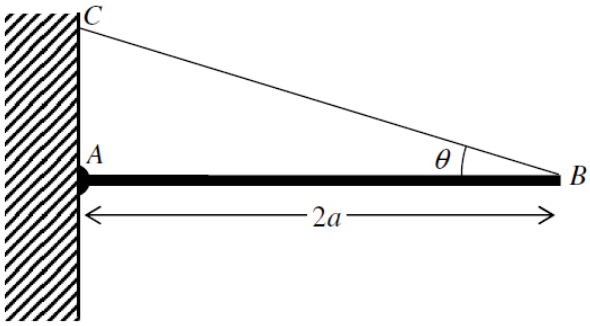
When the wire is inclined at an angle θ to the horizontal, where $\sin\theta = 0.8$, the beam remains horizontal, in **limiting** equilibrium.

Calculate the tension in the wire and the value of the coefficient of friction between the peg and the beam.

$\boxed{}$, $\boxed{T = 575 \text{ N}}$, $\boxed{\mu = \frac{69}{88} \approx 0.784}$



Question 9 (***+)



The figure above shows a uniform rod AB of length $2a$ and of mass m smoothly hinged at the point A, which lies on a vertical wall.

The rod is kept in a horizontal position by a light inextensible string BC, where C lies on the same wall vertically above A.

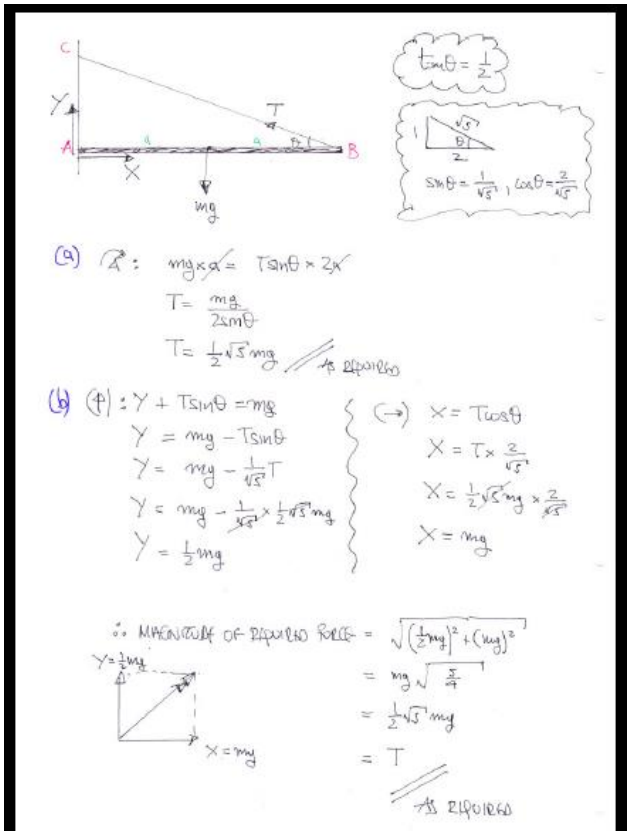
The plane ABC is perpendicular to the wall and the angle ABC is denoted by θ.

Given that $\tan\theta = \frac{1}{2}$, show clearly that …

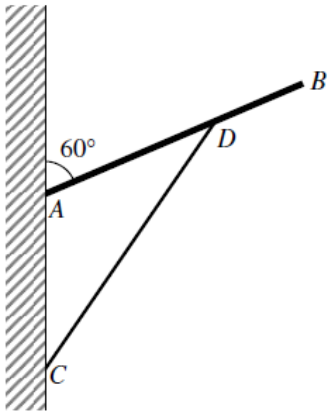
a) … the tension in the string is $\frac{1}{2}\sqrt{5}mg$.

b) … the magnitude of the reaction at the hinge has the same magnitude as the tension in the string.

proof


C
Y
T
A
B
X
mg
$\tan\theta = \frac{1}{2}$
$\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$
(a) $\circlearrowleft A$: $mg \times a = T\sin\theta \times 2a$
$T = \frac{mg}{2\sin\theta}$
$T = \frac{1}{2}\sqrt{5}\,mg$ AS REQUIRED
(b) $(\uparrow)$: $Y + T\sin\theta = mg$
$Y = mg - T\sin\theta$
$Y = mg - \frac{1}{\sqrt{5}}T$
$Y = mg - \frac{1}{\sqrt{5}} \times \frac{1}{2}\sqrt{5}\,mg$
$Y = \frac{1}{2}mg$
$(\rightarrow)$ $X = T\cos\theta$
$X = T \times \frac{2}{\sqrt{5}}$
$X = \frac{1}{2}\sqrt{5}\,mg \times \frac{2}{\sqrt{5}}$
$X = mg$
∴ MAGNITUDE OF REQUIRED FORCE $= \sqrt{\left(\frac{1}{2}mg\right)^2 + (mg)^2}$
$= mg\sqrt{\frac{5}{4}}$
$= \frac{1}{2}\sqrt{5}\,mg$
$= T$
AS REQUIRED
$Y = \frac{1}{2}mg$
$X = mg$

