edexcel

A Level Mathematics

Revision

Contents

1. Proof

Topics	What students need to learn:		
	Content		Guidance
1 **Proof**	1.1	**Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including:**	**Examples of proofs:**
		Proof by deduction	**Proof by deduction** **e.g. using completion of the square, prove that $n^2 - 6n + 10$ is positive for all values of n or, for example, differentiation from first principles for small positive integer powers of x** or proving results for arithmetic and geometric series. **This is the most commonly used method of proof throughout this specification**
		Proof by exhaustion	**Proof by exhaustion** **This involves trying all the options. Suppose x and y are odd integers less than 7. Prove that their sum is divisible by 2.**
		Disproof by counter example	**Disproof by counter example** **e.g. show that the statement "$n^2 - n + 1$ is a prime number for all values of n" is untrue**
		Proof by contradiction (including proof of the irrationality of √2 and the infinity of primes, and application to unfamiliar proofs).	

1. Proof

1 Prove that $\frac{x-y}{\sqrt{x}-\sqrt{y}} \equiv \sqrt{x}+\sqrt{y}$.

P 2 Prove that the sum of two consecutive positive odd numbers less than ten gives an even number.

P 3 Prove that the statement '$n^2 - n + 3$ is a prime number for all values of n' is untrue.

E/P 4 Prove by contradiction that $\sqrt{\frac{1}{2}}$ is an irrational number. **(5 marks)**

P 5 Prove that if q^2 is an irrational number then q is an irrational number.

P 6 Use proof by contradiction to show that there exist no integers a and b for which $21a + 14b = 1$.

Hint Assume the opposite is true, and then divide both sides by the highest common factor of 21 and 14.

1. Proof

Answers

1 $\frac{x-y}{(\sqrt{x}-\sqrt{y})} \times \frac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})} = \frac{(x-y)(\sqrt{x}+\sqrt{y})}{x-y} = \sqrt{x}+\sqrt{y}$

2 $1 + 3 =$ even, $3 + 5 =$ even, $5 + 7 =$ even, $7 + 9 =$ even

3 For example when $n = 6$

4 Assume $\sqrt{\frac{1}{2}}$ is a rational number.

Then $\sqrt{\frac{1}{2}} = \frac{a}{b}$ for some integers a and b.

Further assume that this fraction is in its simplest terms: there are no common factors between a and b.

So $0.5 = \frac{a^2}{b^2}$ or $2a^2 = b^2$.

Therefore b^2 must be a multiple of 2.
We know that this means b must also be a multiple of 2.
Write $b = 2c$, which means $b^2 = (2c)^2 = 4c^2$.
Now $4c^2 = 2a^2$, or $2c^2 = a^2$.
Therefore a^2 must be a multiple of 2, which implies a is also a multiple of 2.
If a and b are both multiples of 2, this contradicts the statement that there are no common factors between a and b.
Therefore, $\sqrt{\frac{1}{2}}$ is an irrational number.

5 Assume there exists a rational number q such that q^2 is irrational.

So write $q = \frac{a}{b}$ where a and b are integers.

$q^2 = \frac{a^2}{b^2}$

As a and b are integers a^2 and b^2 are integers.
So q^2 is rational.

6 Assumption: there exists integers a and b such that $21a + 14b = 1$.
Since 21 and 14 are multiples of 7, divide both sides by 7.
So now $3a + 2b = \frac{1}{7}$
$3a$ is also an integer. $2b$ is also an integer.
The sum of two integers will always be an integer, so $3a + 2b =$ 'an integer'.
This contradicts the statement that $3a + 2b = \frac{1}{7}$.
Therefore there exists no integers a and b for which $21a + 14b = 1$.

2. Algebra and Functions

Topics	What students need to learn:		
	Content		Guidance
2 **Algebra and functions**	2.1	Understand and use the laws of indices for all rational exponents.	$a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$ The equivalence of $a^{\frac{m}{n}}$ and $\sqrt[n]{a^m}$ should be known.
	2.2	Use and manipulate surds, including rationalising the denominator.	Students should be able to simplify algebraic surds using the results $\left(\sqrt{x}\right)^2 = x$, $\sqrt{xy} = \sqrt{x}\sqrt{y}$ and $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right) = x - y$
	2.3	Work with quadratic functions and their graphs. The discriminant of a quadratic function, including the conditions for real and repeated roots. Completing the square. Solution of quadratic equations including solving quadratic equations in a function of the unknown.	The notation f(x) may be used Need to know and to use $b^2 - 4ac > 0$, $b^2 - 4ac = 0$ and $b^2 - 4ac < 0$ $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ Solution of quadratic equations by factorisation, use of the formula, use of a calculator and completing the square. These functions could include powers of x, trigonometric functions of x, exponential and logarithmic functions of x.
	2.4	Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.	The quadratic may involve powers of 2 in one unknown or in both unknowns, e.g. solve $y = 2x + 3$, $y = x^2 - 4x + 8$ or $2x - 3y = 6$, $x^2 - y^2 + 3x = 50$

