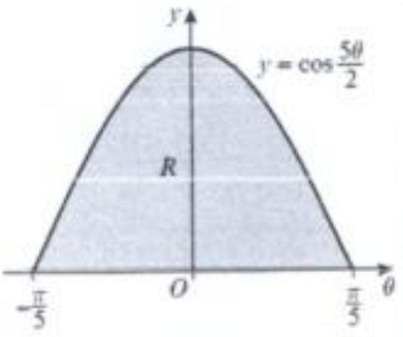
2 The diagram shows the region R bounded by the x-axis and the curve with equation $y = \cos\frac{5\theta}{2}$, $-\frac{\pi}{5} \leqslant \theta \leqslant \frac{\pi}{5}$



The table shows corresponding values of θ and y for $y = \cos\frac{5\theta}{2}$

θ	$-\frac{\pi}{5}$	$-\frac{\pi}{10}$	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$
y	0		1		0

a Complete the table giving the missing values for y to 4 decimal places. **(1 mark)**

b Using the trapezium rule, with all the values for y in the completed table, find an approximation for the area of R, giving your answer to 3 decimal places. **(4 marks)**

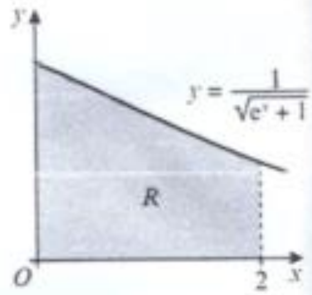
c State, with a reason, whether your approximation in part **b** is an underestimate or an overestimate. **(1 mark)**

d Use integration to find the exact area of R. **(3 marks)**

e Calculate the percentage error in your answer in part **b**. **(2 marks)**

3 The diagram shows a sketch of the curve with equation $y = \frac{1}{\sqrt{e^x + 1}}$

The shaded region R is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.



a Complete the table giving values of y to 3 decimal places. **(2 marks)**

x	0	0.5	1	1.5	2
y	0.707	0.614	0.519		0.345

b Use the trapezium rule, with all the values from your table, to estimate the area of the region R, giving your answer to 2 decimal places. **(4 marks)**

2 a 0.7071, 0.7071 **b** 0.758

c The shape of the graph is concave, so the trapezium lines will underestimate the area.

d 0.8 **e** 5.25%

3 a 0.427 **b** 1.04

Integration – Differential Equations

1 Find general solutions to the following differential equations. Give your answers in the form $y = \mathrm{f}(x)$.

a $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (1 + y)(1 - 2x)$

b $\dfrac{\mathrm{d}y}{\mathrm{d}x} = y \tan x$

c $\cos^2 x \dfrac{\mathrm{d}y}{\mathrm{d}x} = y^2 \sin^2 x$

d $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{x-y}$

2 Find particular solutions to the following differential equations using the given boundary conditions.

a $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sin x \cos^2 x;\ y = 0,\ x = \dfrac{\pi}{3}$

b $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x \sec^2 y;\ y = 0,\ x = \dfrac{\pi}{4}$

c $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos^2 y \cos^2 x;\ y = \dfrac{\pi}{4},\ x = 0$

d $\sin y \cos x \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cos y}{\cos x},\ y = 0,\ x = 0$

3 a Find the general solution to the differential equation $x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} = y + xy$, giving your answer in the form $y = \mathrm{g}(x)$.

Hint Begin by factorising the right-hand side of the equation.

b Find the particular solution to the differential equation that satisfies the boundary condition $y = \mathrm{e}^4$ at $x = -1$.

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1 a $y = A\mathrm{e}^{x-x^2} - 1$ **b** $y = k\sec x$

c $y = \dfrac{-1}{\tan x - x + c}$ **d** $y = \ln|2\mathrm{e}^x + c|$

2 a $\dfrac{1}{24} - \dfrac{\cos^3 x}{3}$ **b** $\sin 2y + 2y = 4\tan x - 4$

c $\tan y = \frac{1}{2}\sin 2x + x + 1$ **d** $y = \arccos(\mathrm{e}^{-\tan x})$

3 a $y = Ax\mathrm{e}^{-\frac{1}{x}}$ **b** $y = -\mathrm{e}^3 x\mathrm{e}^{-\frac{1}{x}} = -x\mathrm{e}^{\frac{3x-1}{x}}$ ← $\dfrac{3x-1}{x}$