1. Factorise completely $x^3 - 4x^2 + 3x$. **(3)**

---

2. The sequence of positive numbers $u_1, u_2, u_3, \ldots$, is given by $u_{n+1} = (u_n - 3)^2$, $u_1 = 1$.

(*a*) Find u_2, u_3 and u_4. **(3)**

(*b*) Write down the value of u_{20}. **(1)**

---

3. The line L has equation $y = 5 - 2x$.

(*a*) Show that the point P (3, –1) lies on L. **(1)**

(*b*) Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

---

4. Given that $y = 2x^2 - \dfrac{6}{x^3}$, $x \neq 0$,

(*a*) find $\dfrac{dy}{dx}$, **(2)**

(*b*) find $\displaystyle\int y \, dx$. **(3)**

---

5. (*a*) Write $\sqrt{45}$ in the form $a\sqrt{5}$, where a is an integer. **(1)**

(*b*) Express $\dfrac{2(3+\sqrt{5})}{(3-\sqrt{5})}$ in the form $b + c\sqrt{5}$, where b and c are integers. **(5)**

6.

Figure 1

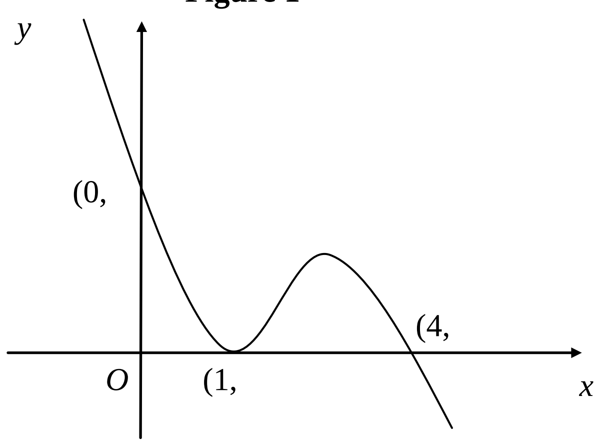

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curve with equation

(*a*) $y = f(x + 1)$, **(3)**

(*b*) $y = 2f(x)$, **(3)**

(*c*) $y = f\left(\dfrac{1}{2}x\right)$. **(3)**

On each diagram show clearly the coordinates of all the points at which the curve meets the axes.

7. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

(*a*) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200. **(1)**

(*b*) Find the amount of Alice's annual allowance on her 18th birthday. **(2)**

(*c*) Find the total of the allowances that Alice had received up to and including her 18th birthday. **(3)**

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(*d*) Find how old Alice was when she received her last allowance. **(7)**

---

8. The curve with equation $y = \mathrm{f}(x)$ passes through the point (1, 6). Given that

$$\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find $\mathrm{f}(x)$ and simplify your answer. **(7)**

---

9. **Figure 2**

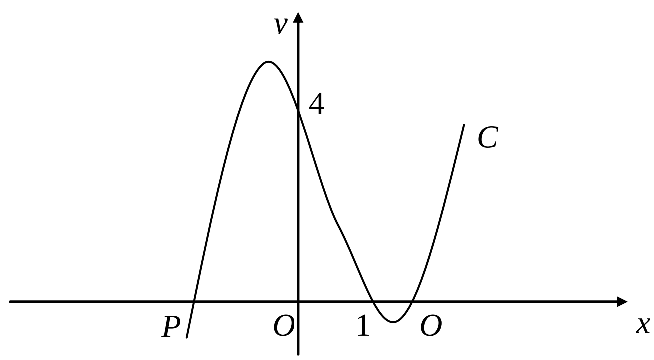

Figure 2 shows part of the curve C with equation $y = (x - 1)(x^2 - 4)$.

The curve cuts the x-axis at the points P, (1, 0) and Q, as shown in Figure 2.

(*a*) Write down the x-coordinate of P and the x-coordinate of Q. **(2)**

(*b*) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4$. **(3)**

(*c*) Show that $y = x + 7$ is an equation of the tangent to C at the point (–1, 6). **(2)**

The tangent to C at the point R is parallel to the tangent at the point (–1, 6).

(*d*) Find the exact coordinates of R. **(5)**

---

10. $$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(*a*) Find the values of the constants a and b. **(2)**

(*b*) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes. **(3)**

(*c*) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (*b*). **(2)**

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(*d*) Find the set of possible values of k, giving your answer in surd form. **(4)**

1. $x(x-3)(x-1)$

2. a) $u_2 = 4$ $u_3 = 1$ $u_4 = 4$ b) $u_{20} = 4$

3. a) $y = 5-(2\times 3) = -1$ b) $x-2y-5=0$

4. a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4x + 18x^{-4}$ b) $\dfrac{2x^3}{3} + 3x^{-2} + C$

5. a) $3\sqrt{5}$ b) $7+3\sqrt{5}$

6. sketches

7. a) 500 + (500 + 200) = 1200 or $S_2 = \frac{1}{2}2\{1000+200\} = 1200$

   b) £1900 c) £9600 d) 26

8. $3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3$

9. a) –2(P) 2(Q) b) and c) proof d) $x = \frac{5}{3}$ $y = -\frac{22}{27}$

10. a) a = 1, b = 2 c) –8. Negative, so curve does not cross x-axis

    d) $-\sqrt{12} < k < \sqrt{12}$