2. Algebra and Functions

Topics	What students need to learn:		
	Content		Guidance
2 **Algebra and functions** *continued*	2.5	**Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically,**	**e.g. solving** $ax + b > cx + d$, $px^2 + qx + r \geq 0$, $px^2 + qx + r < ax + b$ **and interpreting the third inequality as the range of x for which the curve $y = px^2 + qx + r$ is below the line with equation $y = ax + b$**
		including inequalities with brackets and fractions.	**These would be reducible to linear or quadratic inequalities** **e.g.** $\frac{a}{x} < b$ **becomes** $ax < bx^2$
		Express solutions through correct use of 'and' and 'or', or through set notation.	**So, e.g.** $x < a$ **or** $x > b$ **is equivalent to** $\{ x : x < a \} \cup \{ x : x > b \}$ **and** $\{ x : c < x \} \cap \{ x : x < d \}$ **is equivalent to** $x > c$ **and** $x < d$
		Represent linear and quadratic inequalities such as $y > x + 1$ **and** $y > ax^2 + bx + c$ **graphically.**	**Shading and use of dotted and solid line convention is required.**
	2.6	**Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.**	**Only division by** $(ax + b)$ **or** $(ax – b)$ **will be required. Students should know that if** $f(x) = 0$ **when** $x = a$**, then** $(x – a)$ **is a factor of** $f(x)$**.** **Students may be required to factorise cubic expressions such as** $x^3 + 3x^2 – 4$ **and** $6x^3 + 11x^2 – x – 6$**.**
		Simplify rational expressions, including by factorising and cancelling, and algebraic division (by linear expressions only).	Denominators of rational expressions will be linear or quadratic, e.g. $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+a^3}{x^2-a^2}$

2. Algebra and Functions

Topics	What students need to learn:		
	Content		Guidance
2 **Algebra and functions** *continued*	2.7	**Understand and use graphs of functions; sketch curves defined by simple equations including polynomials** The modulus of a linear function. $y = \frac{a}{x}$ **and** $y = \frac{a}{x^2}$ **(including their vertical and horizontal asymptotes)** **Interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations.** **Understand and use proportional relationships and their graphs.**	**Graph to include simple cubic and quartic functions,** **e.g. sketch the graph with equation** $y = x^2(2x - 1)^2$ Students should be able to sketch the graphs of $y = \|ax + b\|$ They should be able to use their graph. For example, sketch the graph with equation $y = \|2x - 1\|$ and use the graph to solve the equation $\|2x - 1\| = x$ or the inequality $\|2x - 1\| > x$ **The asymptotes will be parallel to the axes e.g. the asymptotes of the curve with equation** $y = \frac{2}{x + a} + b$ **are the lines with equations** $y = b$ **and** $x = -a$ **Direct proportion between two variables.** **Express relationship between two variables using proportion "∝" symbol or using equation involving constant** **e.g. the circumference of a semicircle is directly proportional to its diameter so** $C \propto d$ **or** $C = kd$ **and the graph of** C **against** d **is a straight line through the origin with gradient** k**.**

2. Algebra and Functions

Topics	What students need to learn:		
	Content		Guidance
2 **Algebra and functions** *continued*	2.8	Understand and use composite functions; inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\text{f}: x \mapsto$ and $\text{f}(x)$ will be used. Domain and range of functions. Students should know that fg will mean 'do g first, then f' and that if f^{-1} exists, then $\text{f}^{-1}\text{f}(x) = \text{ff}^{-1}(x) = x$ They should also know that the graph of $y = \text{f}^{-1}(x)$ is the image of the graph of $y = \text{f}(x)$ after reflection in the line $y = x$
	2.9	**Understand the effect of simple transformations on the graph of $y = \text{f}(x)$, including sketching associated graphs:** $y = a\text{f}(x)$, $y = \text{f}(x) + a$, $y = \text{f}(x + a)$, $y = \text{f}(ax)$ **and combinations of these transformations**	Students should be able to find the graphs of $y = \lvert \text{f}(x) \rvert$ and $y = \lvert \text{f}(-x) \rvert$, given the graph of $y = \text{f}(x)$. **Students should be able to apply a combination of these transformations to any of the functions in the A Level specification (quadratics, cubics, quartics, reciprocal, $\frac{a}{x^2}$, $\lvert x \rvert$, $\sin x$, $\cos x$, $\tan x$, e^x and a^x) and sketch the resulting graph.** Given the graph of $y = \text{f}(x)$, students should be able to sketch the graph of, e.g. $y = 2\text{f}(3x)$, or $y = \text{f}(-x) + 1$, and should be able to sketch (for example) $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$
	2.10	Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).	Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. Applications to integration, differentiation and series expansions.
2 **Algebra and functions** *continued*	2.11	Use of functions in modelling, including consideration of limitations and refinements of the models.	For example, use of trigonometric functions for modelling tides, hours of sunlight, etc. Use of exponential functions for growth and decay (see Paper 1, Section 6.7). Use of reciprocal function for inverse proportion (e.g. pressure and volume).

