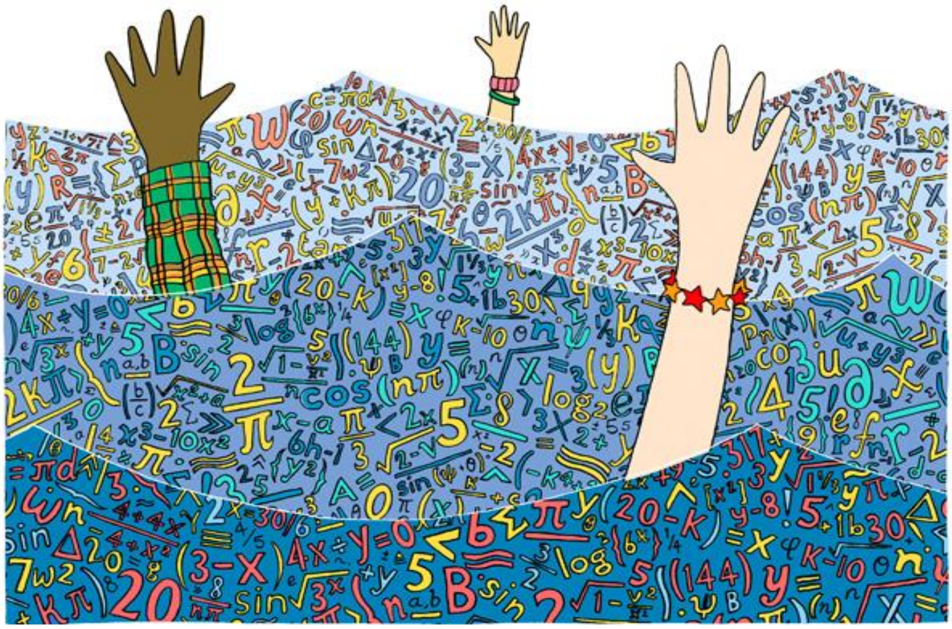
A Level Maths

The Final Countdown



How to get 100% in every exam

The key to success is **honest self-assessment** followed by **remedial action**. If you are honest with yourself about how much you understand the work **and** if you take remedial action to improve your weak areas, you will get a grade A* (unless you make lots of expensive errors).

The Mark Scheme Is Your Enemy

When you work through these papers, do **NOT** use the mark scheme every time you get stuck. Try to work out what to do yourself. You won't have the mark scheme in the real exam! Do the test in 2 hours and then mark it. If you don't get full marks on a question, find out what went wrong (use the textbook or any other form of support from the Internet). Then go back to it a day later and do it again (without looking at the mark scheme). Keep doing this until you can get the question right without help. This is the way you will improve your understanding.

In the mark schemes, the following symbols are used:
M marks: method marks are awarded for 'knowing a method and attempting to apply it'.
A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
B marks are for the correct answer (method not necessary)

Support

If you get stuck there are a number of things you can do

- Refer to your Survival Kit
- Watch videos on YouTube
- Use the textbook and read the bits between the exercises
- Try some of the questions in the exercises to get further help
- Look at videos and resources from other websites
    - Exam Solutions: https://www.youtube.com/user/ExamSolutions
    - Mr Hegarty: https://www.youtube.com/user/HEGARTYMATHS/videos
    - Physics and Maths tutor http://www.physicsandmathstutor.com/
    - MadAsMaths http://www.madasmaths.com/
    - Dr Frost Maths http://www.drfrostmaths.com/
    - Owen's Resource Library https://www.tes.com/teaching-resources/shop/Owen134866

However, the best thing you can do is figure it out yourself.

Formulae To Learn

Make sure you remember **all** the formulae (see later in this booklet)
The formulae that you don't need to remember are in the formula book (also see later in this booklet)

How To Study An Exam Paper

DO NOT simply copy out the mark scheme.

Here is why you must ***study*** the exam papers. By following this programme of exam paper study, you will:

Improve your knowledge of how to solve standard problems. By completing every question from past papers and practice papers you will encounter almost every question that has been put into your real exam.

Improve the accuracy of your algebra… The exam board have told us that the only difference between E grade students and A grade students is that the A grade students make fewer algebraic errors. *Being able to answer the questions is not enough*. You need to be able to answer them without making expensive errors, and this is not something you can learn at the last minute. It takes practice.

