Statistics & Mechanics 1. Sampling

1 **Statistical sampling**	1.1	**Understand and use the terms 'population' and 'sample'.** **Use samples to make informal inferences about the population.**	**Students will be expected to comment on the advantages and disadvantages associated with a census and a sample.**
		Understand and use sampling techniques, including simple random sampling and opportunity sampling.	**Students will be expected to be familiar with: simple random sampling, stratified sampling, systematic sampling, quota sampling and opportunity (or convenience) sampling.**
		Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.	

Statistics & Mechanics 1. Sampling

1 The table shows the daily mean temperature recorded on the first 15 days in May 1987 at Heathrow.

Day of month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Daily mean temp (°C)	14.6	8.8	7.2	7.3	10.1	11.9	12.2	12.1	15.2	11.1	10.6	12.7	8.9	10.0	9.5

a Use an opportunity sample of the first 5 dates in the table to estimate the mean daily mean temperature at Heathrow for the first 15 days of May 1987.

b Describe how you could use the random number function on your calculator to select a simple random sample of 5 dates from this data.

Hint Make sure you describe your sampling frame.

c Use a simple random sample of 5 dates to estimate the mean daily mean temperature at Heathrow for the first 15 days of May 1987.

d Use all 15 dates to calculate the mean daily mean temperature at Heathrow for the first 15 days of May 1987. Comment on the reliability of your two samples.

2 **a** Give one advantage and one disadvantage of using:
i a census **ii** a sample survey.

b It is decided to take a sample of 100 from a population consisting of 500 elements. Explain how you would obtain a simple random sample from this population.

3 **a** Explain briefly what is meant by:
i a population **ii** a sampling frame.

b A market research organisation wants to take a sample of:
i owners of diesel motor cars in the UK
ii persons living in Oxford who suffered injuries to the back during July 1996.
Suggest a suitable sampling frame in each case.

4 Write down one advantage and one disadvantage of using:
a stratified sampling **b** simple random sampling.

5 The managing director of a factory wants to know what the workers think about the factory canteen facilities. 100 people work in the offices and 200 work on the shop floor.
The factory manager decides to ask the people who work in the offices.

a Suggest a reason why this is likely to produce a biased sample.

b Explain briefly how the factory manager could select a sample of 30 workers using:
i systematic sampling **ii** stratified sampling **iii** quota sampling.

6 There are 64 girls and 56 boys in a school.
Explain briefly how you could take a random sample of 15 pupils using:
a simple random sampling **b** stratified sampling.

Statistics & Mechanics 1. Sampling

Answers

1 a 9.6°C

b Sampling frame: first 15 days in May 1987
Allocate each date a number from 1 to 15
Use the random number function on calculator to generate 5 numbers between 1 and 15

c Students' own answers.

d 10.8°C

2 a i Advantage: very accurate; disadvantage: expensive (time consuming).

ii Advantage: easier data collection (quick, cheap); disadvantage: possible bias.

b Assign unique 3-digit identifiers 000, 001, ..., 499 to each member of the population. Work along rows of random number tables generating 3-digit numbers. If these correspond to an identifier then include the corresponding member in the sample; ignore repeats and numbers greater than 499. Repeat this process until the sample contains 100 members.

3 a i Collection of individual items.

ii List of sampling units.

b i List of registered owners from DVLA.

ii List of people visiting a doctor's clinic in Oxford in July 1996.

4 a Advantage – the results are the most representative of the population since the structure of the sample reflects the structure of the population.
Disadvantage – you need to know the structure of the population before you can take a stratified sample.

b Advantage – quick and cheap.
Disadvantage – can introduce bias (e.g. if the sample, by chance, only includes very tall people in an investigation into heights of students).

5 a People not in office not represented.

b i Get a list of the 300 workers at the factory.
$\frac{300}{30} = 10$ so choose one of the first ten workers on the list at random and every subsequent 10th worker on the list, e.g. if person 7 is chosen, then the sample includes workers 7, 17, 27, ..., 297.

ii The population contains 100 office workers ($\frac{1}{3}$ of population) and 200 shop floor workers ($\frac{2}{3}$ of population).
The sample should contain $\frac{1}{3} \times 30 = 10$ office workers and $\frac{2}{3} \times 30 = 20$ shop floor workers.
The 10 office workers in the sample should be a simple random sample of the 100 office workers.
The 20 shop floor workers should be a simple random sample of the 200 shop floor workers.

iii Decide the categories e.g. age, gender, office/non office and set a quota for each in proportion to their numbers in the population. Interview workers until quotas are full.

6 a Allocate a number between 1 and 120 to each pupil.
Use random number tables, computer or calculator to select 15 different numbers between 1 and 120 (or equivalent).
Pupils corresponding to these numbers become the sample.

b Allocate numbers 1–64 to girls and 65–120 to boys.
Select $\frac{64}{120} \times 15 = 8$ different random numbers between 1 and 64 for girls.
Select 7 different random numbers between 65 and 120 for boys. Include the corresponding boys and girls in the sample.

Statistics & Mechanics
2. Data Presentation and Interpretation

2 **Data presentation and interpretation**	2.1	**Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency.** **Connect to probability distributions.**	**Students should be familiar with histograms, frequency polygons, box and whisker plots (including outliers) and cumulative frequency diagrams.**
2 **Data presentation and interpretation** *continued*	2.2	**Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded).**	**Students should be familiar with the terms explanatory (independent) and response (dependent) variables.** Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than x and y may be used. **Use of interpolation and the dangers of extrapolation. Variables other than x and y may be used.** Change of variable may be required, e.g. using knowledge of logarithms to reduce a relationship of the form $y = ax^n$ or $y = kb^x$ into linear form to estimate a and n or k and b.
		Understand informal interpretation of correlation. **Understand that correlation does not imply causation.**	**Use of terms such as positive, negative, zero, strong and weak are expected.**

Statistics & Mechanics
2. Data Presentation and Interpretation

	2.3	**Interpret measures of central tendency and variation, extending to standard deviation.**	**Data may be discrete, continuous, grouped or ungrouped. Understanding and use of coding.** **Measures of central tendency: mean, median, mode.** **Measures of variation: variance, standard deviation, range and interpercentile ranges.** **Use of linear interpolation to calculate percentiles from grouped data is expected.**
		Be able to calculate standard deviation, including from summary statistics.	**Students should be able to use the statistic** $S_{xx} = \Sigma(x-\bar{x})^2 = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$ **Use of standard deviation** $= \sqrt{\frac{S_{xx}}{n}}$ **(or equivalent) is expected but the use of** $S = \sqrt{\frac{S_{xx}}{n-1}}$ **(as used on spreadsheets) will be accepted.**

2 **Data presentation and interpretation** *continued*	2.4	**Recognise and interpret possible outliers in data sets and statistical diagrams.**	**Any rule needed to identify outliers will be specified in the question.** **For example, use of** $Q_1 - 1.5 \times \text{IQR}$ **and** $Q_3 + 1.5 \times \text{IQR}$ **or mean** $\pm 3 \times$ **standard deviation.**
		Select or critique data presentation techniques in the context of a statistical problem.	**Students will be expected to draw simple inferences and give interpretations to measures of central tendency and variation.** Significance tests, other than those mentioned in Section 5, will not be expected.
		Be able to clean data, including dealing with missing data, errors and outliers.	**For example, students may be asked to identify possible outliers on a box plot or scatter diagram.**

Statistics & Mechanics
2. Data Presentation and Interpretation

E/P 1 The table gives the distances travelled to school, in km, of the population of children in a particular region of the United Kingdom.