2. Algebra and Functions

E/P 1 Lynn is selling cushions as part of an enterprise project. On her first attempt, she sold 80 cushions at the cost of £15 each. She hopes to sell more cushions next time. Her adviser suggests that she can expect to sell 10 more cushions for every £1 that she lowers the price.

a The number of cushions sold c can be modelled by the equation $c = 230 - Hp$, where £p is the price of each cushion and H is a constant. Determine the value of H. **(1 mark)**

To model her total revenue, £r, Lynn multiplies the number of cushions sold by the price of each cushion. She writes this as $r = p(230 - Hp)$.

b Rearrange r into the form $A - B(p - C)^2$, where A, B and C are constants to be found. **(3 marks)**

c Using your answer to part **b** or otherwise, show that Lynn can increase her revenue by £122.50 through lowering her prices, and state the optimum selling price of a cushion. **(2 marks)**

E 2 Find the set of values of x for which the curve with equation $y = 2x^2 + 3x - 15$ is below the line with equation $y = 8 + 2x$. **(5 marks)**

E/P 3 **a** Factorise completely $x^3 - 6x^2 + 9x$. **(2 marks)**

b Sketch the curve of $y = x^3 - 6x^2 + 9x$ showing clearly the coordinates of the points where the curve touches or crosses the axes. **(4 marks)**

c The point with coordinates $(-4, 0)$ lies on the curve with equation $y = (x - k)^3 - 6(x - k)^2 + 9(x - k)$ where k is a constant.
Find the two possible values of k. **(3 marks)**

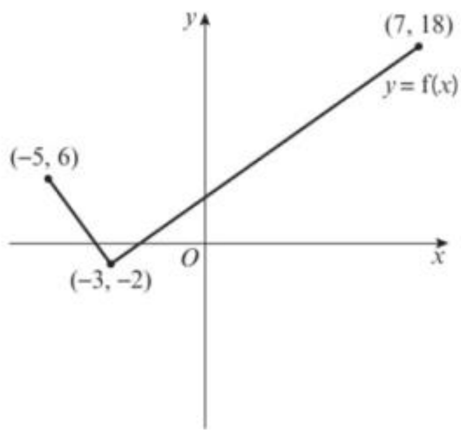
E/P 4 $h(x) = x^3 + 4x^2 + rx + s$. Given $h(-1) = 0$, and $h(2) = 30$:

a find the values of r and s **(6 marks)**

b factorise $h(x)$. **(3 marks)**

E 5 Given that $\dfrac{x^2 + 1}{x(x - 2)} \equiv P + \dfrac{Q}{x} + \dfrac{R}{x - 2}$, find the values of the constants P, Q and R. **(5 marks)**

E/P 6 The function f has domain $-5 \leqslant x \leqslant 7$ and is linear from $(-5, 6)$ to $(-3, -2)$ and from $(-3, -2)$ to $(7, 18)$.
The diagram shows a sketch of the function.



a Write down the range of f. **(1 mark)**

b Find ff(-3). **(2 marks)**

c Sketch the graph of $y = |f(x)|$, marking the points at which the graph meets or cuts the axes. **(3 marks)**

The function g is defined by g: $x \mapsto x^2 - 7x + 10$.

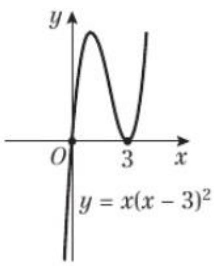
d Solve the equation fg$(x) = 2$. **(3 marks)**

2. Algebra and Functions

Answers

1 **a** $H = 10$
b $r = 1322.5 - 10(p - 11.5)^2$
$A = 1322.5, B = 10, C = 11.5$
c Old revenue is $80 \times £15 = £1200$; new revenue is £1322.50; difference is £122.50. The best selling price of a cushion is £11.50.