Improve the speed of your algebra… *Your real exam will be an algebra sprint.* It is very important that you get used to the speed required.

Studying **an exam paper is not the same as** ***doing*** **an exam paper.**

This what you should not do:

- Sit down and complete an exam paper……
- After an hour you have done all you can do so you mark your work using the mark scheme.
- You realise you've made some errors and you think 'oops! – I won't do that in the exam!'
- You read the solutions to the questions you couldn't do and think 'oh – I see how to do it now'.
- What has this process done to improve your chance of getting a good grade in the real exam?

Do you have any more knowledge? **NO.** You have read the solutions to the questions you couldn't do, but this doesn't mean you can actually do them.

Have you improved the accuracy of your algebra? **NO.** As soon as you finished the paper you went to the mark scheme. You didn't practice the most important part of an exam – looking for and correcting your error.

Have you improved the speed of your algebra? **NO.** You didn't try to complete each question in a fixed number of minutes so you still have no idea whether you were going at exam speed.

This is what you **should** do:

1. Complete the exam paper in exam conditions. This means you continue working for two hours and make a real determined effort to find your errors before the time is up.
2. Don't use
   a. The mark scheme
   b. Textbooks
   c. Any other support
3. Mark the paper carefully using the mark scheme. Add up the marks and determine the grade.
4. Look at all the marks you lost – categorising them as being due to
   a. LU = Lack of understanding (not knowing what to do)
   b. EE = An expensive error (something that seems silly when you realise what you did)
5. Study your mistakes using
   a. The mark scheme
   b. Textbooks
   c. Videos
   d. etc.
6. Wait a day then repeat any question that you lost marks on using the strict timing (number of minutes = number of marks) and looking for your errors **before** you look at the mark scheme.
7. Repeat steps 3 and 4 over and over again until you are confident that if any of those questions are in your real exam, you will be able to do them quickly and accurately.

What has this process done to improve your chance of getting an improved grade in the real exam?

Do you have any more knowledge? YES. You kept going back to the harder questions until you could do them, so if these questions come up in your exam you, unlike some other students, can be confident you will know what to do.

Have you improved the accuracy of your algebra? YES. Not only have you practised finding errors during step one of the process, the fact that you have written them down and categorised them will help you to be more aware of the sorts of errors you make and this will help you, unlike other students, to avoid making them in the real exam.

Have you improved the speed of your algebra? YES. Every time you complete a paper (or question) in the correct time you are training yourself to be more comfortable working at the speed needed in the exam. This means that, unlike some other students, you won't have the problem of running out of time.

The key to success is honest self assessment followed by remedial action.

Revision Tips

Multiple attempts and crossing out

"Crossed-out work should be marked unless the candidate has replaced it with an alternative response."
"Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete."

Circle or highlight key phrases in questions.

Tick off question parts as you go so you don't leave any parts out
Key phrases might include

3 d.p.	Hence	Exact Answer	Write down
Simplified Fraction	Nearest Integer	x is an integer, x>1	
Express the probability as a %		Show – Prove – Verify	

Read the key phrases carefully

"**Give an exact answer**" means leave your answer with a fraction or π or √ or ln or e in it. Don't give a rounded decimal.
"**Hence**" means use what you have just found out.
"**Write down**" means that there is no working – this shouldn't take you very long.
"**Interpret**" means that you need to write a sentence in the context of the question.

Prove or Show

If you're asked to "Prove" or "Show" something, the last line in your working should state the answer.
It's not just enough to write "As required" or "Q.E.D." – you must write out the statement.

e.g. $f(x) = 2x^3 - 7x^2 + 4x - 4$. Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

$$f(2) = 2 \, x \, 8 - 7 \, x \, 4 + 4 \, x \, 4 - 4 = 0$$

∴ *(x – 2)* is a factor of *f(x)*

Therefore $(x-2)$ is a factor.

Prove, Show, Verify

Prove means LHS$\equiv \cdots \equiv \cdots \equiv \cdots \equiv RHS$
e.g. prove that $1 + \tan^2\theta \equiv \sec^2\theta$.......
∴ Proof complete or QED
Don't take any shortcuts with proofs. Write out every step. You may know that $\frac{tan\theta}{sec\theta} = sin\theta$ but if it's part of a proof, you must go through the intermediate step.

