# Second Year Assignment 1

## SECTION A

1 a) Sketch the curve with equation $y = f(x)$ where $f(x) = x^2 - 4$

b) Sketch the graphs of i) $y = f(4x)$, ii) $\frac{1}{3}y = f(x)$, (iii) $y = f(-x)$ (iv) $y = -f(x)$

2. a) Find the distance between (4a,a) and (-3a,2a).

b) Hence find the distance between the following pairs of points:
(4,1) and (-3,2) ii) (12,3) and (-9,6) iii) (-20,-5) and (15,-10)

3. Look at the table showing the mean temperature in Heathrow and Beijing for the first week of 2015.
Find the mean and standard deviation of the temperature in each place.
What conclusions can you draw?

| | Heathrow | Beijing |
|---|---|---|
| 1-May-15 | 9.8 | 17.5 |
| 2-May-15 | 11.0 | 20.0 |
| 3-May-15 | 14.7 | 19.2 |
| 4-May-15 | 15.0 | 18.5 |
| 5-May-15 | 14.3 | 21.1 |
| 6-May-15 | 11.5 | 17.1 |
| 7-May-15 | 13.1 | 18.8 |

4. A particle is projected vertically upwards with a speed of 30 $\mathrm{ms}^{-1}$ from a point A. The point B is *h* metres above A. The particle moves freely under gravity and is above B for 2.4 seconds. Calculate the value of *h.*

5. A particle moves 100 m in a straight line. The particle starts with velocity *u* $\mathrm{ms}^{-1}$ and accelerates to a velocity of 10 $\mathrm{ms}^{-1}$ in 3 s. The velocity of 10 $\mathrm{ms}^{-1}$ is maintained for 7 s and then the particle decelerates to rest in a further 2 s.

a) Sketch a velocity-time graph of the motion of the particle.

b) Find the value of *u*

c) Find the acceleration of the particle in the first 3 s of motion.

6. a) Sketch the graphs of $y = \arcsin(x)$ and $y = \arccos(x)$ on the same diagram, $-1 \leq x \leq 1$

b) Use your sketch to state an approximate solution to the equation

$$\arcsin(x) = \arccos(x), \qquad -1 \leq x \leq 1$$

c) Find an exact solution to the equation

$$\arcsin(x) = \arccos(x), \qquad -1 \leq x \leq 1$$

## SECTION B

1. a) Use the double angle formulae to prove that $\frac{1-cos2x}{1+cos2x} \equiv \tan^2 x$

b) Hence find for $-\pi \leq x \leq \pi$ all the solutions of $\frac{1-cos2x}{1+cos2x} = 3$ , leaving your answer in terms of $\pi$

2. a) Prove, by counter example, that the statement $\sec(A + B) \equiv \sec A + \sec B$ for all A and B is false.

b) Prove that $\tan\theta + \cot\theta \equiv 2\,\mathrm{cosec}\,2\theta, \qquad \theta \pm \frac{n\pi}{2}, \quad n \epsilon Z$

3. A set of data values, $x$, is shown below:
52, 73, 31, 73, 38, 80, 17, 24

a) Code the data using the coding $y = \frac{x-3}{7}$

b) Calculate the mean of the coded data values

c) Use your answer to (b) to calculate the mean of the original data.

4. Copy these sentences and fill in the gaps
a) Newton's ….. law of motion states that an object at rest will …. .. …. and that an object moving with constant …….. will continue to move with constant …….. unless an ……. force acts on the object

b) Newton's …… law of motion states that the ….. needed to ……. a particle is equal to the ……. of the mass of the particle and the …… produced. F = ..

c) Newton's ….. law states that for every …… there is an ….. and …….. reaction

5. A frequency distribution is shown below

| **Class interval** | 1-20 | 21-40 | 41-60 | 61-80 | 81-100 |
|---|---|---|---|---|---|
| **Frequency** | 5 | 10 | 15 | 12 | 8 |

Use interpolation to find an estimate for the interquartile range

6. a) Show that $\left(2+5\sqrt{x}\right)^2$ can be written as $4+k\sqrt{x}+25x$, where k is a constant to be found.

