2nd Year Assignment 27

1. The number of bacteria, n thousand per cm^3, in a sample of liquid is measured over a period of time, t, in hours. The data is shown in the table.

t	3.9	5.5	6.8	8.5	10.6	11.5	13.3	14.7	16.5	17.8
n	10.1	13.1	14.6	20.7	27.9	31.5	40	49.9	64.7	75.6

The data is coded using the changes of variable $x = t$ and $y = \log_{10} n$.

The regression line of y on x is found to be $y = 0.7606 + 0.0635x$.

(a) Given that the data can be modelled by an equation of the form $n = ab^t$ where a and b are constants, find the values of a and b.

(b) Give an interpretation of the constant a in this equation.

(c) Explain why this model is not reliable for estimating the number of bacteria after 24 hours.

2. In a factory, three machinists, Amy, Brad and Ceri, are used to sew shirts.

Amy sews 40% of the shirts.
Brad sews 35% of the shirts.
Ceri sews the rest of the shirts.

It is known that 5% of the shirts sewn by Amy are faulty, 2% of the shirts sewn by Brad are faulty and 3% of the shirts sewn by Ceri are faulty.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities.

A shirt is selected at random.

(b) Calculate the probability that the shirt is sewn by Brad and is not faulty.

(c) Calculate the probability that the shirt is faulty.

(d) Given that the shirt is faulty, find the probability that it was not sewn by Ceri.

3. The heights of a population of men are normally distributed with mean μ cm and standard deviation σ cm. It is known that 20% of the men are taller than 180 cm and 5% are shorter than 170 cm.

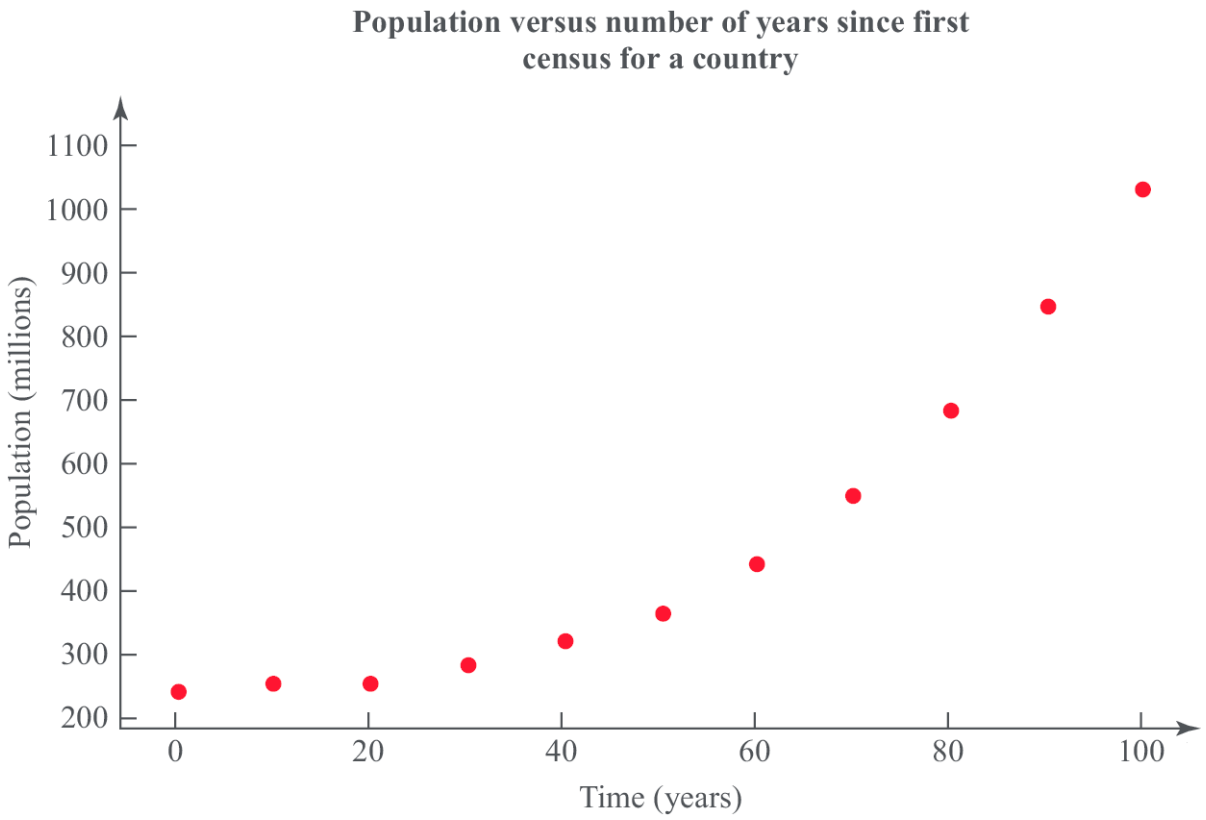
(a) Sketch a diagram to show the distribution of heights represented by this information.

