Assignment 16 Test Version O

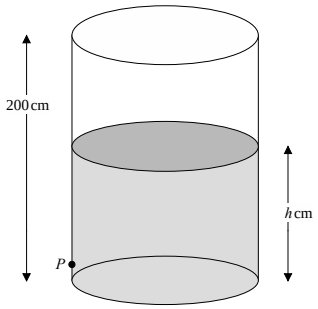


Diagram not drawn to scale

Figure 3

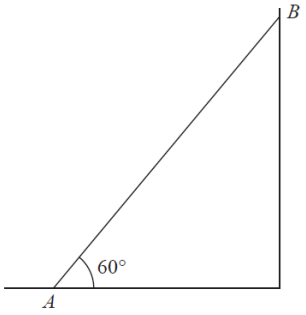
1. The diagram shows a vertical cylindrical tank of height 200 cm containing water. Water is leaking from a hole *P* on the side of the tank. At time *t* minutes after the leaking starts, the height of water in the tank is *h* cm. The height *h* cm of the water in the tank satisfies the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t} = k(h-9)^{\frac{1}{2}}$, $9 < h \leqslant 200$

where *k* is a constant. Given that, when *h* = 130, the height of the water is falling at a rate of 1.1 cm per minute,

(*a*) show that the value of *k* is -0.1

Given that the tank was full of water when the leaking started,

(*b*) solve the differential equation with your value of *k*, to find the value of *t* when *h* = 50 Give your answer to the nearest minute.



2. A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is $\frac{1}{4}$ and the coefficient of friction between the rod and the wall is $\frac{2}{3}$. The rod is on the point of slipping at both ends.

(*a*) Find the magnitude of the vertical component of the force exerted on the rod by the floor. Write your answer in the format kmg where k is a rational number which should be stated.

The centre of mass of the rod is at *G*.

(*b*) Find the distance *AG*. Write your answer in the format pl where p is a real number which should be stated correct to 3 significant figures.

Assignment 16 Test Version P

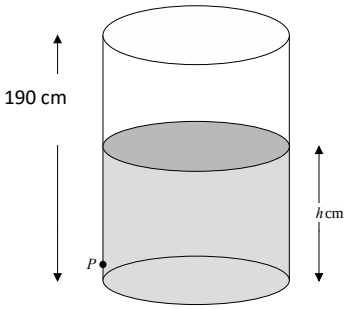


Diagram not drawn to scale

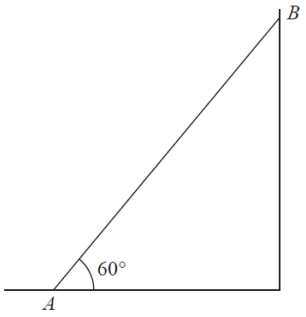
Figure 3

1. The diagram shows a vertical cylindrical tank of height 190 cm containing water. Water is leaking from a hole *P* on the side of the tank. At time *t* minutes after the leaking starts, the height of water in the tank is *h* cm. The height *h* cm of the water in the tank satisfies the differential equation $\frac{dh}{dt} = k(h-10)^{\frac{1}{2}}$ $10 < h \leqslant 190$ where *k* is a constant. Given that, when *h* = 130, the height of the water is falling at a rate of 1.1 cm per minute,

(*a*) show that the value of *k* is -0.1

Given that the tank was full of water when the leaking started,

(*b*) solve the differential equation with your value of *k*, to find the value of *t* when *h* = 60 Give your answer to the nearest minute.



2. A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is $\frac{2}{3}$ and the coefficient of friction between the rod and the wall is $\frac{1}{4}$. The rod is on the point of slipping at both ends.

(*a*) Find the magnitude of the vertical component of the force exerted on the rod by the floor. Write your answer in the format kmg where k is a rational number which should be stated.

The centre of mass of the rod is at *G*.

(*b*) Find the distance *AG*. Write your answer in the format pl where p is a real number which should be stated correct to 3 significant figures.