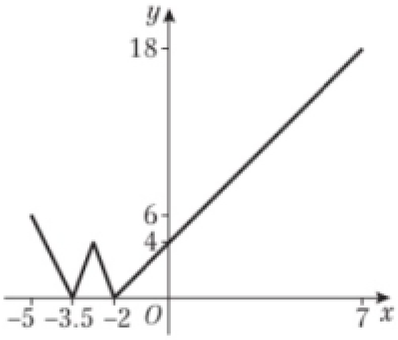
2 $\frac{1}{4}(-1 - \sqrt{185}) < x < \frac{1}{4}(-1 + \sqrt{185})$

3 **a** $x(x - 3)^2$
b


c -4 and -7

4 **a** $r = 3, s = 0$ **b** $x(x + 1)(x + 3)$

5 $P = 1, Q = -\frac{1}{2}, R = \frac{5}{2}$

6 **a** $-2 \leqslant f(x) \leqslant 18$
b 0
c


d $x = \dfrac{7 \pm \sqrt{5}}{2}$

3. Coordinate Geometry

Topics	What students need to learn:		
	Content		Guidance
3 Coordinate geometry in the (x,y) plane	3.1	Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; Gradient conditions for two straight lines to be parallel or perpendicular. Be able to use straight line models in a variety of contexts.	To include the equation of a line through two given points, and the equation of a line parallel (or perpendicular) to a given line through a given point. $m' = m$ for parallel lines and $m' = -\frac{1}{m}$ for perpendicular lines For example, the line for converting degrees Celsius to degrees Fahrenheit, distance against time for constant speed, etc.
	3.2	Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ Completing the square to find the centre and radius of a circle; use of the following properties: • the angle in a semicircle is a right angle • the perpendicular from the centre to a chord bisects the chord • the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.	Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa. Students should also be familiar with the equation $x^2 + y^2 + 2fx + 2gy + c = 0$ Students should be able to find the equation of a circumcircle of a triangle with given vertices using these properties. Students should be able to find the equation of a tangent at a specified point, using the perpendicular property of tangent and radius.

3. Coordinate Geometry

Topics	What students need to learn:		
	Content		Guidance
3 **Coordinate geometry in the (x, y) plane** *continued*	3.3	Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.	For example: $x = 3\cos t,\ y = 3\sin t$ describes a circle centre O radius 3 $x = 2 + 5\cos t,\ y = -4 + 5\sin t$ describes a circle centre $(2, -4)$ with radius 5 $x = 5t,\ y = \frac{5}{t}$ describes the curve $xy = 25$ (or $y = \frac{25}{x}$) $x = 5t, y = 3t^2$ describes the quadratic curve $25y = 3x^2$ and other familiar curves covered in the specification. Students should pay particular attention to the domain of the parameter t, as a specific section of a curve may be described.
	3.4	Use parametric equations in modelling in a variety of contexts.	A shape may be modelled using parametric equations or students may be asked to find parametric equations for a motion. For example, an object moves with constant velocity from $(1, 8)$ at $t = 0$ to $(6, 20)$ at $t = 5$. This may also be tested in Paper 3, section 7 (kinematics).

3. Coordinate Geometry

E/P 1 A is the point $(-1, 5)$. Let (x, y) be any point on the line $y = 3x$.

a Write an equation in terms of x for the distance between (x, y) and $A(-1, 5)$. **(3 marks)**

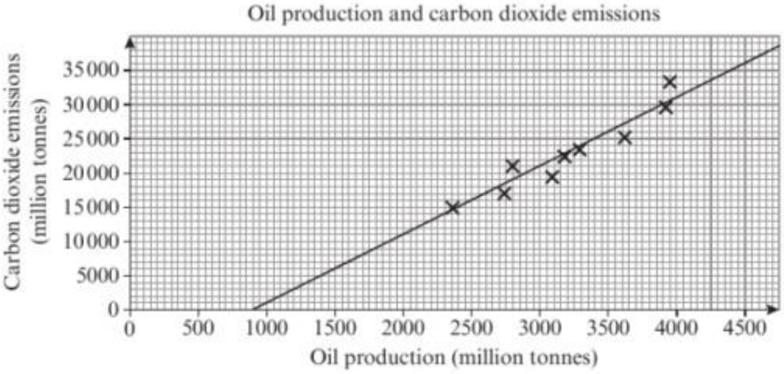
b Find the coordinates of the two points, B and C, on the line $y = 3x$ which are a distance of $\sqrt{74}$ from $(-1, 5)$. **(3 marks)**

c Find the equation of the line l_1 that is perpendicular to $y = 3x$ and goes through the point $(-1, 5)$. **(2 marks)**

d Find the coordinates of the point of intersection between l_1 and $y = 3x$. **(2 marks)**

e Find the area of triangle ABC. **(2 marks)**

E/P 2 The scatter graph shows the oil production P and carbon dioxide emissions C for various years since 1970. A line of best fit has been added to the scatter graph.



a Use two points on the line to calculate its gradient. **(1 mark)**

b Formulate a linear model linking oil production P and carbon dioxide emissions C, giving the relationship in the form $C = aP + b$. **(2 marks)**

c Interpret the value of a in your model. **(1 mark)**

d With reference to your value of b, comment on the validity of the model for small values of P. **(1 mark)**

E/P 3 The circle C has a centre at $(6, 9)$ and a radius of $\sqrt{50}$.