Question 15 (***+)


B
60°
D
A
C

The figure above shows a uniform rod AB, of weight $\sqrt{3}W$ and length $4L$, is freely hinged at the end A to a vertical wall.

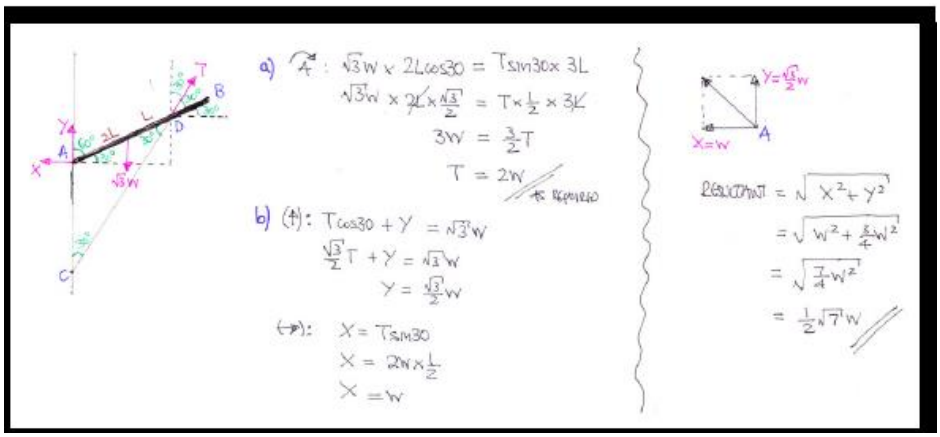
The rod is supported by a light rigid strut CD and rests in equilibrium at an angle of $60°$ to the wall. The strut is freely hinged to the rod at the point D and to the wall at the point C, which is vertically below A. It is further given that $AC = AD = 3L$

The rod and the strut lie in the same vertical plane, which is perpendicular to the wall.

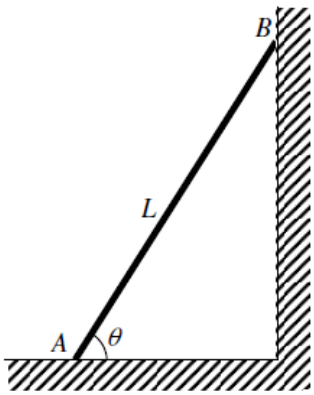
a) Show that the magnitude of the thrust in the strut is $2W$.

b) Find, in terms of W, the magnitude of the force acting on the rod at A.

$$\boxed{\text{resultant} = \frac{1}{2}\sqrt{7}W \text{ N}}$$



Question 28 (****)



The figure above shows a non-uniform rod AB, of mass m and length L, rests in equilibrium with the end A on a rough horizontal floor and the other end B, against a rough vertical wall.

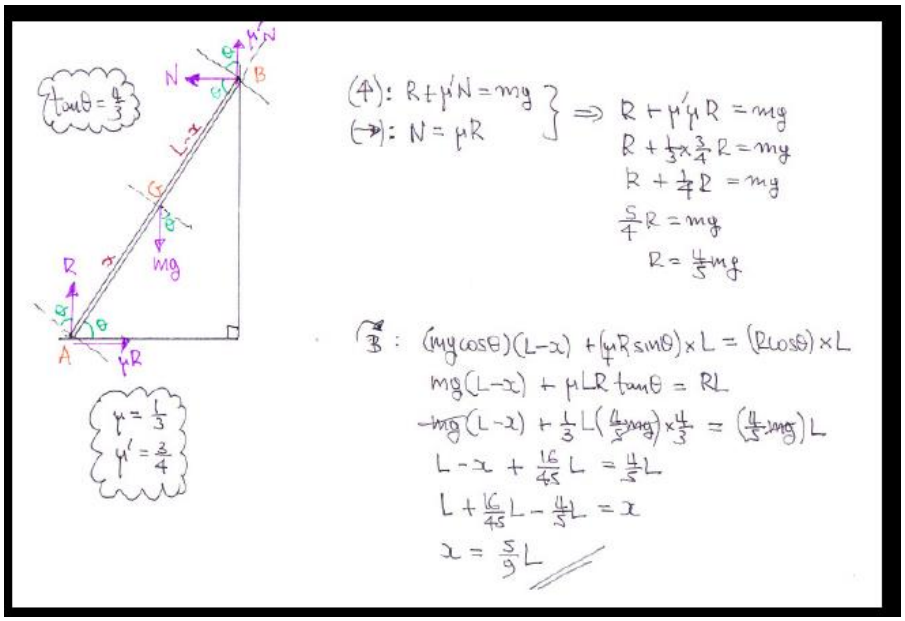
The rod is in a vertical plane perpendicular to the wall and makes an angle θ with the floor, where $\tan\theta = \frac{4}{3}$. The coefficient of friction between the rod and the floor is $\frac{1}{3}$ and the coefficient of friction between the rod and the wall is $\frac{3}{4}$.