Distance, d (km)	$0 \leqslant d < 1$	$1 \leqslant d < 2$	$2 \leqslant d < 3$	$3 \leqslant d < 5$	$5 \leqslant d < 10$	$10 \leqslant d$
Number	2565	1784	1170	756	630	135

A histogram of this data was drawn with distance along the horizontal axis. A bar of horizontal width 1.5 cm and height 5.7 cm represented the 0–1 km group.

Find the widths and heights, in cm, to one decimal place, of the bars representing the following groups:

a $2 \leqslant d < 3$ **b** $5 \leqslant d < 10$ **(5 marks)**

E 2 A manufacturer stores drums of chemicals. During storage, evaporation take place. A random sample of 10 drums was taken and the time in storage, x weeks, and the evaporation loss, y ml, are shown in the table below.

x	3	5	6	8	10	12	13	15	16	18
y	36	50	53	61	69	79	82	90	88	96

a On graph paper, draw a scatter diagram to represent these data. **(3)**

b Give a reason to support fitting a regression model of the form $y = a + bx$ to these data. **(1)**

The equation of the regression line of y on x is $y = 29.02 + 3.9x$.

c Give an interpretation of the value of the gradient in the equation of the regression line. **(1)**

The manufacturer uses this model to predict the amount of evaporation that would take place after 19 weeks and after 35 weeks.

d Comment, with a reason, on the reliability of each of these predictions. **(2)**

Statistics & Mechanics
2. Data Presentation and Interpretation

(E) 3 20 endangered forest owlets were caught for ringing. Their wingspans (x cm) were measured to the nearest centimetre.

The following summary statistics were worked out:

$\Sigma x = 316$ $\Sigma x^2 = 5078$

a Work out the mean and the standard deviation of the wingspans of the 20 birds. **(3 marks)**

One more bird was caught. It had a wingspan of 13 centimetres.

b Without doing any further calculation, say how you think this extra wingspan will affect the mean wingspan. **(1 mark)**

20 giant ibises were also caught for ringing. Their wingspans (y cm) were also measured to the nearest centimetre and the data coded using $z = \frac{y - 5}{10}$.

The following summary statistics were obtained from the coded data:

$\Sigma z = 104$ $S_{zz} = 1.8$

c Work out the mean and standard deviation of the wingspans of the giant ibis. **(5 marks)**

4 A frequency distribution is shown below.

Class interval	1–10	11–20	21–30	31–40	41–50
Frequency	10	20	30	24	16

a Use interpolation to estimate the value of the 30th percentile.

b Use interpolation to estimate the value of the 70th percentile.

c Hence estimate the 30% to 70% interpercentile range.

(E) 5 The table shows some data collected on the temperature in °C of a chemical reaction (t) and the amount of dry residue produced (d grams).

Temperature, t (°C)	38	51	72	83	89	94
Dry residue, d (grams)	4.3	11.7	58.6	136.7	217.0	318.8

The data are coded using the changes of variable $x = t$ and $y = \log d$. The regression line of y on x is found to be $y = -0.635 + 0.0334x$.

a Given that the data can be modelled by an equation of the form $d = ab^t$ where a and b are constants, find the values of a and b. **(3 marks)**

b Explain why this model is not reliable for estimating the amount of dry residue produced when the temperature is 151 °C. **(1 mark)**

Statistics & Mechanics
2. Data Presentation and Interpretation

E 6 Energy consumption is claimed to be a good predictor of Gross National Product.
An economist recorded the energy consumption (x) and the Gross National Product (y) for eight countries. The data is shown in the table.

Energy consumption (x)	3.4	7.7	12.0	75	58	67	113	131
Gross National Product (y)	55	240	390	1100	1390	1330	1400	1900

The equation of the regression line of y on x is $y = 225 + 12.9x$.

The economist uses this regression equation to estimate the energy consumption of a country with a Gross National Product of 3500.

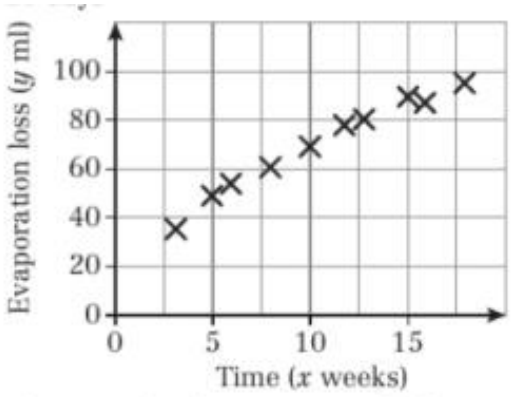
Give two reasons why this may not be a valid estimate. **(2 marks)**

Statistics & Mechanics
2. Data Presentation and Interpretation

Answers

1 **a** width = 1.5 cm, height = 2.6 cm
b width = 7.5 cm, height = 0.28 cm

2 **a**



b The points lie close to a straight line.
c 3.90 ml of the chemicals evaporate each week.
d The estimate for 19 weeks is reasonably reliable, since it is just outside the range of the data.
The estimate for 35 weeks is unreliable, since it is far outside the range of the data.

3 **a** Mean 15.8 cm, standard deviation 2.06 cm
b The mean wingspan will decrease.
c Mean 57 cm, standard deviation 3 cm

4 **a** 20.5 **b** 34.7 **c** 14.2

5 **a** $a = 0.232$ (3 s.f.), $b = 1.08$ (3 s.f.)
b 151 °C is outside the range of the data (extrapolation).

6 (1) 3500 is outside the range of the data (extrapolation).
(2) The regression equation should only be used to predict a value of GNP (y) given energy consumption (x).

Statistics & Mechanics 3. Probability

3 **Probability**	3.1	**Understand and use mutually exclusive and independent events when calculating probabilities.** **Link to discrete and continuous distributions.**	**Venn diagrams or tree diagrams may be used.** Set notation to describe events may be used. Use of $P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$, $P(A \cap B) = P(A)\,P(B)$ in connection with independent events. **No formal knowledge of probability density functions is required but students should understand that area under the curve represents probability in the case of a continuous distribution.**
	3.2	Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables. Understand and use the conditional probability formula $P(A\|B) = \frac{P(A \cap B)}{P(B)}$	Understanding and use of $P(A') = 1 - P(A)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(A)\,P(B \mid A)$.

3 **Probability** *continued*	3.3	Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions.	For example, questioning the assumption that a die or coin is fair.

Statistics & Mechanics 3. Probability

(P) 1 The scores of 250 students in a test are recorded in a table.

Score, s	Frequency (male)	Frequency (female)
$20 \leqslant s < 25$	7	8
$25 \leqslant s < 30$	15	13
$30 \leqslant s < 35$	18	19
$35 \leqslant s < 40$	25	30
$40 \leqslant s < 45$	30	26
$45 \leqslant s < 50$	27	32

One student is chosen at random.

a Find the probability that the student is female.

b Find the probability that the student scored less than 35.

c Find the probability that the student is male with a score s such that $25 \leq s < 35$.

In order to pass the test, students must score 37 or more.

d Estimate the probability that a student chosen at random passes the test. State one assumption you have made in making your estimate.