b) Hence find $\int\left(2+5\sqrt{x}\right)^2 dx$

## SECTION C

1. Solve the following equations in the given intervals

a) $(\sec\theta-\cos\theta)^2=\tan\theta-\sin^2\theta, \quad 0\leq\theta\leq\pi$

b) $3\sec\frac{1}{2}\theta=2\tan^2\frac{1}{2}\theta, \quad 0\leq\theta\leq 360°$

c) $\tan^2 2\theta=\sec 2\theta-1, \quad 0\leq\theta\leq 180°$

d) $\sec^2\theta-\left(1+\sqrt{3}\right)\tan\theta+\sqrt{3}=1, \quad 0\leq\theta\leq 2\pi$

2. a) Sketch the curve with equation $y=(x-4)(2x+3)$

b) The curve cuts the x-axis at A and B and the y-axis at C. State the co-ordinates of A, B and C.

c) Find the area of the region bounded by the curve and the x-axis

3. Two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on a particle of mass m. Find the acceleration of the particle, $\mathbf{a}\ \mathrm{ms}^{-2}$ given that

a) $\mathbf{F}_1=(2\mathbf{i}+7\mathbf{j})\ \mathrm{N}, \quad \mathbf{F}_2=(-3\mathbf{i}+\mathbf{j})\ \mathrm{N}, \quad \mathrm{m}=0.25\ \mathrm{kg}$

b) $\mathbf{F}_1=(\mathbf{3i}-\mathbf{4j})\ \mathrm{N}, \quad \mathbf{F}_2=(\mathbf{2i}+3\mathbf{j})\ \mathrm{N}, \quad \mathrm{m}=6\ \mathrm{kg}$

c) $\mathbf{F}_1=(-40\mathbf{i}-20\mathbf{j})\ \mathrm{N}, \quad \mathbf{F}_2=(25\mathbf{i}+10\mathbf{j})\ \mathrm{N}, \quad \mathrm{m}=15\ \mathrm{kg}$

d) $\mathbf{F}_1=4\mathbf{j}\ \mathrm{N}, \quad \mathbf{F}_2=(-2\mathbf{i}+5\mathbf{j})\ \mathrm{N}, \quad \mathrm{m}=1.5\ \mathrm{kg}$

4. Two particles, P and Q, have masses *k*m and 3m respectively where k < 3. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with P and Q at the same height above a horizontal plane. The system is released from rest. After release, Q descends with acceleration $\frac{1}{3}g$.

a) Calculate the tension in the string as Q descends

b) Show that k = 1.5

c) State how you have used the information that the pulley is smooth
After descending for 1.8 s, the particle Q reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between P and the pulley is such that, in the subsequent motion, P does not reach the pulley.

d) Show that the greatest height, in metres, reached by the P above the plane is 1.26g.

5. The table shows the Daily Maximum Gust in Camborne in September 1987

a) Calculate $\mathrm{Q}_1$, $\mathrm{Q}_2$ and $\mathrm{Q}_3$

b) Use the formula $Q_1$ – 1.5 × IQR and $Q_3$ + 1.5 × IQR to determine whether or not there are any outliers.

c) Draw a box plot for this data.

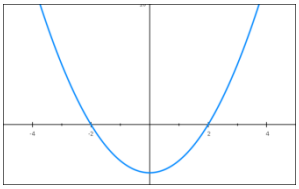
| Date | 1-Sep | 2-Sep | 3-Sep | 4-Sep | 5-Sep | 6-Sep | 7-Sep | 8-Sep | 9-Sep | 10-Sep |
|---|---|---|---|---|---|---|---|---|---|---|
| Max Gust | n/a | 21 | 27 | 30 | 78 | 35 | n/a | n/a | 29 | 24 |
| | | | | | | | | | | |
| Date | 21-Sep | 22-Sep | 23-Sep | 24-Sep | 25-Sep | 26-Sep | 27-Sep | 28-Sep | 29-Sep | 30-Sep |
| Max Gust | 33 | 26 | 26 | 25 | 17 | 26 | 13 | n/a | 25 | 30 |
| | | | | | | | | | | |
| Date | 11-Sep | 12-Sep | 13-Sep | 14-Sep | 15-Sep | 16-Sep | 17-Sep | 18-Sep | 19-Sep | 20-Sep |
| Max Gust | 46 | 38 | n/a | 25 | 20 | 22 | 23 | 19 | 31 | 21 |