The rod is on the point of slipping at both ends.

The centre of mass of the rod is at the point G.

Determine, in terms of L, the distance AG.

$$\boxed{|AG| = \frac{5}{9}L}$$



Projectiles

Question 8 (***+)

A particle is projected from a point O on level horizontal ground with speed of 23.8 ms^{-1} at an angle ψ to the horizontal, where $\tan\psi = \frac{15}{8}$.

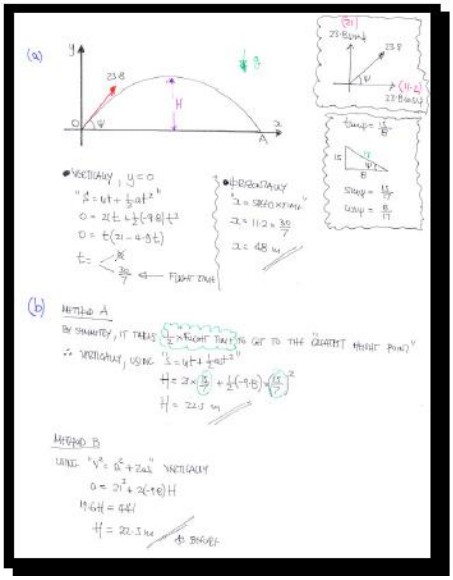
The particle is moving freely under gravity, reaching a greatest height of H m above the ground before it lands on the ground at a point A.

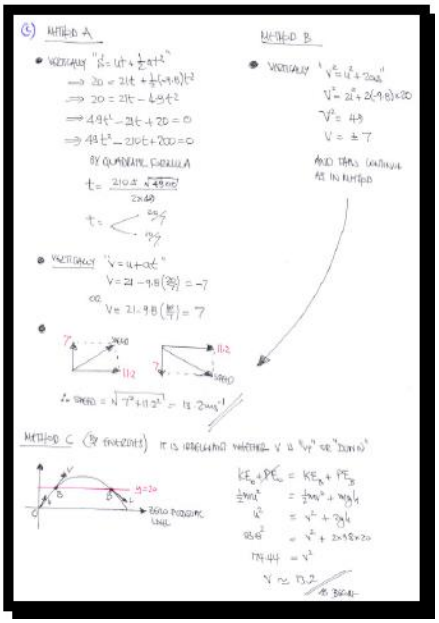
a) Determine the distance OA

b) Find the value of H.

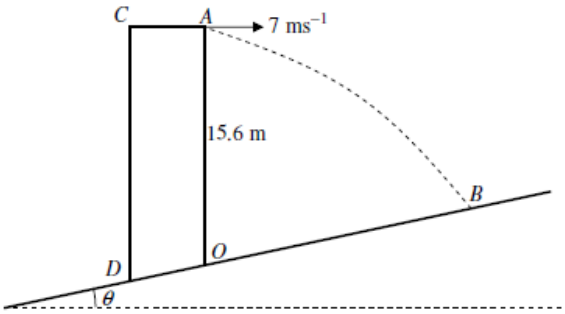
c) Calculate, to three significant figures, the speed of the particle when it is at a height of 20 m above the ground.

$\boxed{|OA| = 48 \text{ m}}$, $\boxed{H = 22.5}$, $\boxed{v \approx 13.2 \text{ ms}^{-1}}$





Question 18 (****)



The figure above shows the cross section of a vertical tower $OACD$ standing on a plane inclined at an angle θ to the horizontal, where $\tan\theta = 0.1$.