(E/P) 2 For events J and K, P(J or K or both) = 0.5, P(K but not J) = 0.2 and P(J but not K) = 0.25.

a Draw a Venn diagram to represent events J and K and the sample space S. **(3 marks)**

b Determine whether events J and K are independent. **(3 marks)**

(P) 3 In a factory, machines A, B and C produce electronic components. Machine A produces 16% of the components, machine B produces 50% of the components and machine C produces the rest. Some of the components are defective. Machine A produces 4%, machine B 3% and machine C 7% defective components.

a Draw a tree diagram to represent this information.

b Find the probability that a randomly selected component is:

i produced by machine B and is defective **ii** defective.

(E/P) 4 J, K and L are three events such that P(J) = 0.25, P(K) = 0.45 and P(L) = 0.15. Given that K and L are independent, J and L are mutually exclusive and P($J \cap K$) = 0.1

a draw a Venn diagram to illustrate this situation. **(2 marks)**

b Find:

i P($J \cup K$) **(1 mark)**

ii P($J' \cap L'$) **(1 mark)**

iii P($J|K$) **(2 marks)**

iv P($K|J' \cap L'$) **(2 marks)**

Statistics & Mechanics 3. Probability

E 5 On a randomly chosen day the probabilities that Bill travels to work by car, by bus or by train are 0.1, 0.6 and 0.3 respectively. The probabilities of being late when using these methods of travel are 0.55, 0.3 and 0.05 respectively.

a Draw a tree diagram to represent this information. **(3 marks)**

b Find the probability that on a randomly chosen day,

i Bill travels by train and is late **(2 marks)**

ii Bill is late. **(2 marks)**

c Given that Bill is late, find the probability that he did not travel by car. **(4 marks)**

E/P 6 A box A contains 7 counters of which 4 are green and 3 are blue.
A box B contains 5 counters of which 2 are green and 3 are blue.
A counter is drawn at random from box A and placed in box B. A second counter is drawn at random from box A and placed in box B.
A third counter is then drawn at random from the counters in box B.

a Draw a tree diagram to show this situation. **(4 marks)**

The event C occurs when the 2 counters drawn from box A are of the same colour.
The event D occurs when the counter drawn from box B is blue.

b Find $P(C)$. **(3 marks)**

c Show that $P(D) = \frac{27}{49}$ **(3 marks)**

d Show that $P(C \cap D) = \frac{11}{49}$ **(2 marks)**

e Hence find $P(C \cup D)$. **(2 marks)**

f Given that all three counters drawn are the same colour, find the probability that they are all green. **(3 marks)**

Statistics & Mechanics 3. Probability

Answers

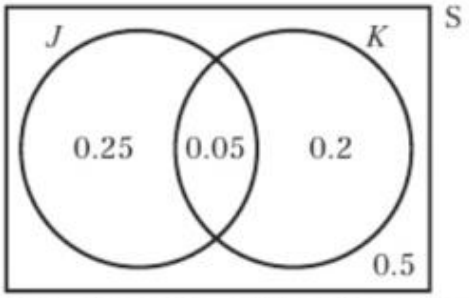
1 **a** $\frac{64}{125}$

b $\frac{8}{25}$

c $\frac{33}{250}$

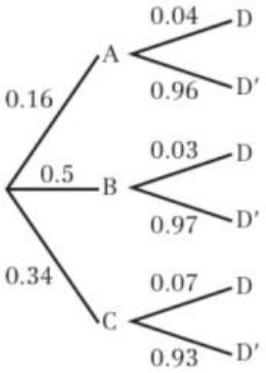
d $\frac{74}{125}$, using interpolation and assuming uniform distribution of scores

2 **a**



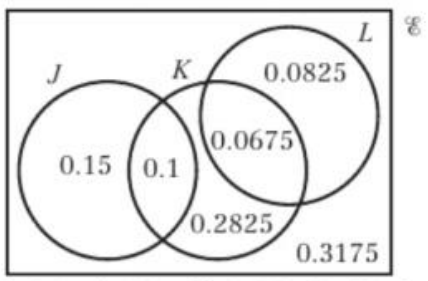
b $P(J) = 0.3$, $P(K) = 0.25$, $P(J \text{ and } K) = 0.05$
$P(J) \times P(K) = 0.075 \neq P(J \text{ and } K)$, so J and K are not independent.

3 **a**



b **i** 0.015 **ii** 0.0452

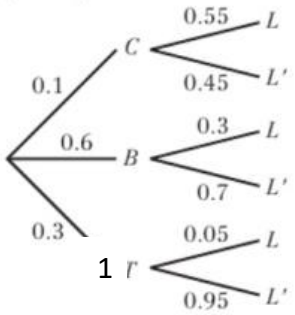
4 **a**



b **i** 0.6 **ii** 0.6 **iii** 0.222 (3 s.f.) **iv** 0.471 (3 s.f.)

Statistics & Mechanics 3. Probability

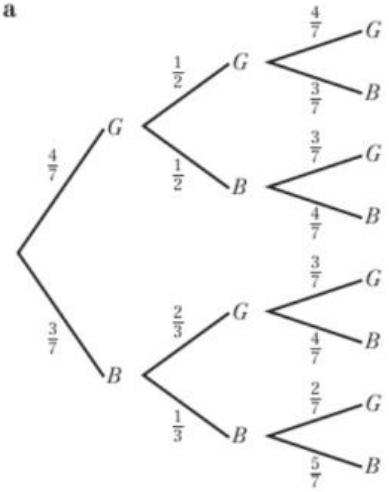
5 **a**



1 Γ

b **i** 0.015 **ii** 0.25 **c** 0.78

6 **a**



b $\frac{3}{7}$

c Adding together the probabilities on the 4 branches of the tree diagram where the counter from box B is blue: $\frac{12}{98} + \frac{16}{98} + \frac{24}{147} + \frac{15}{147} = \frac{27}{49}$

d Adding together the probabilities on the 2 branches of the tree diagram where events C and D both occur. $\frac{12}{98} + \frac{15}{147} = \frac{11}{49}$

e $\frac{37}{49}$ **f** $\frac{8}{13}$

Statistics & Mechanics 4. Distributions

4 **Statistical distributions**	4.1	**Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.**	**Students will be expected to use distributions to model a real-world situation and to comment critically on the appropriateness.** **Students should know and be able to identify the discrete uniform distribution.** **The notation $X \sim \mathrm{B}(n, p)$ may be used.** **Use of a calculator to find individual or cumulative binomial probabilities.**
	4.2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution	The notation $X \sim \mathrm{N}(\mu, \sigma^2)$ may be used. Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Questions may involve the solution of simultaneous equations. Students will be expected to use their calculator to find probabilities connected with the normal distribution.
		Link to histograms, mean, standard deviation, points of inflection	Students should know that the points of inflection on the normal curve are at $x = \mu \pm \sigma$. The derivation of this result is not expected.
		and the binomial distribution.	Students should know that when n is large and p is close to 0.5 the distribution $\mathrm{B}(n, p)$ can be approximated by $\mathrm{N}(np, np[1 - p])$ The application of a continuity correction is expected.

4 **Statistical distributions** *continued*	4.3	Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate.	Students should know under what conditions a binomial distribution or a Normal distribution might be a suitable model.

Statistics & Mechanics 4. Distributions

E/P 1 The random variable X has probability function

$$P(X = x) = \frac{(3x - 1)}{26} \qquad x = 1, 2, 3, 4.$$

a Construct a table giving the probability distribution of X. **(2 marks)**

b Find $P(2 < X \leqslant 4)$. **(2 marks)**

P 2 Records kept in a hospital show that 3 out of every 10 patients who visit the accident and emergency department have to wait more than half an hour. Find, to 3 decimal places, the probability that of the first 12 patients who come to the accident and emergency department:

a none

b more than 2

will have to wait more than half an hour.