(b) Find the value of μ and σ.

(c) Three men are selected at random, find the probability that they are all taller than 175 cm.

4. The data and scatter diagram show the population, p, in millions, of a country taken t years since their first census.

t	0	10	20	30	40	50	60	70	80	90	100
p	238.4	252.1	251.3	279	318.7	361.1	439.2	548.2	683.3	846.4	1028.7



(a) Give a reason why the data is coded using the changes of variable $x = t$ and $y = \log_{10} p$.

(b) The product moment correlation coefficient for the coded data is $r = 0.9735$. Comment on r for this model.

(c) With reference to your answer to part (b), state whether a model in the form $p = ab^t$, where a and b are constants, is a good fit for this data.

5. A group of students were surveyed by a principal and $\frac{2}{3}$ were found to always hand in assignments on time. When questioned about their assignments $\frac{3}{5}$ said they always start their assignments on the day they are issued and, of those who always start their assignments on the day they are issued, $\frac{11}{20}$ hand them in on time.

(a) Draw a tree diagram to represent this information.

(b) Find the probability that a randomly selected student

(i) always start their assignments on the day they are issued and hand them in on time,

(ii) does not always hand in assignments on time and does not start their assignments on the day they are issued.

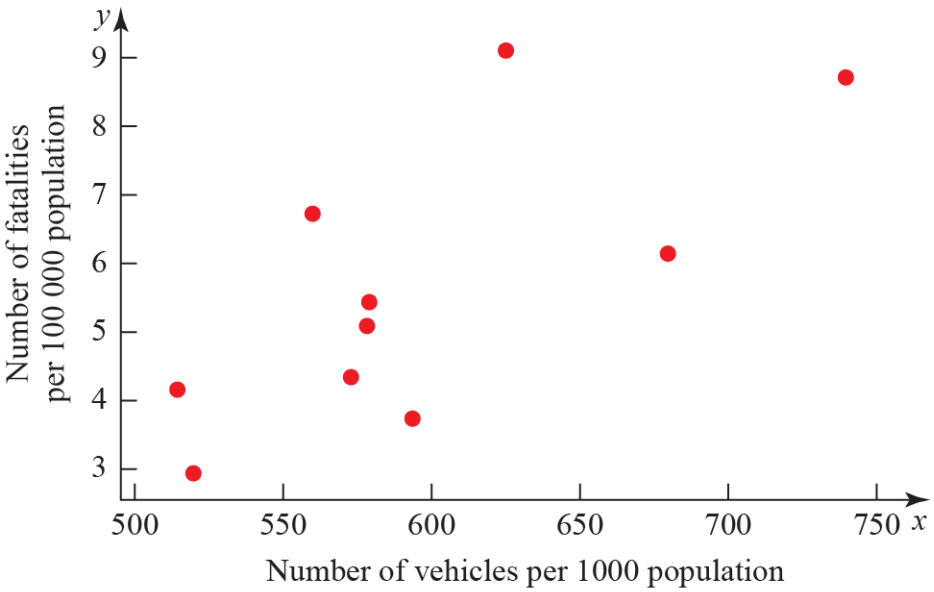
(c) Determine whether or not always starting assignments on the day they are issued and handing them in on time are statistically independent. Give reasons for your answer.

