Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3
Advanced Level

Thursday 18 January 2007 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(*b*) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

2.

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(*a*) Show that $f(x) = \frac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$.

(4)

(*b*) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(*c*) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

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3. The curve C has equation $x = 2\sin y$.

(*a*) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(*b*) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(*c*) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

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4. (i) The curve C has equation $y = \frac{x}{9 + x^2}$.

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that $y = (1 + e^{2x})^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)

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5. **Figure 1**

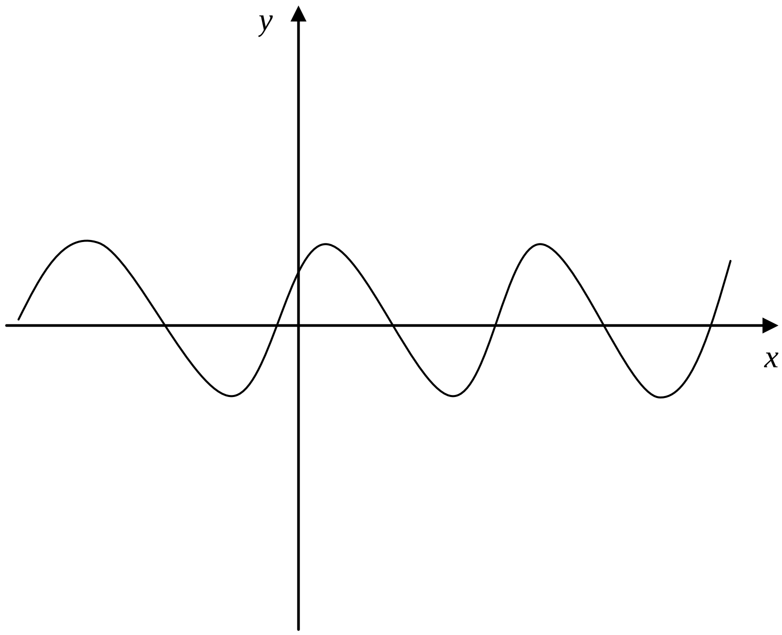

Figure 1 shows an oscilloscope screen.

The curve on the screen satisfies the equation $y = \sqrt{3}\cos x + \sin x$.

(*a*) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Find the values of x, $0 \le x < 2\pi$, for which $y = 1$.

(4)

6. The function f is defined by

$$\mathrm{f} : x \mapsto \ln(4 - 2x), \quad x < 2 \text{ and } x \in \mathbb{R}.$$

(*a*) Show that the inverse function of f is defined by

$$\mathrm{f}^{-1} : x \mapsto 2 - \tfrac{1}{2}\mathrm{e}^{x}$$

and write down the domain of f^{-1}.

(4)

(*b*) Write down the range of f^{-1}.

(1)

(*c*) Sketch the graph of $y = \mathrm{f}^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = \mathrm{f}^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\tfrac{1}{2}\mathrm{e}^{x_n}, \quad x_0 = -0.3,$$

is used to find an approximate value for k.

(*d*) Calculate the values of x_1 and x_2, giving your answer to 4 decimal places.

(2)

(*e*) Find the values of k to 3 decimal places.

(2)

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7. $$\mathrm{f}(x) = x^4 - 4x - 8.$$

(*a*) Show that there is a root of $\mathrm{f}(x) = 0$ in the interval $[-2, -1]$.

(3)

(*b*) Find the coordinates of the turning point on the graph of $y = \mathrm{f}(x)$.

(3)

(*c*) Given that $\mathrm{f}(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants a, b and c.

(3)

(*d*) Sketch the graph of $y = \mathrm{f}(x)$.

(3)

(*e*) Hence sketch the graph of $y = |\mathrm{f}(x)|$.

(1)

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8. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi,$$

(*a*) express $\arcsin x$ in terms of y.

(2)

(*b*) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)

TOTAL FOR PAPER: 75 MARKS

END