P 3 A completely unprepared student is given a true/false-type test with 10 questions. Assuming that the student answers all the questions at random:

a find the probability that the student gets all the answers correct.

It is decided that a pass will be awarded for 8 or more correct answers.

b Find the probability that the student passes the test.

E/P 4 The time a mobile phone battery lasts before needing to be recharged is assumed to be normally distributed with a mean of 48 hours and a standard deviation of 8 hours.

a Find the probability that a battery will last for more than 60 hours. **(2 marks)**

b Find the probability that the battery lasts less than 35 hours. **(1 mark)**

A random sample of 30 phone batteries is taken.

c Find the probability that 3 or fewer last less than 35 hours. **(2 marks)**

E 5 The owner of a local corner shop calculates that the probability of a customer buying a newspaper is 0.40.

A random sample of 100 customers is recorded.

a Give two reasons why a normal approximation may be used in this situation. **(2 marks)**

b Write down the parameters of the normal distribution used. **(2 marks)**

c Use this approximation to estimate the probability that at least half the customers bought a newspaper. **(2 marks)**

E/P 6 A herbalist claims that a particular remedy is successful in curing a particular disease in 52% of cases.

A random sample of 25 people who took the remedy is taken.

a Find the probability that more than 12 people in the sample were cured. **(2 marks)**

A second random sample of 300 people was taken and 170 were cured.

b Assuming the herbalist's claim is true, use a suitable approximation to find the probability that at least 170 people were cured. **(4 marks)**

c Using your answer to part **b**, comment on the herbalist's claim. **(1 mark)**

Statistics & Mechanics 4. Distributions

Answers

1 **a**

x	1	2	3	4
$P(X = x)$	0.0769	0.1923	0.3077	0.4231

b $\frac{19}{26}$

2 **a** 0.014 (3 d.p.) **b** 0.747 (3 d.p.)

3 **a** 0.000977 **b** 0.0547

4 **a** 0.0668 **b** 0.0521 **c** 0.9315

5 **a** n is large and p is close to 0.5.
b $\mu = 40, \sigma^2 = 24$
c 0.0262

6 **a** 0.5801 **b** 0.0594
c Assuming that the claim is correct, there is a greater than 5% chance that 170 people out of 300 would be cured, therefore there is insufficient evidence to reject the herbalist's claim.

Statistics & Mechanics 5. Hypothesis Testing

5 **Statistical hypothesis testing**	5.1	**Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value;** extend to correlation coefficients as measures of how close data points lie to a straight line. and be able to interpret a given correlation coefficient using a given *p*-value or critical value (calculation of correlation coefficients is excluded).	**An informal appreciation that the expected value of a binomial distribution is given by *np* may be required for a 2-tail test.** Students should know that the product moment correlation coefficient r satisfies $\lvert r\rvert \leq 1$ and that a value of $r = \pm 1$ means the data points all lie on a straight line. Students will be expected to calculate a value of r using their calculator but use of the formula is not required. Hypotheses should be stated in terms of ρ with a null hypothesis of $\rho = 0$ where ρ represents the population correlation coefficient. Tables of critical values or a *p*-value will be given.
5 **Statistical hypothesis testing** *continued*	5.2	**Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.**	
		Understand that a sample is being used to make an inference about the population. **and** **appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.**	**Hypotheses should be expressed in terms of the population parameter *p*** **A formal understanding of Type I errors is not expected.**
	5.3	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context.	Students should know that: If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ and that a test for μ can be carried out using: $\frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1^2)$. No proofs required. Hypotheses should be stated in terms of the population mean μ. Knowledge of the Central Limit Theorem or other large sample approximations is not required.

Statistics & Mechanics 5. Hypothesis Testing

E/P 1 The manager of a superstore thinks that the probability of a person buying a certain make of computer is only 0.2.
To test whether this hypothesis is true the manager decides to record the make of computer bought by a random sample of 50 people who bought a computer.

a Find the critical region that would enable the manager to test whether or not there is evidence that the probability is different from 0.2. The probability of each tail should be as close to 2.5% as possible. **(4 marks)**

b Write down the significance level of this test. **(2 marks)**

15 people buy that certain make.

c Comment on this observation in light of your critical region. **(2 marks)**

E/P 2 A pharmaceutical company claims that 85% of patients suffering from a chronic rash recover when treated with a new ointment.

A random sample of 20 patients with this rash is taken from hospital records.

a Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new ointment. **(2 marks)**

b Given that the claim is correct, find the probability that the ointment will be successful for exactly 16 patients. **(2 marks)**

The hospital believes that the claim is incorrect and the percentage who will recover is lower. From the records an administrator took a random sample of 30 patients who had been prescribed the ointment. She found that 20 had recovered.

c Stating your hypotheses clearly, test, at the 5% level of significance, the hospital's belief. **(6 marks)**

E 3 As part of a survey in a particular profession, age, x years, and yearly salary, £y thousands, were recorded. The values of x and y for a randomly selected sample of ten members of the profession are as follows:

x	30	52	38	48	56	44	41	25	32	27
y	22	38	40	34	35	32	28	27	29	41

a Calculate, to 3 decimal places, the product moment correlation coefficient between age and salary. **(1 mark)**

It is suggested that there is no correlation between age and salary.

b Test this suggestion at the 5% significance level, stating your null and alternative hypotheses clearly. **(3 marks)**

Statistics & Mechanics 5. Hypothesis Testing

E 4 A meteorologist believes that there is a positive correlation between daily mean windspeed and daily maximum gust. She collects data from the large data set for 5 days during August 2015 in the town of Hurn.

Mean windspeed (knots)	4	7	7	8	5
Daily maximum gust (knots)	14	22	18	20	17

By calculating the product moment correlation coefficient for these data, test at the 5% level of significance whether there is evidence to support the meteorologist's claim. State your hypotheses clearly. **(4 marks)**

E 5 Climbing rope produced by a manufacturer is known to be such that one-metre lengths have breaking strengths that are normally distributed with mean 170.2 kg and standard deviation 10.5 kg. Find, to 3 decimal places, the probability that:

a a one-metre length of rope chosen at random from those produced by the manufacturer will have a breaking strength of 175 kg to the nearest kg **(2 marks)**

b a random sample of 50 one-metre lengths will have a mean breaking strength of more than 172.4 kg. **(3 marks)**

A new component material is added to the ropes being produced. The manufacturer believes that this will increase the mean breaking strength without changing the standard deviation. A random sample of 50 one-metre lengths of the new rope is found to have a mean breaking strength of 172.4 kg.

c Perform a significance test at the 5% level to decide whether this result provides sufficient evidence to confirm the manufacturer's belief that the mean breaking strength is increased. State clearly the null and alternative hypotheses that you are using. **(3 marks)**

E/P 6 The random variable X is normally distributed with mean μ and variance σ^2.

a Write down the distribution of the sample mean $\overline{X}$ of a random sample of size n. **(1 mark)**

A construction company wishes to determine the mean time taken to drill a fixed number of holes in a metal sheet.

b Determine how large a random sample is needed so that the expert can be 95% certain that the sample mean time will differ from the true mean time by less than 15 seconds. Assume that it is known from previous studies that $\sigma = 40$ seconds. **(4 marks)**

Statistics & Mechanics 5. Hypothesis Testing

Answers

1 **a** Critical region is $X \leqslant 4$ and $X \geqslant 16$
b 0.0493
c There is insufficient evidence to reject H_0.
There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

2 **a** $X \sim B(20, 0.85)$
b 0.1821
c Test statistic is proportion of patients who recover.
H_0: $p = 0.85$, H_1: $p < 0.85$
$P(X \leqslant 20) = 0.00966$
$0.00966 < 0.05$ so there is enough evidence to reject H_0. The percentage of patients who recover after treatment with the new ointment is lower than 85%.