The line l_1 with equation $x + y - 21 = 0$ intersects the circle at the points P and Q.

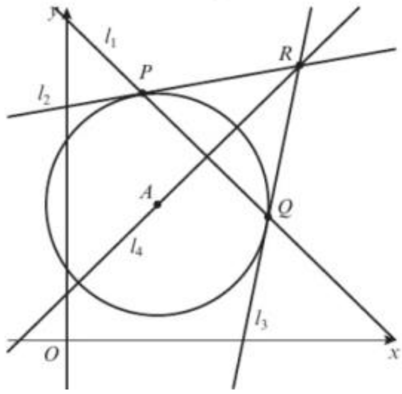
a Find the coordinates of the point P and the point Q. **(5 marks)**

b Find the equations of l_2 and l_3, the tangents at the points P and Q respectively. **(4 marks)**

c Find the equation of l_4, the perpendicular bisector of the chord PQ. **(4 marks)**

d Show that the two tangents and the perpendicular bisector intersect and find the coordinates of R, the point of intersection. **(2 marks)**

e Calculate the area of the kite $APRQ$. **(3 marks)**



(P) 4 The line $y = -3x + 12$ meets the coordinate axes at A and B.

a Find the coordinates of A and B.

b Find the coordinates of the midpoint of AB.

c Find the equation of the circle that passes through A, B and O, where O is the origin.

(E/P) 5 A mountaineer's climb at time t hours can be modelled with the following parametric equations

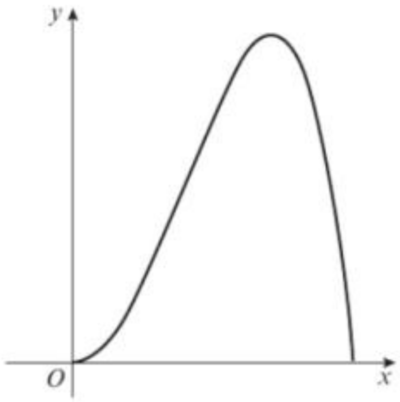
$$x = 300\sqrt{t}, \quad y = 244t(4 - t), \quad 0 < t < k$$

where x represents the distance travelled horizontally in metres and y represents the height above sea level in metres.

a Find the height of the peak and the time at which the mountaineer reaches it. **(3 marks)**

Given that the mountaineer completes her climb when she gets back to sea level,

b find the horizontal distance from the beginning of her climb to the end. **(2 marks)**



6 A BMX cyclist's position on a ramp at time t seconds can be modelled with the parametric equations

$$x = 3(e^t - 1), \quad y = 10(t - 1)^2, \quad 0 \leqslant t \leqslant 1.3$$

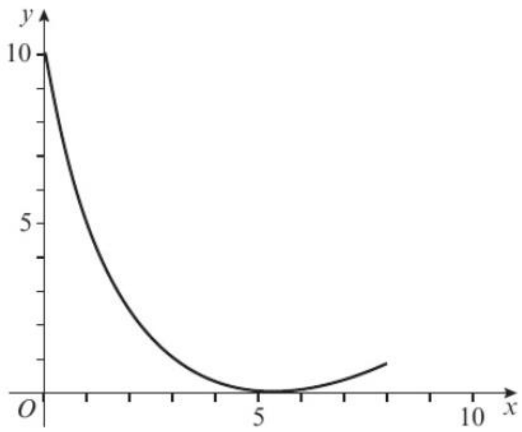
where x is the horizontal distance travelled in metres and y is the height above ground level in metres.

a Find the initial height of the cyclist.

b Find the time the cyclist is at her lowest height.

Given that after 1.3 seconds, the cyclist is at the end of the ramp,

c find the height at which the cyclist leaves the ramp.



3. Coordinate Geometry

Answers

1 **a** $d = \sqrt{10x^2 - 28x + 26}$

b $B\left(-\frac{6}{5}, -\frac{18}{5}\right)$ and $C(4, 12)$

c $y = -\frac{1}{3}x + \frac{14}{3}$

d $\left(\frac{7}{5}, \frac{21}{5}\right)$

e 20.8

2 **a** gradient = 10

b $C = 10P - 9000$

c When the oil production increases by 1 million tonnes, the carbon dioxide emissions increase by 10 million tonnes.

d The model is not valid for small values of P, as it is not possible to have a negative amount of carbon dioxide emissions. It is always dangerous to extrapolate beyond the range on the model in this way.