6. A researcher wishes to investigate if there is a positive correlation between the number of vehicles and the number of road fatalities in European countries.

He selects a random sample of 10 European countries and records the number of vehicles, v per 1000 people, and the number of road fatalities, r per 100 000 population, for a particular year. These are shown in the table and scatter diagrams.

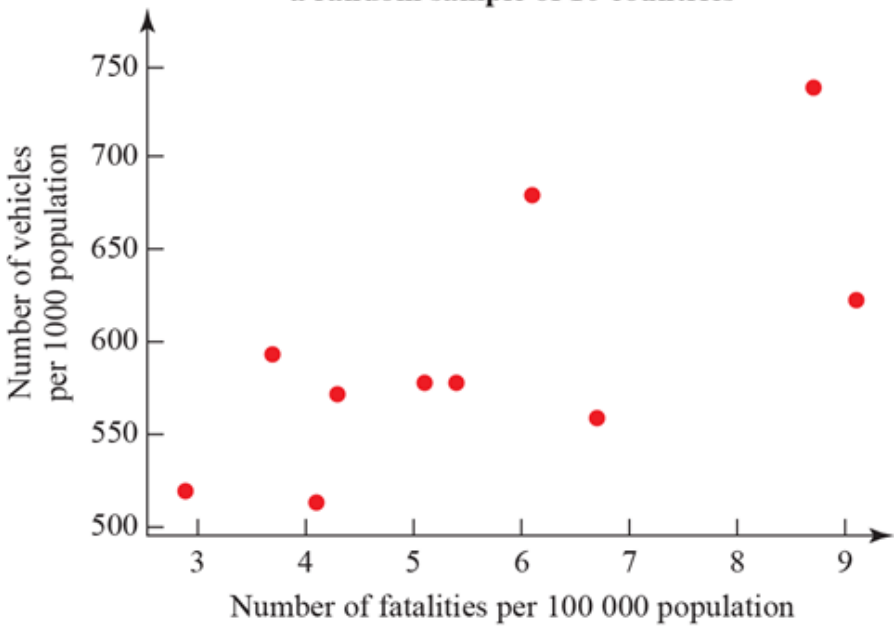
Country	***v***	***r***
Austria	578	5.4
Belgium	559	6.7
France	578	5.1
Germany	572	4.3
Greece	624	9.1
Ireland	513	4.1
Italy	679	6.1
Luxembourg	739	8.7
Spain	593	3.7
UK	519	2.9

Number of road fatalities versus Number of vehicles for a random sample of 10 countries



$r = -7.0 + 0.02y$

Number of vehicles versus fatalities for a random sample of 10 countries



$v = 460.6 + 24.0r$

(a) What is the definition of a critical value?

(b) The product moment correlaton coefficient for v and r is 0.714. Use this value to test for positive correlation at the 5% significance level. Interpret your result in context.

(c) The researcher wishes to predict the number of road fatalities for a country with 650 vehicles per 1000 people. Write down the regression model he should use.

(d) State the dependent variable for the regression model in part (c).

(e) Monaco has 899 vehicles per 1000 people. Explain why the model stated in part (c) is not reliable for estimating the number of road fatalities in Monaco.

7. An object has three different forces $\boldsymbol{F_1}$ N, $\boldsymbol{F_2}$ N and $\boldsymbol{F_3}$ N acting on its centre of mass.

$\boldsymbol{F_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and , $\boldsymbol{F_2} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$. The object is in equilibrium.

Find $\boldsymbol{F_3}$

8. A 0.1 kg inflatable ball floats on the surface of the sea. The current from the water underneath the ball exerts a force $\mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ N and the wind exerts a force of $\mathbf{W} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ N on the ball.