3 **a** 0.340 (3 d.p.)
b H_0: $\rho = 0$, H_1: $\rho \neq 0$, critical value = ±0.6319. Accept H_0. There is not enough evidence that there is a correlation between age and salary.

4 $r = 0.843$ (3 s.f.), H_0: $\rho = 0$, H_1: $\rho > 0$, critical value 0.8054.
Reject H_0. There is evidence that mean windspeed and daily maximum gust are positively correlated.

5 **a** 0.0342 **b** Accept $\leqslant 0.069$
c Test statistic = 1.4815... < 1.6449
Not significant so accept H_0. Insufficient evidence of an increase in the mean breaking strength of climbing rope.

6 **a** $X \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ **b** Need $n = 28$ or more

Statistics & Mechanics 6. Units (Modelling)

6 **Quantities and units in mechanics**	6.1	**Understand and use fundamental quantities and units in the S.I. system: length, time, mass.** **Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.**	Students may be required to convert one unit into another e.g. km h^{-1} into m s^{-1}

(P) 1 A diver dives from a diving board into a swimming pool with a depth of 4.5 m. The height of the diver above the water, h m, can be modelled using $h = 10 - 0.58x^2$ for $0 \leqslant x \leqslant 5$, where x m is the horizontal distance from the end of the diving board.

a Find the height of the diver when $x = 2$ m.

b Find the horizontal distance from the end of the diving board to the point where the diver enters the water.

In this model the diver is modelled as particle.

c Describe the effects of this modelling assumption.

d Comment on the validity of this modelling assumption for the motion of the diver after she enters the water.

2 A man throws a bowling ball in a bowling alley.

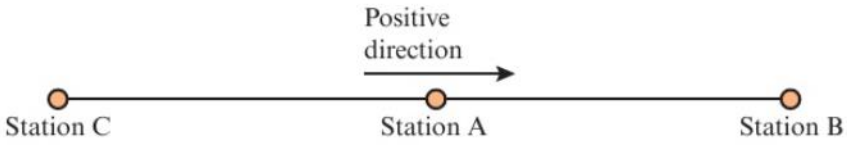
a Make a list of the assumptions you might make to create a simple model of the motion of the bowling ball.

b Taking the direction that the ball travels in as the positive direction, state with a reason whether each of the following are likely to be positive or negative:

i the velocity **ii** the acceleration.

Statistics & Mechanics 6. Units (Modelling)

3 A train engine pulling a truck starts at station A then travels in a straight line to station B. It then moves back from station B to station A and on to station C as shown in the diagram.



Hint The **sign** of something means whether it is positive or negative.

What is the sign of the velocity and displacement on the journey from:

a station A to station B **b** station B to station A **c** station A to station C?

4 The acceleration of a boat is given by $\mathbf{a} = -0.05\mathbf{i} + 0.15\mathbf{j}\,\text{m s}^{-2}$. Find:

a the magnitude of the acceleration

b the angle the direction of the acceleration vector makes with the unit vector $\mathbf{i}$.

5 The velocity of a toy car is given by $\mathbf{v} = 3.5\mathbf{i} - 2.5\mathbf{j}\,\text{m s}^{-1}$. Find:

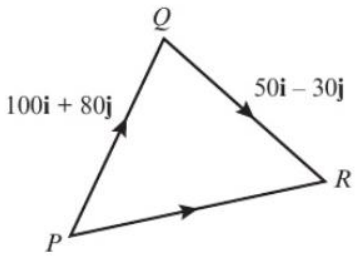
a the speed of the toy car

b the angle the direction of motion of the toy car makes with the unit vector $\mathbf{j}$.

6 A plane flies from P to Q and then from Q to R.

The displacement from P to Q is $100\mathbf{i} + 80\mathbf{j}$ m.

The displacement from Q to R is $50\mathbf{i} - 30\mathbf{j}$ m.



a Find the magnitude of the displacement from P to R.

b Find the total distance the plane has travelled in getting from P to R.

c Find the angle the vector $\overrightarrow{PQ}$ makes with the unit vector $\mathbf{j}$.

Statistics & Mechanics 6. Units (Modelling)

Answers

1 **a** 7.68 m **b** 4.15 m
c Ignore the effects of air resistance on the diver and rotational effects of external forces.
d Assumption not valid, diver experiences drag and buoyancy in the water.

2 **a** Model ball as a particle. Assume the floor is smooth.
b **i** Positive – the positive direction is defined as the direction in which the ball is travelling.
ii Negative – the ball will be slowing down.

3 **a** Velocity is positive, displacement is positive
b Velocity is negative, displacement is positive
c Velocity is negative, displacement is negative

4 **a** $0.158\,\text{ms}^{-2}$ **b** 108.4°

5 **a** $4.3\,\text{ms}^{-1}$ **b** 125.5°

6 **a** 158.1 m **b** 186.4 m **c** 51.3°

Statistics & Mechanics 7. Kinematics

7 **Kinematics**	7.1	**Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.**	Students should know that distance and speed must be positive.
	7.2	**Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.**	**Graphical solutions to problems may be required.**
	7.3	**Understand, use and derive the formulae for constant acceleration for motion in a straight line.** Extend to 2 dimensions using vectors.	**Derivation may use knowledge of sections 7.2 and/or 7.4** **Understand and use *suvat* formulae for constant acceleration in 2-D,** e.g. $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with vectors given in $\mathbf{i} - \mathbf{j}$ or column vector form. Use vectors to solve problems.
	7.4	**Use calculus in kinematics for motion in a straight line:** $v = \frac{dr}{dt}, a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$ $r = \int v \ dt$, $v = \int a \ dt$ Extend to 2 dimensions using vectors.	**The level of calculus required will be consistent with that in Sections 7 and 8 in Paper 1** and Sections 6 and 7 in Paper 2. Differentiation and integration of a vector with respect to time. e.g. Given $\mathbf{r} = t^2\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.
	7.5	Model motion under gravity in a vertical plane using vectors; projectiles.	Derivation of formulae for time of flight, range and greatest height and the derivation of the equation of the path of a projectile may be required.

Statistics & Mechanics 7. Kinematics

E/P 1 A train starts from rest at station A and accelerates uniformly at $3x\ \text{m s}^{-2}$ until it reaches a velocity of $30\ \text{m s}^{-1}$. For the next T seconds the train maintains this constant velocity. The train then decelerates uniformly at $x\ \text{m s}^{-2}$ until it comes to rest at a station B. The distance between the stations is 6 km and the time taken from A to B is 5 minutes.

a Sketch a velocity–time graph to illustrate this journey. **(2 marks)**

b Show that $\frac{40}{x} + T = 300$. **(4 marks)**

c Find the value of T and the value of x. **(2 marks)**

d Calculate the distance the train travels at constant velocity. **(2 marks)**

e Calculate the time taken from leaving A until reaching the point halfway between the stations. **(3 marks)**

P 2 At a time t seconds after launch, the space shuttle can be modelled as a particle moving in a straight line with acceleration, $a\ \text{m s}^{-2}$, given by the equation:

$$a = (6.77 \times 10^{-7})t^3 - (3.98 \times 10^{-4})t^2 + 0.105t + 0.859, \quad 124 \leqslant t \leqslant 446$$

a Suggest two reasons why the space shuttle might experience variable acceleration during its launch phase.