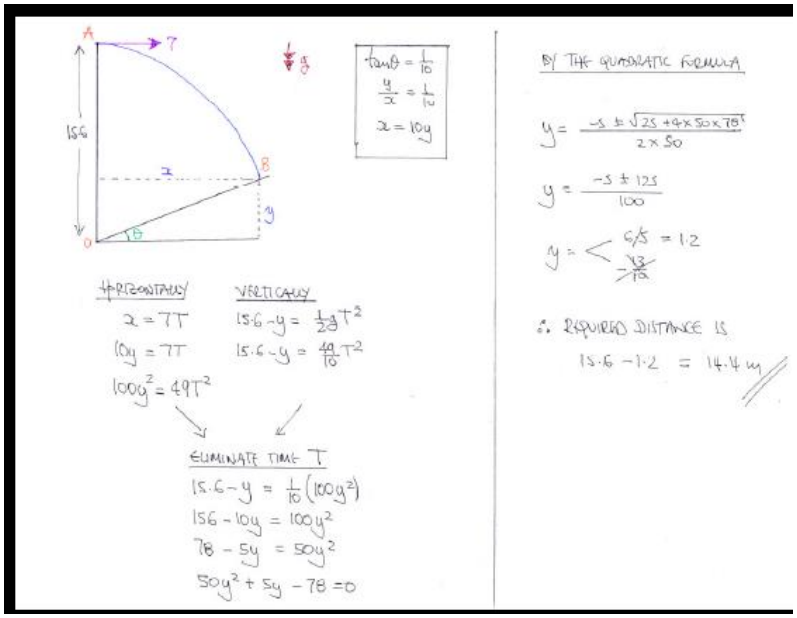
A particle is projected horizontally from A hitting the incline plane at the point B.

The journey of the particle is in a vertical plane containing O, A and B.

Given that $|OA| = 15.6$ m determine the vertical distance through which the particle falls as it travels from A to B.

You may assume that the only force acting on the particle is its weight.

$\boxed{14.4 \text{ m}}$



Vectors

Question 7 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A particle P, of mass 2 kg, is moving under the action of a single force $\mathbf{F}\text{ N}$.

At time t s the velocity of P is $\mathbf{v}\text{ ms}^{-1}$.

When $t=0$, $\mathbf{v}=(-3\mathbf{i}+\mathbf{j})$ and when $t=4$, $\mathbf{v}=(5\mathbf{i}+5\mathbf{j})$.

a) Find, in degrees, the bearing of the direction of motion of P, when $t=0$.

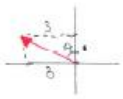
b) Calculate the magnitude of $\mathbf{F}$.

c) Determine, in terms of t, an expression for the velocity of P.

d) Find the time when P is moving parallel to the vector $3\mathbf{i}+2\mathbf{j}$.

$\boxed{288°}$, $\boxed{|\mathbf{F}|=\sqrt{20}\approx 4.47\text{ N}}$, $\boxed{\mathbf{v}=(2t-3)\mathbf{i}+(t+1)\mathbf{j}}$, $\boxed{t=9\text{ s}}$

(a)



$\tan\theta = \frac{3}{1}$

$\theta = 71.56°$

$\therefore$ BEARING° $= 360 - \theta$

$= 288°$

(b) $\underline{v} = \underline{u} + \underline{a}t$

$5\underline{i} + 5\underline{j} = -3\underline{i} + \underline{j} + \underline{a} \times 4$

$8\underline{i} + 4\underline{j} = 4\underline{a}$

$\underline{a} = 2\underline{i} + \underline{j}$

NOW $\underline{F} = m\underline{a}$

$\underline{F} = 2(2\underline{i} + \underline{j})$

$\underline{F} = 4\underline{i} + 2\underline{j}$

$|\underline{F}| = \sqrt{4^2 + 2^2}$

$|\underline{F}| = \sqrt{20} \approx 4.47\text{N}$

(c) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = (-3\underline{i} + \underline{j}) + (2\underline{i} + \underline{j})t$

$\underline{v} = (2t-3)\underline{i} + (t+1)\underline{j}$

(d) PARALLEL TO $3\underline{i} + 2\underline{j}$

$\frac{2t-3}{t+1} = \frac{3}{2}$

$4t - 6 = 3t + 3$

$t = 9$

Question 4 (***)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A ship P is sailing with constant velocity $(2\mathbf{i}-6\mathbf{j})\ \text{km h}^{-1}$.

a) Calculate the speed of P.

