Exercise 11G

Integration 7 – Partial Fractions

1 Use partial fractions to integrate the following:

a $\dfrac{3x+5}{(x+1)(x+2)}$ **b** $\dfrac{3x-1}{(2x+1)(x-2)}$ **c** $\dfrac{2x-6}{(x+3)(x-1)}$ **d** $\dfrac{3}{(2+x)(1-x)}$

2 Find the following integrals.

a $\int \dfrac{2(x^2+3x-1)}{(x+1)(2x-1)}\,dx$ **b** $\int \dfrac{x^3+2x^2+2}{x(x+1)}\,dx$ **c** $\int \dfrac{x^2}{x^2-4}\,dx$ **d** $\int \dfrac{x^2+x+2}{3-2x-x^2}\,dx$

3 $f(x) = \dfrac{4}{(2x+1)(1-2x)}$, $x \neq \pm\dfrac{1}{2}$

a Given that $f(x) = \dfrac{A}{2x+1} + \dfrac{B}{1-2x}$, find the value of the constants A and B.

b Hence find $\int f(x)\,dx$, writing your answer as a single logarithm.

c Find $\int_1^2 f(x)\,dx$, giving your answer in the form $\ln k$ where k is a rational constant.

Exercise 11H

Integration 8 – Finding areas

1 Find the area of the finite region R bounded by the curve with equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$.

a $f(x) = \dfrac{2}{1+x}$; $a = 0$, $b = 1$ **b** $f(x) = \sec x$; $a = 0$, $b = \dfrac{\pi}{3}$ **c** $f(x) = \ln x$; $a = 1$, $b = 2$

d $f(x) = \sec x \tan x$; $a = 0$, $b = \dfrac{\pi}{4}$ **e** $f(x) = x\sqrt{4-x^2}$; $a = 0$, $b = 2$

2 Find the exact area of the finite region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ where:

a $f(x) = \dfrac{4x-1}{(x+2)(2x+1)}$; $a = 0$, $b = 2$ **b** $f(x) = \dfrac{x}{(x+1)^2}$; $a = 0$, $b = 2$

c $f(x) = x \sin x$; $a = 0$, $b = \dfrac{\pi}{2}$ **d** $f(x) = \cos x\sqrt{2\sin x + 1}$; $a = 0$, $b = \dfrac{\pi}{6}$

e $f(x) = xe^{-x}$; $a = 0$, $b = \ln 2$

Exercise 11G

1 **a** $\ln|(x+1)^2(x+2)| + c$ **b** $\ln|(x-2)\sqrt{2x+1}| + c$

c $\ln\left|\dfrac{(x+3)^3}{x-1}\right| + c$ **d** $\ln\left|\dfrac{2+x}{1-x}\right| + c$

2 **a** $x + \ln|(x+1)^2\sqrt{2x-1}| + c$ **b** $\dfrac{x^2}{2} + x + \ln\left|\dfrac{x^2}{(x+1)^3}\right| + c$

c $x + \ln\left|\dfrac{x-2}{x+2}\right| + c$ **d** $-x + \ln\left|\dfrac{(3+x)^2}{1-x}\right| + c$

3 **a** $A = 2$, $B = 2$ **b** $\ln\left|\dfrac{2x+1}{1-2x}\right| + c$ **c** $\ln\frac{5}{9}$, so $k = \frac{5}{9}$

Exercise 11H

1 **a** $2\ln 2$ **b** $\ln(2+\sqrt{3})$ **c** $2\ln 2 - 1$

d $\sqrt{2} - 1$ **e** $\frac{8}{3}$

2 **a** $\ln\frac{8}{5}$ **b** $\ln 3 - \frac{2}{3}$ **c** 1

d $\dfrac{2\sqrt{2}-1}{3}$ **e** $\frac{1}{2}(1 - \ln 2)$

3 $\ln 4$