## Exercise 3F

1 For each of the following binomial random variables, $X$:
   i state, with reasons, whether $X$ can be approximated by a normal distribution.
   ii if appropriate, write down the normal approximation to $X$ in the form $N(\mu, \sigma^2)$, giving the values of $\mu$ and $\sigma$.

   **a** $X \sim B(120, 0.6)$ **b** $X \sim B(6, 0.5)$ **c** $X \sim B(250, 0.52)$
   **d** $X \sim B(100, 0.98)$ **e** $X \sim B(400, 0.48)$ **f** $X \sim B(1000, 0.58)$

2 The random variable $X \sim B(150, 0.45)$. Use a suitable approximation to estimate:
   **a** $P(X \leqslant 60)$ **b** $P(X > 75)$ **c** $P(65 \leqslant X \leqslant 80)$

3 The random variable $X \sim B(200, 0.53)$. Use a suitable approximation to estimate:
   **a** $P(X < 90)$ **b** $P(100 \leqslant X < 110)$ **c** $P(X = 105)$

4 The random variable $X \sim B(100, 0.6)$. Use a suitable approximation to estimate:
   **a** $P(X > 58)$ **b** $P(60 < X \leqslant 72)$ **c** $P(X = 70)$

5 A fair coin is tossed 70 times. Use a suitable approximation to estimate the probability of obtaining more than 45 heads.

6 The probability of a roulette ball landing on red when the wheel is spun is $\frac{50}{101}$.
   On one day in a casino, the wheel is spun 1200 times.
   Estimate the probability that the ball lands on red in at least half of these spins.

7 **a** Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution. **(2 marks)**
   A company sells orchids of which 45% produce pink flowers.

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### Exercise 3F

1 **a** i Yes, $n$ is large (> 50) and $p$ is close to 0.5.
      ii $X \sim N(72, 5.37^2)$
   **b** i No, $n$ is not large enough (< 50).
   **c** i Yes, $n$ is large (> 50) and $p$ is close to 0.5.
      ii $X \sim N(130, 7.90^2)$
   **d** i No, $p$ is too far from 0.5.
   **e** i Yes, $n$ is large (> 50) and $p$ is close to 0.5.
      ii $X \sim N(192, 9.99^2)$
   **f** i Yes, $n$ is large (> 50) and $p$ is close to 0.5.
      ii $X \sim N(580, 15.6^2)$

2 **a** 0.1253 **b** 0.0946 **c** 0.6723
3 **a** 0.0097 **b** 0.5115 **c** 0.0559
4 **a** 0.6203 **b** 0.4540 **c** 0.0102
5 0.006
6 0.3767
7 **a** $n$ large, $p$ close to 0.5. **b** 0.1593
   **c** 0.5772 **d** 115
8 **a** 0.6277 **b** 0.8456
9 **a** 0.0784 (3 s.f.) **b** 0.31%

### Exercise 3G

1 **a** Not significant. Do not reject $H_0$.
   **b** Significant. Reject $H_0$.
   **c** Not significant. Do not reject $H_0$.
   **d** Significant. Reject $H_0$.
   **e** Not significant. Do not reject $H_0$.
2 **a** $\bar{X} < 119.4$
   **b** $\bar{X} > 13.2$
   **c** $\bar{X} < 84.3$
   **d** $\bar{X} > 0.877$ or $\bar{X} < -0.877$
   **e** $\bar{X} > -7.31$ or $\bar{X} < -8.69$
3 Result is significant so reject $H_0$. There is evidence that the new formula is an improvement.
4 **a** $\bar{X} > 103.29$
   **b** $102.5 < 103.29$, so there is not enough evidence to reject the null hypothesis

1 In each part, a random sample of size $n$ is taken from a population having a normal distribution with mean $\mu$ and variance $\sigma^2$. Test the hypotheses at the stated levels of significance.

**a** $H_0: \mu = 21$, $H_1: \mu \neq 21$, $n = 20$, $\bar{x} = 21.2$, $\sigma = 1.5$, at the 5% level

**b** $H_0: \mu = 100$, $H_1: \mu < 100$, $n = 36$, $\bar{x} = 98.5$, $\sigma = 5.0$, at the 5% level

**c** $H_0: \mu = 5$, $H_1: \mu \neq 5$, $n = 25$, $\bar{x} = 6.1$, $\sigma = 3.0$, at the 5% level

**d** $H_0: \mu = 15$, $H_1: \mu > 15$, $n = 40$, $\bar{x} = 16.5$, $\sigma = 3.5$, at the 1% level

**e** $H_0: \mu = 50$, $H_1: \mu \neq 50$, $n = 60$, $\bar{x} = 48.9$, $\sigma = 4.0$, at the 1% level

2 In each part, a random sample of size $n$ is taken from a population having a $N(\mu, \sigma^2)$ distribution. Find the critical regions for the test statistic $\overline{X}$ in the following tests.

**a** $H_0: \mu = 120$, $H_1: \mu < 120$, $n = 30$, $\sigma = 2.0$, at the 5% level

**b** $H_0: \mu = 12.5$, $H_1: \mu > 12.5$, $n = 25$, $\sigma = 1.5$, at the 1% level

**c** $H_0: \mu = 85$, $H_1: \mu < 85$, $n = 50$, $\sigma = 4.0$, at the 10% level

**d** $H_0: \mu = 0$, $H_1: \mu \neq 0$, $n = 45$, $\sigma = 3.0$, at the 5% level

**e** $H_0: \mu = -8$, $H_1: \mu \neq -8$, $n = 20$, $\sigma = 1.2$, at the 1% level

3 The times taken for a capful of stain remover to remove a standard chocolate stain from a baby's bib are normally distributed with a mean of 185 seconds and a standard deviation of 15 seconds. The manufacturers of the stain remover claim to have developed a new formula which will shorten the time taken for a stain to be removed. A random sample of 25 capfuls of the new formula are tested and the mean time for the sample is 179 seconds.

Test, at the 5% level, whether or not there is evidence that the new formula is an improvement.

**Hint** You are testing for an improvement, so use a **one-tailed** test.

4 The IQ scores of a population are normally distributed with a mean of 100 and a standard deviation of 15. A psychologist wishes to test the theory that eating chocolate before sitting an IQ test improves your score. A random sample of 80 people are selected and they are each given an identical bar of chocolate to eat before taking an IQ test.

**a** Find, at the 2.5% level, the critical region for this test, stating your hypotheses clearly.

The mean score on the test for the sample of 80 people was 102.5.

**b** Comment on this observation in light of the critical region.

5 The diameters of circular cardboard drinks mats produced by a certain machine are normally distributed with a mean of 9 cm and a standard deviation of 0.15 cm. After the machine is serviced a random sample of 30 mats is selected and their diameters are measured to see if the mean diameter has altered.

The mean of the sample was 8.95 cm.

**a** Test, at the 5% level, whether there is significant evidence of a change in the mean diameter of mats produced by the machine.

**b** State the $p$-value for your test.

**Hint** You are testing for an alteration in either direction, so use a **two-tailed** test. Work out $P(\overline{X} < 8.95)$, and multiply this probability by 2 to find the $p$-value.