

Appendix 1: Formulae

Formulae that students are expected to know for A Level Mathematics are given below and will not appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper.

Pure Mathematics

Quadratic Equations

$$ax^2 + bx + c = 0 \text{ has roots } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Indices

$$a^x a^y \equiv a^{x+y}$$

$$a^x \div a^y \equiv a^{x-y}$$

$$(a^x)^y \equiv a^{xy}$$

Laws of Logarithms

$$x = a^n \Leftrightarrow n = \log_a x \text{ for } a > 0 \text{ and } x > 0$$

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y} \right)$$

$$k \log_a x \equiv \log_a (x^k)$$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation $y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Trigonometry

In the triangle ABC

$$\text{Sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sec^2 A \equiv 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Mensuration

Circumference and area of circle, radius r and diameter d :

$$C = 2\pi r = \pi d \quad A = \pi r^2$$

Pythagoras' theorem:

In any right-angled triangle where a , b and c are the lengths of the sides and c is the hypotenuse, $c^2 = a^2 + b^2$

Area of a trapezium = $\frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section \times length

For a circle of radius r , where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \quad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k} \sin kx + c$
$\sin kx$	$-\frac{1}{k} \cos kx + c$
e^{kx}	$\frac{1}{k} e^{kx} + c$
$\frac{1}{x}$	$\ln x + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$
Area under a curve	$= \int_a^b y \, dx \quad (y \geq 0)$

Vectors

$$|xi + yj + zk| = \sqrt{(x^2 + y^2 + z^2)}$$

Statistics

The mean of a set of data: $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{X - \mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$

Mechanics

Forces and Equilibrium

Weight = mass \times g

Friction: $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{dr}{dt} \quad a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$r = \int v dt \quad v = \int a dt$$

Appendix 2: Notation

The tables below set out the notation that must be used in A Level Mathematics examinations. Students will be expected to understand this notation without need for further explanation.

1	Set notation	
1.1	\in	is an element of
1.2	\notin	is not an element of
1.3	\subseteq	is a subset of
1.4	\subset	is a proper subset of
1.5	$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
1.6	$\{x: \dots\}$	the set of all x such that \dots
1.7	$n(A)$	the number of elements in set A
1.8	\emptyset	the empty set
1.9	\mathcal{E}	the universal set
1.10	A'	the complement of the set A
1.11	\mathbb{N}	the set of natural numbers, $\{1, 2, 3, \dots\}$
1.12	\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
1.13	\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
1.14	\mathbb{Z}_0^+	the set of non-negative integers, $\{0, 1, 2, 3, \dots\}$
1.15	\mathbb{R}	the set of real numbers
1.16	\mathbb{Q}	the set of rational numbers, $\left\{\frac{p}{q}: p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	\cup	union
1.18	\cap	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2	Miscellaneous symbols	
2.1	$=$	is equal to
2.2	\neq	is not equal to
2.3	\equiv	is identical to or is congruent to
2.4	\approx	is approximately equal to
2.5	∞	infinity
2.6	\propto	is proportional to
2.7	\therefore	therefore
2.8	\because	because
2.9	$<$	is less than
2.10	\leq, \leq	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	\geq, \geq	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

3	Operations	
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b, ab, a \cdot b$	a multiplied by b
3.4	$a \div b, \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
3.6	$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
3.7	\sqrt{a}	the non-negative square root of a
3.8	$ a $	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \dots \times 2 \times 1, n \in \mathbb{N}; 0! = 1$
3.10	$\binom{n}{r}, {}^n C_r, {}_n C_r$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leq n$ or $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$

4	Functions	
4.1	$f(x)$	the value of the function f at x
4.2	$f : x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $gf(x) = g(f(x))$
4.5	$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
4.6	$\Delta x, \delta x$	an increment of x
4.7	$\frac{dy}{dx}$	the derivative of y with respect to x
4.8	$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
4.9	$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ..., n^{th} derivatives of $f(x)$ with respect to x

4		
Functions		
4.10	\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to t
4.11	$\int y \, dx$	the indefinite integral of y with respect to x
4.12	$\int_a^b y \, dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$

5		
Exponential and Logarithmic Functions		
5.1	e	base of natural logarithms
5.2	$e^x, \exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x, \log_e x$	natural logarithm of x

6		
Trigonometric Functions		
6.1	$\sin, \cos, \tan,$ $\operatorname{cosec}, \sec, \cot$	the trigonometric functions
6.2	$\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1} \\ \arcsin, \arccos, \arctan \end{array} \right\}$	the inverse trigonometric functions
6.3	$^\circ$	degrees
6.4	rad	radians