(a) Find the resultant force exerted on the ball.

(b) Calculate the acceleration of the ball.

Initially, the ball is at the origin and has velocity $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ N m s^{-1}.

(c) Find the x and y coordinates of the ball t s later.

(d) Find the distance travelled by the ball when $t = 10$ s.

9. Figure 3 shows Alice, who weighs 50 kg, sitting on the right-hand end of a light see-saw. Bob, who weighs 80 kg, stands on the opposite side at a distance x m from the end. The length of the see-saw is 4 m and it pivots about its centre.

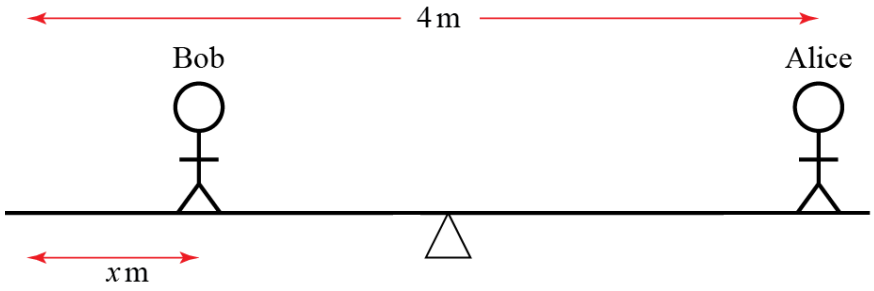


Figure 3

(a) Draw a diagram showing the forces acting on the see-saw due to the two people. Label the value of each force in newtons.

(b) Write down the total clockwise moment about the centre in terms of x.

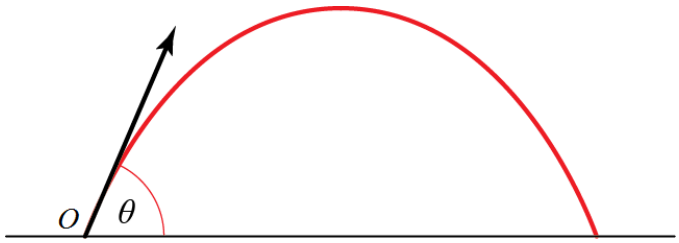
(c) Find the value of x for which the see-saw is in equilibrium.

(d) Given that Bob remains on the opposite side to Alice, describe with inequalities the range of x for which the see-saw tilts towards Alice.

(e) Describe one limitation of this model.

10. A ball is launched from the origin with speed 1 m s^{-1}. Its velocity vector makes an angle θ above the horizontal. It travels over flat ground and is modelled as a particle moving freely under gravity, as shown in the diagram.

(In this question, take $g = 10$ m s^{-2}.)



(a) Find the horizontal and vertical displacements of the particle at time t seconds. You should give your answer in terms of θ and t.

(b) Show that the horizontal distance travelled by the particle before it hits the ground is $\frac{\sin 2\theta}{10}$.

(c) Find the value θ for which the horizontal distance travelled is a maximum.

(d) Describe one limitation of this model.

11. Figure 5 shows a cylindrical object with mass 8 kg resting on two cylindrical bars of equal radius. The lines connecting the centre of each of the bars to the centre of the object make an angle of 40° to the vertical.

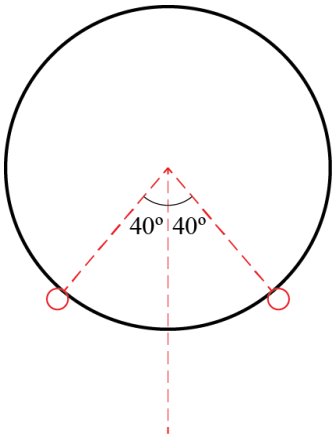


Figure 5

(a) Draw a diagram showing all the forces acting on the object. Describe each of the forces using words.

(b) Calculate the magnitude of the force on each of the bars due to the cylindrical object.