Assignment 16 Test Version Q

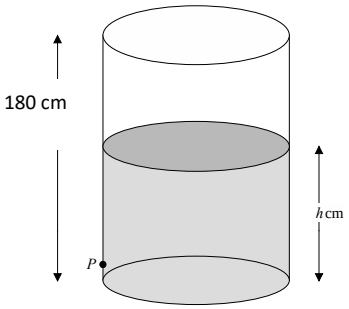


Diagram not drawn to scale

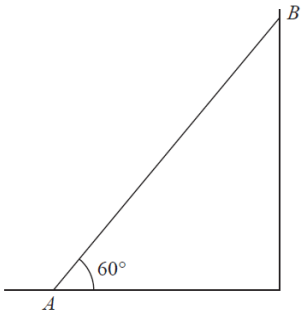
Figure 3

1. The diagram shows a vertical cylindrical tank of height 180 cm containing water. Water is leaking from a hole *P* on the side of the tank. At time *t* minutes after the leaking starts, the height of water in the tank is *h* cm. The height *h* cm of the water in the tank satisfies the differential equation $\frac{dh}{dt} = k(h-10)^{\frac{1}{2}}$ $10 < h \leqslant 180$ where *k* is a constant. Given that, when *h* = 130, the height of the water is falling at a rate of 1.1 cm per minute,

(*a*) show that the value of *k* is -0.1

Given that the tank was full of water when the leaking started,

(*b*) solve the differential equation with your value of *k*, to find the value of *t* when *h* = 70 Give your answer to the nearest minute.



2. A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is $\frac{1}{4}$ and the coefficient of friction between the rod and the wall is $\frac{1}{2}$. The rod is on the point of slipping at both ends.

(*a*) Find the magnitude of the vertical component of the force exerted on the rod by the floor. Write your answer in the format kmg where k is a rational number which should be stated.

The centre of mass of the rod is at *G*.

(*b*) Find the distance *AG*. Write your answer in the format pl where p is a real number which should be stated correct to 3 significant figures.

Assignment 16 Test Version R

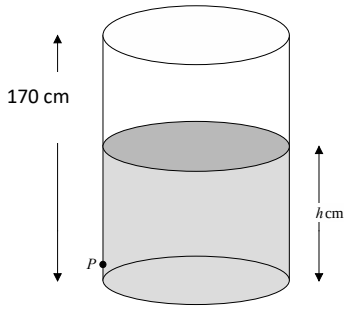


Diagram not drawn to scale

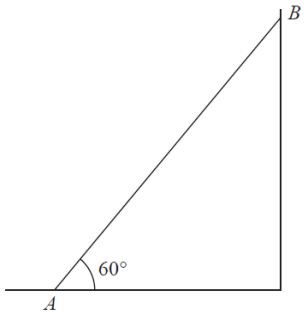
Figure 3

1. The diagram shows a vertical cylindrical tank of height 170 cm containing water. Water is leaking from a hole *P* on the side of the tank. At time *t* minutes after the leaking starts, the height of water in the tank is *h* cm. The height *h* cm of the water in the tank satisfies the differential equation $\frac{dh}{dt} = k(h-10)^{\frac{1}{2}}$ $10 < h \leqslant 170$ where *k* is a constant. Given that, when *h* = 130, the height of the water is falling at a rate of 1.1 cm per minute,

(*a*) show that the value of *k* is -0.1

Given that the tank was full of water when the leaking started,

(*b*) solve the differential equation with your value of *k*, to find the value of *t* when *h* = 40 Give your answer to the nearest minute.



2. A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is $\frac{1}{2}$ and the coefficient of friction between the rod and the wall is $\frac{2}{3}$. The rod is on the point of slipping at both ends.

(*a*) Find the magnitude of the vertical component of the force exerted on the rod by the floor. Write your answer in the format kmg where k is a rational number which should be stated.

The centre of mass of the rod is at *G*.

(*b*) Find the distance *AG*. Write your answer in the format pl where p is a real number which should be stated correct to 3 significant figures.

Answers Version O

1. b) 148

2a) $\frac{6}{7}mg$ b) 1.03 *l*

Answers Version P

1. b) 127

2a) $\frac{6}{7}mg$ b) 2.27 *l*

Answers Version Q

1. b) 106

2a) $\frac{8}{9}mg$ b) 0.99 *l*

Answers Version R

1. b) 143

2a) $\frac{3}{4}mg$ b) 1.80 *l*