Given that the velocity of the space shuttle at time $t = 124$ is $974\ \text{m s}^{-1}$:

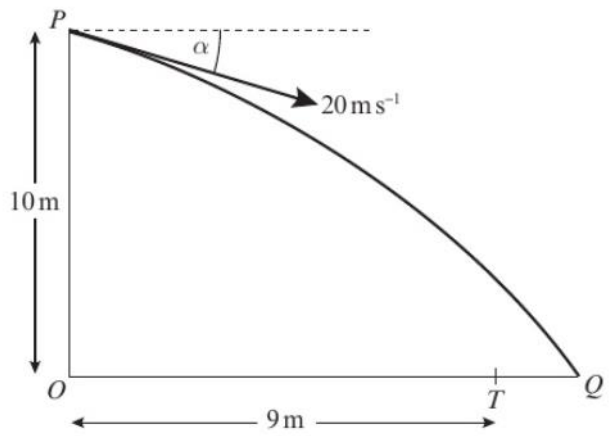
b find an expression for the velocity $v\ \text{m s}^{-1}$ of the space shuttle at time t. Give your coefficients to 3 significant figures.

c Hence find the velocity of the space shuttle at time $t = 446$, correct to 3 s.f.

From $t = 446$, the space shuttle maintains a constant acceleration of $28.6\ \text{m s}^{-2}$ until it reaches its escape velocity of $7.85\ \text{km s}^{-1}$. It then cuts its main engines.

d Calculate the time at which the space shuttle cuts its main engines.

E/P 3 In this question use $g = 10\ \text{m s}^{-2}$.
A stone is thrown from a point P at a target, which is on horizontal ground. The point P is 10 m above the point O on the ground. The stone is thrown from P with speed $20\ \text{m s}^{-1}$ at an angle of α below the horizontal, where $\tan\alpha = \frac{3}{4}$.



The stone is modelled as a particle and the target as a point T. The distance OT is 9 m. The stone misses the target and hits the ground at the point Q, where OTQ is a straight line, as shown in the diagram. Find:

a the time taken by the ball to travel from P to Q **(5 marks)**

b the distance TQ. **(4 marks)**

The point A is on the path of the ball vertically above T.

c Find the speed of the ball at A. **(5 marks)**

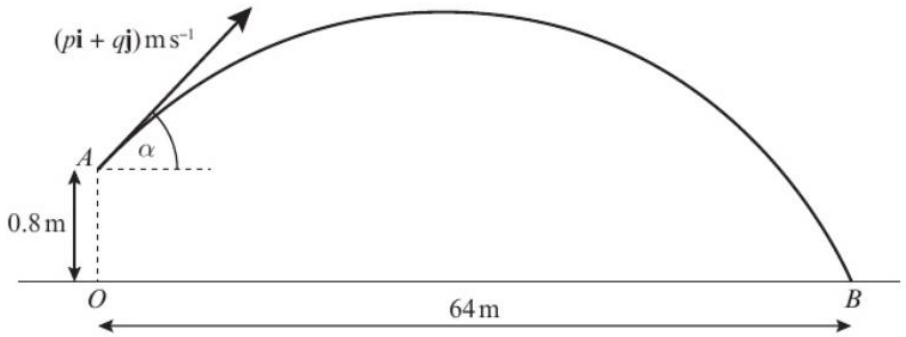
Statistics & Mechanics 7. Kinematics

E/P 4 A vertical mast is 32 m high. Two balls P and Q are projected simultaneously. Ball P is projected horizontally from the top of the mast with speed $18\,\text{m s}^{-1}$. Ball Q is projected from the bottom of the mast with speed $30\,\text{m s}^{-1}$ at an angle α above the horizontal. The balls move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each ball,

a prove that $\cos\alpha = \frac{3}{5}$ **(4 marks)**

b Find the time which elapses between the instant when the balls are projected and the instant when they collide. **(4 marks)**

E/P 5



A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})\,\text{m s}^{-1}$, at an angle α above the horizontal. **i** and **j** are the unit vectors horizontally and vertically upwards respectively. The point A is 0.8 m vertically above the point O, which is on horizontal ground.

The ball takes 4 seconds to travel from A to B, where B is on the ground and $OB = 64$ m, as shown in the diagram. By modelling the motion of the ball as that of a particle moving freely under gravity,

a find the value of p and the value of q **(5 marks)**

b find the initial speed of the ball **(2 marks)**

c find the exact value of $\tan\alpha$ **(1 mark)**

d find the length of time for which the cricket ball is at least 5 m above the ground. **(6 marks)**

e State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic. **(1 mark)**

E/P 6 In this question **i** and **j** are horizontal unit vectors due east and due north respectively.

A clockwork train is moving on a flat, horizontal floor. At time $t = 0$, the train is at a fixed point O and is moving with velocity $3\mathbf{i} + 13\mathbf{j}\,\text{m s}^{-1}$. The velocity of the train at time t seconds is $\mathbf{v}\,\text{m s}^{-1}$, and its acceleration, $\mathbf{a}\,\text{m s}^{-2}$, is given by $\mathbf{a} = 2t\mathbf{i} + 3\mathbf{j}$.

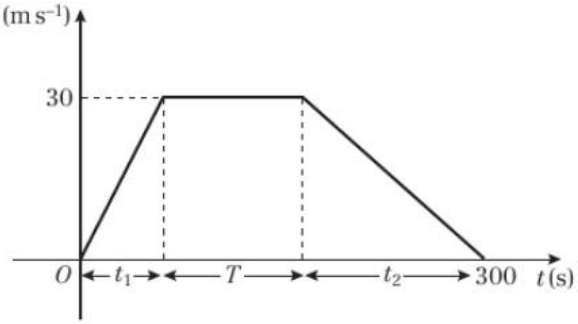
a Find $\mathbf{v}$ in terms of t. **(3 marks)**

b Find the value of t when the train is moving in a north-east direction. **(3 marks)**

Statistics & Mechanics 7. Kinematics

Answers

1 **a**



b $\frac{30}{t_1} = 3x \Rightarrow t_1 = \frac{1}{x}, \frac{-30}{t_2} = -x \Rightarrow t_2 = \frac{30}{x}$

So $\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300$

c $T = 100, x = 0.2$ **d** 3 km **e** 125 s

2 **a** Mass is not constant as fuel is used.
Gravity is not constant so weight not constant.
Thrust may not be constant.

b $v = (1.69 \times 10^{-7})\, t^4 - (1.33 \times 10^{-4})\, t^3 + 0.0525\, t^2 + 0.859\, t + 273 \text{ m s}^{-1}$

c $v = 5990 \text{ m s}^{-1}$

d 510 seconds (2 s.f.) after launch

3 **a** 0.65 s **b** 1.5 m **c** 23.8 m s^{-1}

4 **a** Particle P: $x = 18t$, Particle Q: $x = 30 \cos \alpha \times t$
When particles collide: $18t = 30 \cos \alpha \times t \Rightarrow \cos\alpha = \frac{3}{5}$

b $\frac{4}{3}$ s

5 **a** $p = 16, q = 19.4$ **b** 25.1 m s^{-1}

c $\frac{97}{80}$ **d** 3.50 s (3 s.f.)