Show means use the information to get an answer
e.g. Show that $x = 1.41$ is a solution to the equation $x^2 = 2$ correct to 3 s.f.
∴ $x = 1.41$ is a solution to the equation $x^2 = 2$

Verify means substitute a value
e.g. verify that $x = 7$ is a solution to $x^3 - 2x - 329 = 0$
When $x = 7, x^3 - 2x - 329 = 7^3 - 2x7 - 329 = 343 - 14 - 329 = 0$
∴ $x = 7$ is a solution to $x^3 - 2x - 329 = 0$

Scan the paper.
You can do the questions in any order. Start with all the familiar questions first. There may be questions that look unfamiliar. Do them last.

Read The Question

Read the question **after** you've finished it, to check you've done what it asks you to.

> (*c*) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (*a*) and hence find an approximate value for $\sqrt{2}$. Give your answer in the form $\frac{p}{q}$, where p and q are integers. **(2)**

Write on the exam paper
You can write on the exam paper. Draw diagrams to help you (e.g. area under a graph)
Do a sketch for those questions that involve tangents and normal. Don't try to hold the information in your head. If in doubt, sketch it out.
Write in black ink only. Don't use tippex or highlighters as your answer paper is scanned.

Attempt every part of every question
If you can't do part (a) don't give up. Make up a value for your answer to part (a) and then use it in subsequent parts of the question to earn method marks.

Beware of taking shortcuts with your working.

It doesn't take that long to write out an intermediate step.

$$x^2 + 9x + 4 = 2(x - 4)$$

$$x^2 + 7x - 4 = 0$$

Improve Levels of Accuracy
Always write the full calculator display down first.
Then check the level of accuracy required in the question.
Assume 3 significant figures if no accuracy stated.
Always use <u>unrounded</u> answers in any subsequent calculations.
Remember g = 9.8

Avoid basic arithmetic and algebraic errors.

$$5 - (2 - x) = 3 - x$$

$$3^2 + 5 = 11$$

$$2 + 5(-1)^3 = 7$$

Check your work
In many questions, it's possible to take your answer and substitute values back into the question.
Use the differentiation and integration buttons with limits.

Know your Calculator!
Angle units

- Pure – radians unless you see the degrees sign °
- Mechanics – normally degrees

Use brackets to tell the calculator the order of operations.
Use all the functions

- Statistics (mean, standard deviation)
- Distributions (Normal, Binomial)
- Solving quadratic equations
- Solving simultaneous equations
- Integration & Differentiation

Write in the correct space

Write within the space given for each question. Don't do question 5 in question 6's space as the papers are scanned in question by question. If you need additional paper ask for some. Label parts of questions clearly (a), (b), (c) etc.

Timing

Don't spend too long on one question.
Keep a close eye on the time.
Put a watch on your desk rather than keep looking at the clock on the wall. (Buy a cheap watch if you don't have one.)
Make sure you know how many marks for each question and aim for a minute per mark.

Use the correct notation

Integration

$$\int sin^2 x \; dx = \frac{1}{2}x - \frac{1}{4}sin\, 2x + c \qquad \textbf{NOT} \qquad \int sin^2 x = \frac{1}{2}x - \frac{1}{4}sin\, 2x$$

Differentiation

$$y = 3x^2 \quad \therefore \; \frac{dy}{dx} = 6x \qquad \textbf{NOT} \qquad \frac{d}{dx} = 6x$$

Formulae

Learn the formulae.
Quote the general form first then substitute in the correct values.
Make it clear what values you are substituting.

e.g. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

$$\frac{d}{dx}\left(\frac{e^{2x}}{\cos x}\right) = \frac{2e^{2x}\cos x \; - e^{2x}(-\sin x)}{\cos^2 x}$$

$$\text{When } x = 0, \qquad \frac{dy}{dx} = \frac{2+0}{1} = 2$$

Calculator guidance

Calculators may always be used to check answers.

Students should write down any equations that they are solving, so these can be checked and credit given.

We will always include instructions in questions to indicate that the use of a calculator is **not** permitted. Phrases used to signal that calculators should **not** be used include:

- Solutions relying entirely on calculator technology are not acceptable.
- Solutions based entirely on graphical or numerical methods are not acceptable.
- Numerical (calculator) integration/differentiation is not accepted in this question.
- Use algebraic integration/differentiation to …
- Use algebra to …
- Show that …
- Prove that …

Rate how good you think you are on each of these topics

Topic	****	***	**	*
P1. Proof				
P2. Algebra And Functions				
P3. Coordinate Geometry				
P4. Sequences And Series				
P5. Trigonometry				
P6. Exponentials And Logarithms				
P7. Differentiation				
P8. Integration				
P9. Numerical Methods				
P10. Vectors				
S1. Sampling				
S2. Data Presentation and Interpretation				
S3. Probability				
S4. Distributions				
S5. Hypothesis Testing				
M1. Units				
M2. Kinematics				
M3. Forces				
M4. Moments				

Learn these formulae (they are in the specification pages 49-52).

