2nd Year Assignment 25

1. Prove by exhaustion that $1+2+3+...+n \equiv \dfrac{n(n+1)}{2}$ for positive integers from 1 to 6 inclusive.

2. (a) When θ is small, show that the equation $\dfrac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta - 1}$ can be written as $\dfrac{1}{1-3\theta}$.

(b) Hence write down the value of $\dfrac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta - 1}$ when θ is small.

3. A stone is thrown from the top of a building. The path of the stone can be modelled using the parametric equations $x=10t$, $y=8t-4.9t^2+10$, $t \geq 0$, where x is the horizontal distance from the building in metres and y is the vertical height of the stone above the level ground in metres.

(a) Find the horizontal distance the stone travels before hitting the ground.

(b) Find the greatest vertical height.

4. Given that $x=\sec 4y$, find

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{k}{x\sqrt{x^2-1}}$, where k is a constant which should be found.

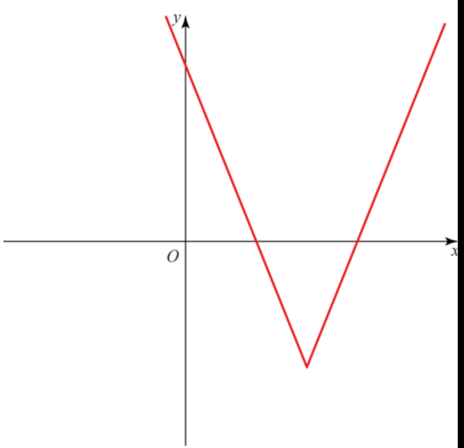
5.

$$\mathrm{f}(x)=\frac{6}{x}+\frac{3}{x^2}-7x^{\frac{5}{2}}$$

(a) Find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(b) Evaluate $\int_4^9 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $m+n\ln p$, where m, n and p are rational numbers.

6. The diagram shows a sketch of part of the graph y = f(x) where $\mathrm{f}(x)=3|x-4|-5$



(a) State the range of f.

(b) Given that $\mathrm{f}(x)=-\dfrac{1}{3}x+k$, where k is a constant has two distinct roots, state the possible values of k.

7. $$\mathrm{f}(x) \equiv \frac{9x^2 + 25x + 16}{9x^2 - 16}$$

Show that $\mathrm{f}(x)$ can be written in the form $A + \frac{B}{3x - 4} + \frac{C}{3x + 4}$, where *A*, *B* and *C* are constants to be found.

8. A ball is dropped from a height of 80 cm. After each bounce it rebounds to 70% of its previous maximum height.

(a) Write a recurrence relation to model the maximum height in centimetres of the ball after each subsequent bounce.

(b) Find the height to which the ball will rebound after the fifth bounce.

(c) Find the total vertical distance travelled by the ball before it stops bouncing.

(d) State one limitation with the model.

9. Solve $6\sin(\theta + 60) = 8\sqrt{3}\cos\theta$ in the range $0 \leqslant \theta \leqslant 360°$. Round your answer to 1 decimal place.

10. Use proof by contradiction to show that there is no greatest positive rational number.

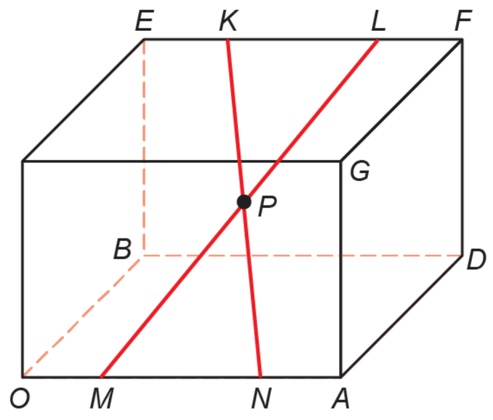
11. The first three terms in the binomial expansion of $(a + bx)^{\frac{1}{3}}$ are $4 - \frac{1}{8}x + cx^2 + \ldots$.

(a) Find the values of *a* and *b*.

(b) State the range of values of *x* for which the expansion is valid.

(c) Find the value of *c*.

12. The diagram shows a cuboid whose vertices are *O*, *A*, *B*, *C*, *D*, *E*, *F* and *G*. **a**, **b** and **c** are the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. The points *M* and *N* lie on *OA* such that $OM : MN : NA = 1:2:1$. The points *K* and *L* lie on *EF* such that $EK : KL : LF = 1:2:1$.



Prove that the diagonals *KN* and *ML* bisect each other at *P*.

13. The value of a computer, V, decreases over time, t, measured in years. The rate of decrease of the value is proportional to the remaining value.

Given that the initial value of the computer is V_0 ,

(a) show that $V = V_0 e^{-kt}$.

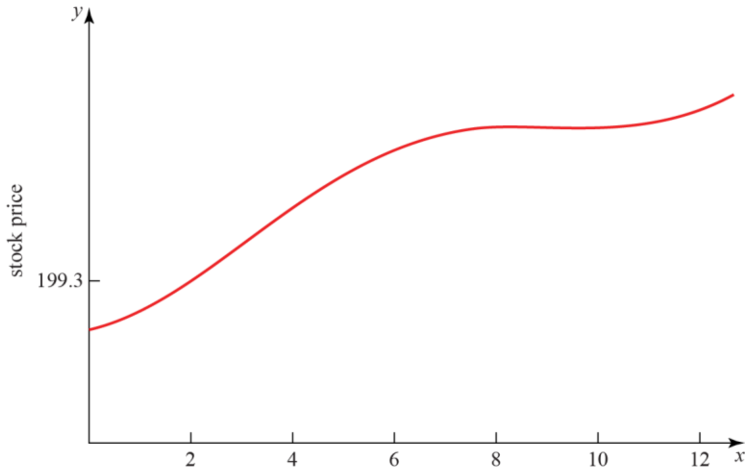
After 10 years the value of the computer is $\frac{1}{5}V_0$.

(b) Find the exact value of k.

(c) How old is the computer when its value is only 5% of its original value? Give your answer to 3 significant figures.

14.

$$p(t) = \frac{1}{10}\ln(t+1) - \cos\left(\frac{t}{2}\right) + \frac{1}{10}t^{\frac{3}{2}} + 199.3, \quad 0 \leqslant t \leqslant 12 .$$



The diagram is a graph of the price of a stock during a 12-hour trading window. The equation of the curve is given above.

(a) Show that the price reaches a local maximum in the interval $8.5 < t < 8.6$.

Figure 3 shows that the price reaches a local minimum between 9 and 11 hours after trading begins.

(b) Using the Newton–Raphson procedure once and taking $t_0 = 9.9$ as a first approximation, find a second approximation of when the price reaches a local minimum.