e e.g. weight of the ball, air resistance

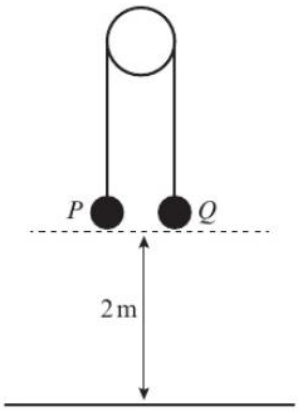
6 **a** $\mathbf{v} = (t^2 + 3)\mathbf{i} + (3t + 13)\mathbf{j}$ **b** 5 s

Statistics & Mechanics 8. Forces

8 **Forces and Newton's laws**	8.1	**Understand the concept of a force; understand and use Newton's first law.**	**Normal reaction, tension, thrust or compression, resistance.**
	8.2	**Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors);** extend to situations where forces need to be resolved (restricted to 2 dimensions).	**Problems will involve motion in a straight line with constant acceleration in scalar form, where the forces act either parallel or perpendicular to the motion.** Extend to problems where forces need to be resolved, e.g. a particle moving on an inclined plane. **Problems may involve motion in a straight line with constant acceleration in vector form, where the forces are given in i – j form or as column vectors.**
	8.3	**Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy.**	**The default value of g will be 9.8 m s^{-2} but some questions may specify another value, e.g. g = 10 m s^{-2}** **The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location.**
	8.4	**Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles;** resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.	**Problems may be set where forces need to be resolved (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).** **Connected particle problems could include problems with particles in contact e.g. lift problems.** Problems may be set where forces need to be resolved, e.g. at least one of the particles is moving on an inclined plane.
	8.5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane.	Students may be required to resolve a vector into two components or use a vector diagram, e.g. problems involving two or more forces, given in magnitude-direction form.
8 **Forces and Newton's laws** *continued*	8.6	Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.	An understanding of $F = \mu R$ when a particle is moving. An understanding of $F \leq \mu R$ in a situation of equilibrium.
9 **Moments**	9.1	Understand and use moments in simple static contexts.	Equilibrium of rigid bodies. Problems involving parallel and non-parallel coplanar forces, e.g. ladder problems.

Statistics & Mechanics 8. Forces

E/P 1 Two particles P and Q have masses 0.5 kg and 0.4 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 2 m above the floor, as shown. The particles are released from rest and in the subsequent motion Q does not reach the pulley.



a i Write down an equation of motion for P. **(2 marks)**

ii Write down an equation of motion for Q. **(2 marks)**

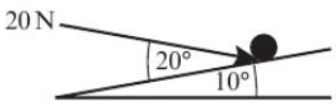
b Find the tension in the string immediately after the particles are released. **(2 marks)**

c Find the acceleration of A immediately after the particles are released. **(2 marks)**

When the particles have been moving for 0.2 s, the string breaks.

d Find the further time that elapses until Q hits the floor. **(9 marks)**

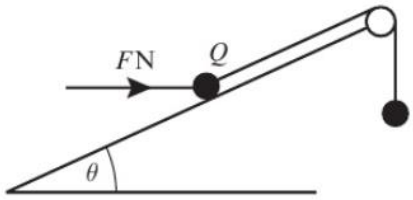
E 2 A particle of mass 5 kg sits on a smooth slope that is inclined at 10° to the horizontal. A force of 20 N acts on the particle at an angle of 20° to the plane, as shown in the diagram. Find the acceleration of the particle. **(5 marks)**



E/P 3 A particle of mass 0.5 kg is being pulled up a rough slope that is angled at 30° to the horizontal by a force of 5 N. The force acts at an angle of 30° above the slope. Given that the coefficient of friction is 0.1, calculate the acceleration of the particle. **(7 marks)**

Statistics & Mechanics 8. Forces

E/P 4 A particle Q of mass 5 kg rests in equilibrium on a smooth inclined plane. The plane makes an angle θ with the horizontal, where $\tan\theta = \frac{3}{4}$.



Q is attached to one end of a light inextensible string which passes over a smooth pulley as shown. The other end of the string is attached to a particle of mass 2 kg.

The particle Q is also acted upon by a force of magnitude F N acting horizontally, as shown in the diagram.

Find the magnitude of:

a the force F **(5 marks)**

b the normal reaction between particle Q and the plane. **(3 marks)**

The plane is now assumed to be rough.

c State, with a reason, which of the following statements is true:

1. F will be larger 2. F will be smaller 3. F could be either larger or smaller. **(2 marks)**

E 5 A sledge of mass 50 kg sits on a snowy hill that is angled at 40° to the horizontal. The sledge is held in place by a rope that is angled at 30° above the line of greatest slope of the hill.

a By modelling the sledge as a particle, the hill as a smooth slope and the rope as a light inextensible string, work out the tension in the rope. **(4 marks)**

b Give one criticism of this model. **(1 mark)**

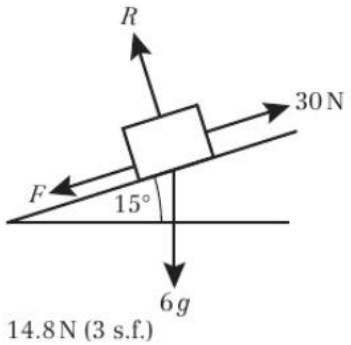
E/P 6 A trailer of mass 20 kg sits at rest on a rough horizontal plane. A force of 20 N pulls the trailer at an angle of 30° above the horizontal. Given that the trailer is in limiting equilibrium, work out the value of the coefficient of friction. **(6 marks)**

Statistics & Mechanics 8. Forces

Answers

1 **a** **i** $0.5g - T = 0.5a$ **ii** $T - 0.4g = 0.4a$
b $\frac{4}{9}g$N **c** $\frac{1}{9}g$ms^{-2} **d** 0.66s (2 s.f.)

2 **a**



b 14.8N (3 s.f.)

3 3.41 ms^{-2} (3 s.f.)

4 **a** 12.25N **b** 46.6N (3 s.f.)
c F will be smaller as friction is acting up the slope.

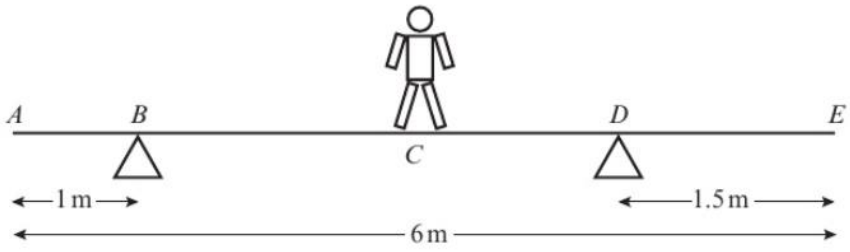
5 104N, 64.5N, 0.620

6 $\mu = \frac{5\sqrt{3}}{93}$

Statistics & Mechanics 9. Moments

9 **Moments**	9.1	Understand and use moments in simple static contexts.	Equilibrium of rigid bodies. Problems involving parallel and non-parallel coplanar forces, e.g. ladder problems.