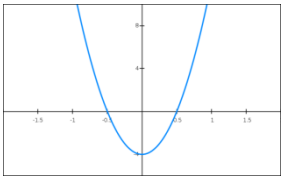
6. a) Show that $x - 2$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$

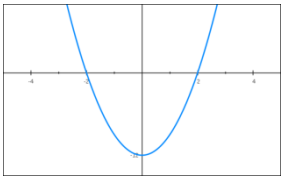
b) Hence find four real solutions to the equation $9x^4 - 18x^3 - x^2 + 2x = 0$

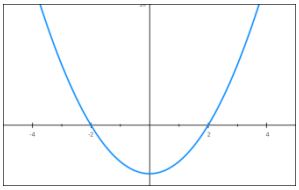
# Answers

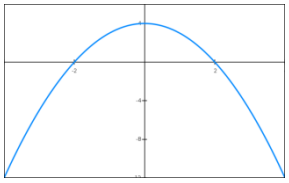
## SECTION A

1a 

1b 

1c 

1d 

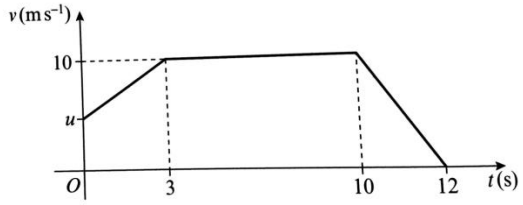
1e 

2a) $5a\sqrt{2}$ bi) $5\sqrt{2}$ bii) $15\sqrt{2}$ biii) $25\sqrt{2}$

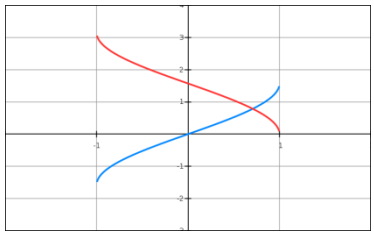
3 Heathrow: mean=12.7, s.d.=1.88
Beijing: mean = 18.9, s.d.=1.28
Beijing hotter. Heathrow more variable

4. h = 39 (2 s.f.)

5. a)




b) $u = \frac{10}{3}$, c) $\frac{20}{9}$

6. a) 

b) $\approx 0.7$
c) $\frac{\sqrt{2}}{2}$

# SECTION B

1 b) $\frac{\pi}{3}, \ -\frac{\pi}{3}$

3a) 7, 10, 4, 10, 5, 11, 2, 3 b) 6.5 c) 48.5

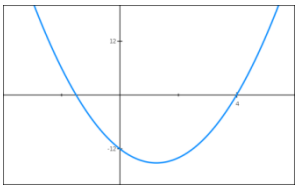
5. 37.5

6. a) k = 20 b) $4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c$

# SECTION C

1. a) $0, \frac{\pi}{4}, \pi$ b)120° c) 0°, 180°, d) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{4}, \frac{4\pi}{3}$

2. a)



b) $A\left(-\frac{3}{2}, 0\right)$, $B(4,0)$ $C(0,-12)$
c) $55\frac{11}{24}$ $\left(=\frac{1331}{24}\right)$

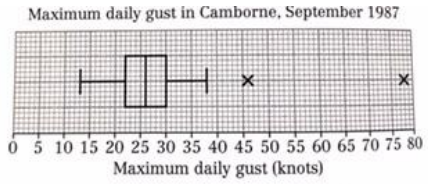
3.

a) $(-4\mathbf{i} + 32\mathbf{j})\text{ms}^{-2}$ b) $\left(\frac{5}{6}\mathbf{i} - \frac{1}{6}\mathbf{j}\right)\text{ms}^{-2}$

c) $\left(-\mathbf{i} - \frac{2}{3}\mathbf{j}\right)\text{ms}^{-2}$ d) $\left(-\frac{4}{3}\mathbf{i} + 6\mathbf{j}\right)\text{ms}^{-2}$

4. a) 2mg N
c) No friction means that the magnitude of the acceleration is the same in objects connected by a taut inextensible string

5. a) $Q_1 = 22, \ Q_2 = 26, \ Q_3 = 30$ b) 46 and 78
c)




6 b) $0, \ 2, \ -\frac{1}{3} and \frac{1}{3}$