3 **a** $P(5, 16)$ and $Q(13, 8)$

b l_2: $y = \frac{1}{7}x + \frac{107}{7}$ and l_3: $y = 7x - 83$

c l_4: $y = x + 3$

d All 3 equations have solution $x = \frac{43}{3}$, $y = \frac{52}{3}$ so $R(\frac{43}{3}, \frac{52}{3})$

e $\frac{200}{3}$

4 **a** (4,0), (0,12)

b (2,6)

c $(x - 2)^2 + (y - 6)^2 = 40$

5 **a** 976 m, 2 hours **b** 600 m

6 **a** 10 m **b** 1 second **c** 0.9 m

4. Sequences And Series

Topics	What students need to learn: Content		Guidance
4 **Sequences and series**	**4.1**	**Understand and use the binomial expansion of $(a+bx)^n$ for positive integer n; the notations $n!$ and ^nC_r link to binomial probabilities.**	**Use of Pascal's triangle.** **Relation between binomial coefficients.** **Also be aware of alternative notations such as $\binom{n}{r}$ and ^nC_r** **Considered further in Paper 3 Section 4.1.**
		Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\| < 1$ (proof not required)	May be used with the expansion of rational functions by decomposition into partial fractions May be asked to comment on the range of validity.
4 **Sequences and series** *continued*	4.2	Work with sequences including those given by a formula for the n th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$; increasing sequences; decreasing sequences; periodic sequences.	For example $u_n = \frac{1}{3n+1}$ describes a decreasing sequence as $u_{n+1} < u_n$ for all integer n $u_n = 2^n$ is an increasing sequence as $u_{n+1} > u_n$ for all integer n $u_{n+1} = \frac{1}{u_n}$ for $n > 1$ and $u_1 = 3$ describes a periodic sequence of order 2
	4.3	Understand and use sigma notation for sums of series.	Knowledge that $\sum_1^n 1 = n$ is expected
	4.4	Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms	The proof of the sum formula for an arithmetic sequence should be known including the formula for the sum of the first n natural numbers.
	4.5	Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation	The proof of the sum formula should be known. Given the sum of a series students should be able to use logs to find the value of n. The sum to infinity may be expressed as S_∞
	4.6	Use sequences and series in modelling.	Examples could include amounts paid into saving schemes, increasing by the same amount (arithmetic) or by the same percentage (geometric) or could include other series defined by a formula or a relation.

4. Sequences And Series

E 1 **a** Expand $(1 + 2x)^{12}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(4 marks)**

b By substituting a suitable value for x, which must be stated, into your answer to part **a**, calculate an approximate value of 1.02^{12}. **(3 marks)**

c Use your calculator, writing down all the digits in your display, to find a more exact value of 1.02^{12}. **(1 mark)**

d Calculate, to 3 significant figures, the percentage error of the approximation found in part **b**. **(1 mark)**

E/P 2 **a** Find the first three terms, in ascending powers of x of the binomial expansion of $(2 + px)^7$, where p is a constant. **(2 marks)**

The first 3 terms are 128, $2240x$ and qx^2, where q is a constant.

b Find the value of p and the value of q. **(4 marks)**

E/P 3 The fourth term of an arithmetic series is $3k$, where k is a constant, and the sum of the first six terms of the series is $7k + 9$.

a Show that the first term of the series is $9 - 8k$. **(3 marks)**

b Find an expression for the common difference of the series in terms of k. **(2 marks)**

Given that the seventh term of the series is 12, calculate:

c the value of k **(2 marks)**

d the sum of the first 20 terms of the series. **(2 marks)**

E/P 4 The adult population of a town is 25 000 at the beginning of 2012.
A model predicts that the adult population of the town will increase by 2% each year, forming a geometric sequence.

a Show that the predicted population at the beginning of 2014 is 26 010. **(1 mark)**

The model predicts that after n years, the population will first exceed 50 000.

b Show that $n > \dfrac{\log 2}{\log 1.02}$ **(3 marks)**

c Find the year in which the population first exceeds 50 000. **(2 marks)**

d Every member of the adult population is modelled to visit the doctor once per year. Calculate the number of appointments the doctor has from the beginning of 2012 to the end of 2019. **(4 marks)**

e Give a reason why this model for doctors' appointments may not be appropriate. **(1 mark)**

E/P 5 **a** Use the binomial theorem to expand $(4+x)^{-\frac{1}{2}}$, $|x|<4$, in ascending powers of x, up to and including the x^3 term, giving each answer as a simplified fraction. **(5 marks)**

b Use your expansion, together with a suitable value of x, to obtain an approximation to $\frac{\sqrt{2}}{2}$. Give your answer to 4 decimal places. **(3 marks)**