E 1 A plank AE, of length 6 m and weight 100 N, rests in a horizontal position on supports at B and D, where $AB = 1$ m and $DE = 1.5$ m. A child of weight 145 N stands at C, the midpoint of AE, as shown in the diagram. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate:



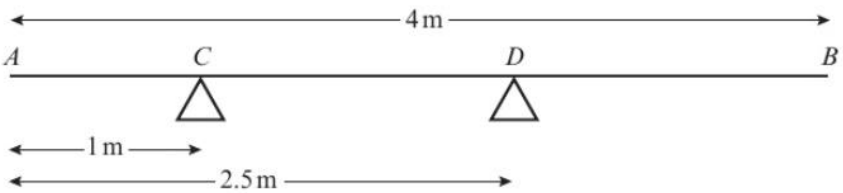
a the magnitude of the force exerted by the support on the plank at B **(3 marks)**

b the magnitude of the force exerted by the support on the plank at D. **(2 marks)**

The child now stands at a different point F on the plank. The plank is in equilibrium and on the point of tilting about D.

c Calculate the distance DF. **(4 marks)**

E/P 2 A uniform rod AB has length 4 m and weight 150 N. The rod rests in equilibrium in a horizontal position, smoothly supported at points C and D, where $AC = 1$ m and $AD = 2.5$ m as shown in the diagram. A particle of weight W N is attached to the rod at a point E where $AE = x$ metres. The rod remains in equilibrium and the magnitude of the reaction at C is now equal to the magnitude of the reaction at D.

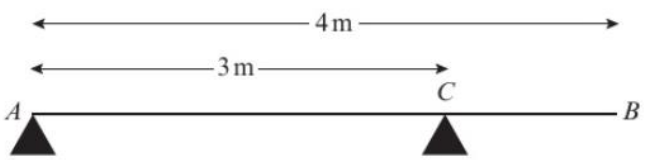


a Show that $W = \dfrac{150}{7 - 4x}$ **(6 marks)**

b Hence deduce the range of possible values of x. **(3 marks)**

Statistics & Mechanics 9. Moments

E 3 A uniform plank AB has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots. One pivot is at the end A and the other is at the point C where $AC = 3$ m, as shown in the diagram.



A man of mass 80 kg stands on the plank which remains in equilibrium. The magnitude of the reaction at A is twice the magnitude of the reaction at C. The magnitude of the reaction at C is R N. The plank is modelled as a rod and the man is modelled as a particle.

a Find the value of R. **(2 marks)**

b Find the distance of the man from A. **(3 marks)**

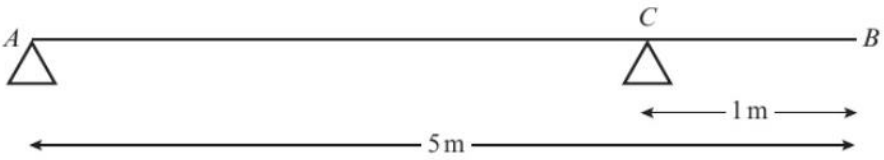
c State how you have used the modelling assumption that:

i the plank is uniform

ii the plank is a rod

iii the man is a particle. **(3 marks)**

E 4 A beam AB has weight 200 N and length 5 m. The beam rests in equilibrium in a horizontal position on two smooth supports.



One support is at end A and the other is at a point C on the beam, where $BC = 1$ m, as shown in the diagram. The beam is modelled as a uniform rod.

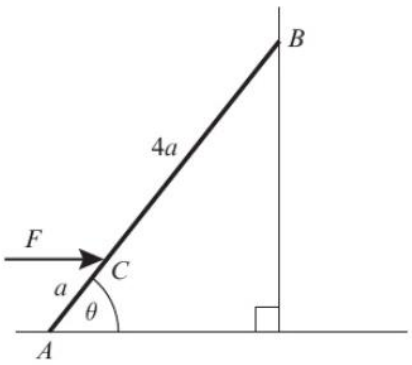
a Find the reaction on the beam at C. **(4 marks)**

A woman of weight 500 N stands on the beam at the point D. The beam remains in equilibrium. The reactions on the beam at A and C are now equal.

b Find the distance AD. **(5 marks)**

Statistics & Mechanics 9. Moments

E/P 5 A uniform ladder AB has one end A on smooth horizontal ground. The other end B rests against a smooth vertical wall. The ladder is modelled as a uniform rod of mass m and length $5a$. The ladder is kept in equilibrium by a horizontal force F acting at a point C of the ladder where $AC = a$. The force F and the ladder lie in a vertical plane perpendicular to the wall. The ladder is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{9}{5}$, as shown in the diagram.



Show that $F = \frac{25mg}{72}$. **(8 marks)**

E/P 6 A uniform ladder, of weight W and length 5 m, has one end on rough horizontal ground and the other touching a smooth vertical wall. The coefficient of friction between the ladder and the ground is 0.3.

The top of the ladder touches the wall at a point 4 m vertically above the level of the ground.

a Show that the ladder cannot rest in equilibrium in this position. **(6 marks)**

In order to enable the ladder to rest in equilibrium in the position described above, a brick is attached to the bottom of the ladder.

Assuming that this brick is at the lowest point of the ladder, but not touching the ground,

b show that the horizontal frictional force exerted by the ladder on the ground is independent of the mass of the brick **(4 marks)**

c find, in terms of W and g, the smallest mass of the brick for which the ladder will rest in equilibrium. **(3 marks)**

Statistics & Mechanics 9. Moments

Answers

1 **a** 105 N **b** 140 N
c 1.03 m to the right of D

2 **a** R(↑) gives reaction at C = reaction at $D = \frac{150 + W}{2}$
M(C): $(1 \times 150) + W(x - 1) = 1.5\left(\frac{150 + W}{2}\right)$
$150 + Wx - W = 112.5 + 0.75W$
$37.5 = 1.75W - Wx \Rightarrow 150 = 7W - 4Wx$
$W = \frac{150}{7 - 4x}$
b $0 \leqslant x < \frac{7}{4}$

3 **a** 40 g **b** $x = \frac{1}{2}$
c **i** The weight acts at the centre of the plank.
ii The plank remains straight.
iii The man's weight acts at a single point.

4 **a** 125 N **b** 1.8 m

5 R(→): $F = N$
Taking moments about A
$Fa\sin\theta + \frac{5}{2}mga\cos\theta = 5aN\sin\theta$
$\frac{5}{2}mg\cos\theta = 4F\sin\theta$
$\frac{5}{8}mg = F\tan\theta$

6 **a** Taking moments about point where ladder touches the ground
R(↑): $R = W$, R(→): $N = 0.3R$
$1.5W = 1.2W$. This cannot be true so the ladder cannot rest in this position.
b R(→): $F = N$
Taking moments about point where ladder touches the ground $1.5W = 4N$, $F = N = \frac{3W}{8}$
c $\frac{W}{4g}$

Extra Questions

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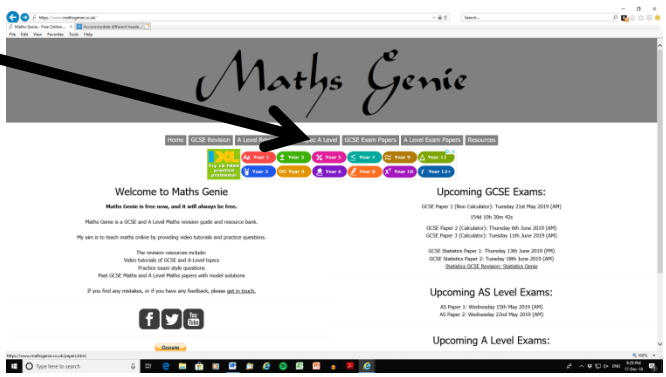

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Forces 17	Videos	2D Vectors F = ma

A Level Pure Mathematics

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Sequences and Series 4	Videos	Recurrence Relations Arithmetic Sequences and Series Geometric Sequences and Series The Binomial Expansion
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Moments	18	Videos	Moments Statics of Rigid Bodies