Appendix 1: Formulae

Formulae that students are expected to know for A Level Mathematics are given below and will not appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation $y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$u_n = a + (n-1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Mensuration

Circumference and area of circle, radius r and diameter d:

$C = 2\pi r = \pi d \qquad A = \pi r^2$

Pythagoras' theorem:

In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse, $c^2 = a^2 + b^2$

Area of a trapezium = $\dfrac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:

$s = r\theta \qquad A = \dfrac{1}{2}r^2\theta$

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \dfrac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c,\ x\neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $=\int_a^b y\ dx\ (y\geqslant 0)$

Vectors

$|x\mathbf{i}+y\mathbf{j}+z\mathbf{k}|=\sqrt{(x^2+y^2+z^2)}$

Statistics

The mean of a set of data: $\bar{x}=\frac{\sum x}{n}=\frac{\sum fx}{\sum f}$

The standard Normal variable: $Z=\frac{X-\mu}{\sigma}$ where $X\sim N(\mu,\sigma^2)$

Mechanics

Forces and Equilibrium

Weight = mass × g

Friction: $F\leqslant \mu R$

Newton's second law in the form: $F=ma$

Kinematics

For motion in a straight line with variable acceleration:

$v=\frac{dr}{dt}$ $\quad a=\frac{dv}{dt}=\frac{d^2r}{dt^2}$

$r=\int v\,dt$ $\quad v=\int a\,dt$

Know your way around the formula book (but try to learn these if you can)

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$$S_n = \tfrac{1}{2}n(a + l) = \tfrac{1}{2}n[2a + (n-1)d]$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Small angle approximations

$\sin\theta \approx \theta$

$\cos\theta \approx 1 - \dfrac{\theta^2}{2}$

$\tan\theta \approx \theta$

where θ is measured in radians

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0}\frac{f(x+h) - f(x)}{h}$$

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k\sec^2 kx$
$\sec kx$	$k\sec kx \tan kx$
$\cot kx$	$-k\,\text{cosec}^2\, kx$
$\text{cosec}\, kx$	$-k\,\text{cosec}\, kx \cot kx$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan kx$	$\frac{1}{k}\ln\lvert\sec kx\rvert$
$\cot kx$	$\frac{1}{k}\ln\lvert\sin kx\rvert$
$\operatorname{cosec} kx$	$-\frac{1}{k}\ln\lvert\operatorname{cosec} kx + \cot kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx)\rvert$
$\sec kx$	$\frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$, $\frac{1}{k}\ln\lvert\tan(\frac{1}{2}kx + \frac{1}{4}\pi)\rvert$

Numerical Methods

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0 : x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Statistics

Probability

$\mathrm{P}(A') = 1 - \mathrm{P}(A)$

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A)$

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \mid A)\mathrm{P}(A)}{\mathrm{P}(B \mid A)\mathrm{P}(A) + \mathrm{P}(B \mid A')\mathrm{P}(A')}$$

For independent events A and B,

$\mathrm{P}(B \mid A) = \mathrm{P}(B)$
$\mathrm{P}(A \mid B) = \mathrm{P}(A)$
$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

Standard deviation

Standard deviation = √(Variance)

Interquartile range = $\mathrm{IQR} = \mathrm{Q}_3 - \mathrm{Q}_1$

For a set of n values $x_1, x_2, \ldots x_i, \ldots x_n$

$$\mathrm{S}_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Standard deviation = $\sqrt{\dfrac{\mathrm{S}_{xx}}{n}}$ or $\sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$

Discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

Sampling distributions

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$$

Statistical tables

The following statistical tables are required for A Level Mathematics:

Binomial Cumulative Distribution Function (see page 29)

Percentage Points of The Normal Distribution (see page 34)

Critical Values for Correlation Coefficients: Product Moment Coefficient (see page 37)

Random Numbers (see page 38)

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$