At noon P is at the point with position vector $(4\mathbf{i}+2\mathbf{j})$ km.

At time t hours after noon the position vector of P is $\mathbf{p}$ km.

b) Determine an expression for $\mathbf{p}$, in terms of t.

The position vector of another ship Q, $\mathbf{q}$ km t hours after noon, is given by

$$\mathbf{q}=7\mathbf{i}-4\mathbf{j}+(-4\mathbf{i}-2\mathbf{j})t.$$

c) Calculate the value of t when Q is west of P.

d) Find the distance between the two ships when Q is west of P.

$\boxed{\text{speed}=\sqrt{40}\approx 6.32\ \text{km h}^{-1}}$, $\boxed{\mathbf{p}=\left[(2t+4)\mathbf{i}+(2-6t)\mathbf{j}\right]\text{km}}$, $\boxed{t=1.5}$, $\boxed{d=6\ \text{km}}$

---

a) SPEED $= |2\underline{i}-6\underline{j}| = \sqrt{2^2+(-6)^2} = \sqrt{40} \simeq 6.32\ \text{ms}^{-1}$

b) $\underline{p} = (4\underline{i}+2\underline{j}) + (2\underline{i}-6\underline{j})t$

$\underline{p} = (2t+4)\underline{i} + (2-6t)\underline{j}$

c) $\underline{q} = 7\underline{i} - 4\underline{j} + (-4\underline{i}-2\underline{j})t$

$\underline{q} = (7-4t)\underline{i} + (-4-2t)\underline{j}$

"DUE WEST" $\Rightarrow$ SAME $\underline{j}$ COMPONENT

$\Rightarrow 2-6t = -4-2t$

$6 = 4t$

$t = \frac{3}{2}$

d) WHEN $t = \frac{3}{2}$ $\quad \underline{p} = 7\underline{i} - 7\underline{j}$

$\underline{q} = \underline{i} - 7\underline{j}$

$\therefore$ DISTANCE IS 6 km

Question 15 (****)

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

A small unmanned boat is drifting in the sea with **constant** velocity.

At 10.00 a.m. it is observed at the point with position vector $(-2\mathbf{i}+3\mathbf{j})$ km and at 10.45 a.m. it has drifted to the point with position vector $(-5\mathbf{i}+3.75\mathbf{j})$ km.

a) Determine the velocity of the boat.

The position vector of the boat, t hours after 10.00 a.m., is $\mathbf{b}$ km.

b) Find an expression for $\mathbf{b}$ in terms of t.

At 11.30 a.m. a patrol boat leaves from the point with position vector $(2\mathbf{i}+\mathbf{j})$ km and intercepts the small unmanned boat at 11.45 a.m. The patrol boat is moving with **constant** velocity, $\mathbf{V}$ km h^{-1}.

c) Find $\mathbf{V}$, giving the answer in the form $a\mathbf{i}+b\mathbf{j}$, where a and b are constants to be found.

a) $\underline{r} = \underline{r}_0 + \underline{v}t \Rightarrow (-5\underline{i}+3.75\underline{j}) = (-2\underline{i}+3\underline{j}) + \underline{v}\times\frac{3}{4}$

$\Rightarrow (-20\underline{i}+15\underline{j}) = (-8\underline{i}+12\underline{j}) + 3\underline{v}$

$\Rightarrow -12\underline{i}+3\underline{j} = 3\underline{v}$

$\Rightarrow \underline{v} = -4\underline{i}+\underline{j}$

b) $\underline{b} = (-2\underline{i}+3\underline{j}) + (-4\underline{i}+\underline{j})t$

$\underline{b} = (-4t-2)\underline{i} + (t+3)\underline{j}$

c) AT 11.45 FOR THE DRIFTING BOAT $t=\frac{7}{4}$

$\underline{b} = (-4\times\frac{7}{4}-2)\underline{i} + (\frac{7}{4}+3)\underline{j}$

$\underline{b} = -9\underline{i} + \frac{19}{4}\underline{j}$

AT 11.45 FOR THE PATROL BOAT $t=\frac{1}{4}$ SINCE IT STARTS AT 11:30

THUS $(-9\underline{i}+\frac{19}{4}\underline{j}) = (2\underline{i}+\underline{j}) + \underline{V}_p\times\frac{1}{4}$

$(-36\underline{i}+19\underline{j}) = (8\underline{i}+4\underline{j}) + \underline{V}_p$

$-44\underline{i}+15\underline{j} = \underline{V}_p$

$\underline{V}_p = -44\underline{i}+15\underline{j}$