7		
Vectors		
7.1	$\mathbf{a}, \underline{a}, \underline{\underline{a}}$	the vector $\mathbf{a}, \underline{a}, \underline{\underline{a}}$; these alternatives apply throughout section 9
7.2	\overline{AB}	the vector represented in magnitude and direction by the directed line segment AB
7.3	$\hat{\mathbf{a}}$	a unit vector in the direction of \mathbf{a}
7.4	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
7.5	$ \mathbf{a} , a$	the magnitude of \mathbf{a}
7.6	$ \overline{AB} , AB$	the magnitude of \overline{AB}

7	Vectors	
7.7	$\begin{pmatrix} a \\ b \end{pmatrix}, ai + bj$	column vector and corresponding unit vector notation
7.8	\mathbf{r}	position vector
7.9	\mathbf{s}	displacement vector
7.10	\mathbf{v}	velocity vector
7.11	\mathbf{a}	acceleration vector

8	Probability and Statistics	
8.1	$A, B, C, \text{ etc.}$	events
8.2	$A \cup B$	union of the events A and B
8.3	$A \cap B$	intersection of the events A and B
8.4	$P(A)$	probability of the event A
8.5	A'	complement of the event A
8.6	$P(A B)$	probability of the event A conditional on the event B
8.7	$X, Y, R, \text{ etc.}$	random variables
8.8	$x, y, r, \text{ etc.}$	values of the random variables X, Y, R etc.
8.9	x_1, x_2, \dots	observations
8.10	f_1, f_2, \dots	frequencies with which the observations x_1, x_2, \dots occur
8.11	$p(x), P(X=x)$	probability function of the discrete random variable X
8.12	p_1, p_2, \dots	probabilities of the values x_1, x_2, \dots of the discrete random variable X
8.13	$E(X)$	expectation of the random variable X
8.14	$\text{Var}(X)$	variance of the random variable X
8.15	\sim	has the distribution
8.16	$B(n, p)$	binomial distribution with parameters n and p , where n is the number of trials and p is the probability of success in a trial
8.17	q	$q = 1 - p$ for binomial distribution
8.18	$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2

8	Probability and Statistics	
8.19	$Z \sim N(0,1)$	standard Normal distribution
8.20	ϕ	probability density function of the standardised Normal variable with distribution $N(0, 1)$
8.21	Φ	corresponding cumulative distribution function
8.22	μ	population mean
8.23	σ^2	population variance
8.24	σ	population standard deviation
8.25	\bar{x}	sample mean
8.26	s^2	sample variance
8.27	s	sample standard deviation
8.28	H_0	Null hypothesis
8.29	H_1	Alternative hypothesis
8.30	r	product moment correlation coefficient for a sample
8.31	ρ	product moment correlation coefficient for a population

9	Mechanics	
9.1	kg	kilograms
9.2	m	metres
9.3	km	kilometres
9.4	$m/s, m s^{-1}$	metres per second (velocity)
9.5	$m/s^2, m s^{-2}$	metres per second per second (acceleration)
9.6	F	Force or resultant force
9.7	N	Newton
9.8	N m	Newton metre (moment of a force)
9.9	t	time
9.10	s	displacement
9.11	u	initial velocity
9.12	v	velocity or final velocity
9.13	a	acceleration
9.14	g	acceleration due to gravity
9.15	μ	coefficient of friction

Appendix 3: Use of calculators

Students may use a calculator in all A Level Mathematics examinations. Students are responsible for making sure that their calculators meet the guidelines set out in this appendix.

The use of technology permeates the study of A Level Mathematics. Calculators used **must** include the following features:

- an iterative function
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

In addition, students **must** be told these regulations before sitting an examination:

<p>Calculators must be:</p> <ul style="list-style-type: none">• of a size suitable for use on the desk• either battery- or solar powered• free of lids, cases and covers that have printed instructions or formulas.	<p>Calculators must not:</p> <ul style="list-style-type: none">• be designed or adapted to offer any of these facilities<ul style="list-style-type: none">o language translatorso symbolic algebra manipulationo symbolic differentiation or integrationo communication with other machines or the internet• be borrowed from another student during an examination for any reason*• have retrievable information stored in them – this includes<ul style="list-style-type: none">o databankso dictionarieso mathematical formulaso text.
<p>The student is responsible for the following:</p> <ul style="list-style-type: none">• the calculator's power supply• the calculator's working condition• clearing anything stored in the calculator.	

Advice: *an invigilator may give a student a replacement calculator.