2nd Year Assignment 26

1. Show that $\dfrac{6(x+7)}{(5x-1)(2x+5)}$ can be written in the form $\dfrac{A}{5x-1}+\dfrac{B}{2x+5}$

Find the values of the constants *A* and *B*.

2. Use proof by contradiction to show that there exist no integers *a* and *b* for which $25a + 15b = 1$.

3. A curve has parametric equations $x = \cos 2t$, $y = \sin t$, $-\pi \leqslant t \leqslant \pi$.

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of *t*.
Leave your answer as a single trigonometric ratio.

(b) Find an equation of the normal to the curve at the point *A* where $t = -\dfrac{5\pi}{6}$.

4. Showing all steps, find $\int \cot 3x \, dx$.

5. A triangle has vertices $A(-2, 0, -4)$, $B(-2, 4, -6)$ and $C(3, 4, 4)$.

By considering the side lengths of the triangle, show that the triangle is a right-angled triangle.

6. The functions p and q are defined by $p : x \to x^2$ and $q : x \to 5 - 2x$.

(a) Given that pq(*x*) = qp(*x*), show that $3x^2 - 10x + 10 = 0$

(b) Explain why $3x^2 - 10x + 10 = 0$ has no real solutions.

7. Prove by contradiction that there are infinitely many prime numbers.

8. In a rainforest, the area covered by trees, *F*, has been measured every year since 1990. It was found that the rate of loss of trees is proportional to the remaining area covered by trees.

Write down a differential equation relating *F* to *t*, where *t* is the numbers of years since 1990.

9. At the beginning of each month Kath places £100 into a bank account to save for a family holiday. Each subsequent month she increases her payments by 5%. Assuming the bank account does not pay interest, find

(a) the amount of money in the account after 9 months.

Month *n* is the first month in which there is more than £6000 in the account.

(b) Show that $n > \dfrac{\log 4}{\log 1.05}$

Maggie begins saving at the same time as Kath. She initially places £50 into the same account and plans to increase her payments by a constant amount each month.

(c) Given that she would like to reach a total of £6000 in 29 months, by how much should Maggie increase her payments each month?

10. Find $\int \cos^2 6x \, dx$.

11. (a) Prove that $\dfrac{\tan x - \sec x}{1 - \sin x} \equiv -\sec x, \quad x \neq (2n+1)\dfrac{\pi}{2}$.

(b) Hence solve, in the interval $0 \leq x \leq 2\pi$, the equation $\dfrac{\tan x - \sec x}{1 - \sin x} = \sqrt{2}$.

12. A large arch is planned for a football stadium. The parametric equations of the arch are $x = 8(t+10)$, $y = 100 - t^2$, $-19 \leq t \leq 10$ where x and y are distances in metres. Find

(a) the cartesian equation of the arch,
(b) the width of the arch,
(c) the greatest possible height of the arch.

13. $$\frac{x^3+8x^2-9x+12}{x+6} = Ax^2 + Bx + C + \frac{D}{x+6}$$

Find the values of the constants *A*, *B*, *C* and *D*.

14. The volume of a sphere V cm^3 is related to its radius r cm by the formula $V = \dfrac{4}{3}\pi r^3$. The surface area of the sphere is also related to the radius by the formula $S = 4\pi r^2$. Given that the rate of decrease in surface area, in cm^2 s^{-1}, is $\dfrac{dS}{dt} = -12$, find the rate of decrease of volume $\dfrac{dV}{dt}$

15. Find $\int \sin^3 x \, dx$.

16. $$\mathrm{h}(t) = 40\, ln(t+1) + 40\, sin\left(\frac{t}{5}\right) - \frac{1}{4}t^2, \quad t \geqslant 0.$$

The graph $y = \mathrm{h}(t)$ models the height of a rocket t seconds after launch.

(a) Show that the rocket returns to the ground between 19.3 and 19.4 seconds after launch.

(b) Using $t_0 = 19.35$ as a first approximation to α, apply the Newton–Raphson procedure once to $\mathrm{h}(t)$ to find a second approximation to α, giving your answer to 3 decimal places.

(c) By considering the change of sign of $\mathrm{h}(t)$ over an appropriate interval, determine if your answer to part (b) is correct to 3 decimal places.

17. (a) Show that in ΔKLM with $\overrightarrow{KL} = 3\mathbf{i} + 0\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{LM} = 2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$, $\angle KLM = 66.4°$ to one decimal place.

(b) Hence find $\angle LKM$ and $\angle LMK$.