E/P 6 $\text{f}(x) = \frac{12x+5}{(1+4x)^2}$, $|x| < \frac{1}{4}$

For $x \neq -\frac{1}{4}$, $\frac{12x+5}{(1+4x)^2} = \frac{A}{1+4x} + \frac{B}{(1+4x)^2}$, where A and B are constants.

a Find the values of A and B. **(3 marks)**

b Hence, or otherwise, find the series expansion of $\text{f}(x)$, in ascending powers of x, up to and including the term x^2, simplifying each term. **(6 marks)**

4. Sequences And Series

Answers

1 **a** $1 + 24x + 264x^2 + 1760x^3$ **b** 1.268 16
c 1.268 241 795 **d** 0.006 45% (3 sf)

2 **a** $128 + 448px + 672p^2x^2$
b $p = 5$, $q = 16\,800$

3 **a** $a + 3d = 3k$, $3(2a + 5d) = 7k + 9 \Rightarrow$

$6a + 15d = 7k + 9$

$6a + 15\left(\dfrac{3k - a}{3}\right) = 7k + 9$

$6a + 15k - 5a = 7k + 9 \Rightarrow a = 9 - 8k$

b $\dfrac{11k - 9}{3}$ **c** 1.5 **d** 415

4 **a** $25000 \times 1.02^2 = 26010$
b $25000 \times 1.02^n > 50000$

$1.02^n > 2 \Rightarrow n \log 1.02 > \log 2 \Rightarrow n > \dfrac{\log 2}{\log 1.02}$

c 2047
d 214574
e People may visit the doctor more frequently than once a year, some may not visit at all, depends on state of health

5 **a** $\dfrac{1}{2} - \dfrac{x}{16} + \dfrac{3x^2}{256} - \dfrac{5x^3}{2048}$ **b** 0.6914

6 **a** $A = 3$ and $B = 2$ **b** $5 - 28x + 144x^2$

5. Trigonometry

Topics	What students need to learn:		
	Content		Guidance
5 **Trigonometry**	5.1	**Understand and use the definitions of sine, cosine and tangent for all arguments;**	**Use of x and y coordinates of points on the unit circle to give cosine and sine respectively,**
		the sine and cosine rules;	**including the ambiguous case of the sine rule.**
		the area of a triangle in the form $\frac{1}{2}ab\sin C$	
		Work with radian measure, including use for arc length and area of sector.	Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for arc lengths and areas of sectors of a circle.
	5.2	Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$ Where θ is in radians.	Students should be able to approximate, e.g. $\frac{\cos 3x - 1}{x\sin 4x}$ when x is small, to $-\frac{9}{8}$
	5.3	**Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.**	**Knowledge of graphs of curves with equations such as $y = \sin x$, $y = \cos(x + 30°)$, $y = \tan 2x$ is expected.**
		Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof.	
	5.4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.	Angles measured in both degrees and radians.

5. Trigonometry

Topics	What students need to learn:		
		Content	Guidance
5 Trigonometry *continued*	5.5	**Understand and use** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **Understand and use** $\sin^2\theta + \cos^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$	**These identities may be used to solve trigonometric equations and angles may be in degrees** or radians. **They may also be used to prove further identities.**
	5.6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, and $\tan(A \pm B)$, understand geometrical proofs of these formulae. Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$	To include application to half angles. Knowledge of the $\tan(\frac{1}{2}\theta)$ formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval.
	5.7	**Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.**	**Students should be able to solve equations such as** $\sin(x + 70°) = 0.5$ for $0 < x < 360°$, $3 + 5\cos 2x = 1$ for $-180° < x < 180°$ $6\cos^2 x + \sin x - 5 = 0$, $0 \le x < 360°$ These may be in degrees or radians and this will be specified in the question.
	5.8	Construct proofs involving trigonometric functions and identities.	Students need to prove identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.
	5.9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	Problems could involve (for example) wave motion, the height of a point on a vertical circular wheel, or the hours of sunlight throughout the year. Angles may be measured in degrees or in radians.

5. Trigonometry

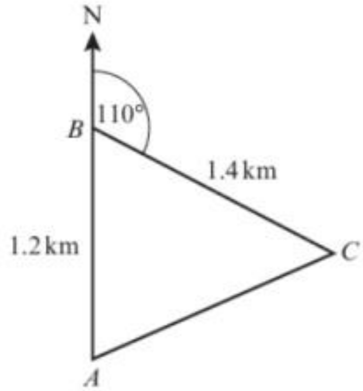
E/P 1 A park is in the shape of a triangle ABC as shown.

A park keeper walks due north from his hut at A until he reaches point B. He then walks on a bearing of 110° to point C.

a Find how far he is from his hut when at point C. Give your answer in km to 3 s.f. **(3 marks)**

b Work out the bearing of the hut from point C. Give your answer to the nearest degree. **(3 marks)**

c Work out the area of the park. **(3 marks)**



E 2 Find, in degrees, the values of θ in the interval $0 \leqslant \theta \leqslant 360°$ for which

$2\cos^2\theta - \cos\theta - 1 = \sin^2\theta$

Give your answers to 1 decimal place, where appropriate. **(6 marks)**

E 3 **a** Express $4\sin\theta - \cos\left(\frac{\pi}{2} - \theta\right)$ as a single trigonometric function. **(1 mark)**

b Hence solve $4\sin\theta - \cos\left(\frac{\pi}{2} - \theta\right) = 1$ in the interval $0 \leqslant \theta \leqslant 2\pi$. Give your answers to 3 significant figures. **(3 marks)**

E/P 4 **a** Given that $\sec x + \tan x = -3$, use the identity $1 + \tan^2 x \equiv \sec^2 x$ to find the value of $\sec x - \tan x$. **(3 marks)**

b Deduce the values of:

i $\sec x$ **ii** $\tan x$ **(3 marks)**

c Hence solve, in the interval $-180° \leqslant x \leqslant 180°$, $\sec x + \tan x = -3$. **(3 marks)**

E/P 5 **a** Prove that $\dfrac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan\theta$. **(4 marks)**

b Verify that $\theta = 180°$ is a solution of the equation $\sin 2\theta = 2 - 2\cos 2\theta$. **(1 mark)**

c Using the result in part **a**, or otherwise, find the two other solutions, $0 < \theta < 360°$, of the equation $\sin 2\theta = 2 - 2\cos 2\theta$. **(3 marks)**

E/P 6 **a** Express $1.4\sin\theta - 5.6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Round R and α to 3 decimal places. **(4 marks)**

b Hence find the maximum value of $1.4\sin\theta - 5.6\cos\theta$ and the smallest positive value of θ for which this maximum occurs. **(3 marks)**

The length of daylight, $d(t)$ at a location in northern Scotland can be modelled using the equation

$$d(t) = 12 - 5.6\cos\left(\frac{360t}{365}\right)^{\circ} + 1.4\sin\left(\frac{360t}{365}\right)^{\circ}$$

where t is the numbers of days into the year.

c Calculate the minimum number of daylight hours in northern Scotland as given by this model. **(2 marks)**

d Find the value of t when this minimum number of daylight hours occurs. **(1 mark)**

5. Trigonometry

Answers

1 **a** 1.50 km **b** 241° **c** 0.789 km^2

2 131.8°, 228.2°

3 **a** $3 \sin\theta$ **b** 0.340, 2.80

4 **a** $-\frac{1}{3}$ **b** **i** $-\frac{5}{3}$, **ii** $-\frac{4}{3}$ **c** 126.9°

5 **a** Use $\cos 2\theta = 1 - 2\sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$.
b $\sin 360° = 0$, $2 - 2\cos(360°) = 2 - 2 = 0$
c 26.6°, 206.6°

6 **a** $R = 5.772$, $\alpha = 75.964°$ **b** 5.772 when $\theta = 166.0°$
c 6.228 hours **d** 350.8 days

Extra Questions

Topics		What students need to learn: Content	Guidance
6 Exponentials and logarithms	6.1	**Know and use the function a^x and its graph, where a is positive.** **Know and use the function e^x and its graph.**	**Understand the difference in shape between $a < 1$ and $a > 1$** To include the graph of $y = e^{ax+b} + c$
	6.2	**Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.**	**Realise that when the rate of change is proportional to the y value, an exponential model should be used.**
	6.3	**Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x > 0$.** **Know and use the function $\ln x$ and its graph.** **Know and use $\ln x$ as the inverse function of e^x**	$a \neq 1$ **Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.**
	6.4	**Understand and use the laws of logarithms:** $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$ $k \le \log_a x = \log_a x^k$ **(including, for example, $k = -1$ and $k = -\frac{1}{2}$)**	**Includes $\log_a a = 1$**
	6.5	**Solve equations of the form $a^x = b$**	**Students may use the change of base formula. Questions may be of the form, e.g. $2^{3x-1} = 3$**
	6.6	**Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y**	**Plot $\log y$ against $\log x$ and obtain a straight line where the intercept is $\log a$ and the gradient is n** **Plot $\log y$ against x and obtain a straight line where the intercept is $\log k$ and the gradient is $\log b$**
6 Exponentials and logarithms *continued*	6.7	**Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.**	**Students may be asked to find the constants used in a model.** **They need to be familiar with terms such as initial, meaning when $t = 0$.** **They may need to explore the behaviour for large values of t or to consider whether the range of values predicted is appropriate.** **Consideration of a second improved model may be required.**