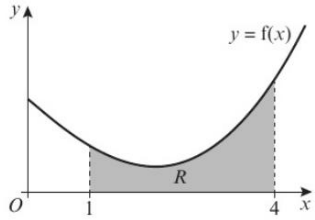
2nd Year Assignment 24

1. The diagram shows the sketch of the curve $y = f(x)$, where $f(x) = \frac{1}{5}x^2 ln\, x - x + 2, \;\; x > 0.$

The region R, shown in the diagram, is bounded by the curve, the $x - axis$ and the lines with equations $x = 1$ and $x = 4$.

The table below shows the corresponding values of x and y with the y values given to 4 decimal places as appropriate.



x	1	1.5	2	2.5	3	3.5	4
y	1	0.6825	0.5545	0.6454		1.5693	2.4361

a) Complete the table with the missing value of y

b) Use the trapezium rule, with all the values in the table, to obtain an estimate for the area of R, giving your answer to 3 decimal places

c) C) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of R

d) Show that the exact area of R can be written in the form $\frac{a}{b} + \frac{c}{d}\ln e$, where a, b, c, d and e are integers

e) Find the percentage error in the answer in part b).

2. Find the particular solution of each differential equation

a. $\frac{dy}{dx} = \frac{x}{2y}, \quad y = 3 \text{ when } x = 4$

b. $\frac{dy}{dx} = (y+1)^3, \quad y = 0 \text{ when } x = 2$

c. $(\tan^2 x)\frac{dy}{dx} = y, \quad y = 1 \text{ when } x = \frac{\pi}{2}$

d. $\frac{dy}{dx} = \frac{y+2}{x-1}, \quad y = 6 \text{ when } x = 3$

e. $\frac{dy}{dx} = x^2 \tan y, \quad y = \frac{\pi}{6} \text{ when } x = 0$

f. $\frac{dy}{dx} = \sqrt{\frac{y}{x+3}}, \quad y = 16 \text{ when } x = 1$

g. $(e^x)\frac{dy}{dx} = x \operatorname{cosec} y, \quad y = \pi \text{ when } x = -1$

h. $\frac{dy}{dx} = \frac{1+\cos y}{2x^2 \sin y}, \quad y = \frac{\pi}{3} \text{ when } x = 1$

3. a) Express $\frac{8x-18}{(3x-8)(x-2)}$ in partial fractions

b) Given that $x \geq 3$, find the general solution to the differential equation

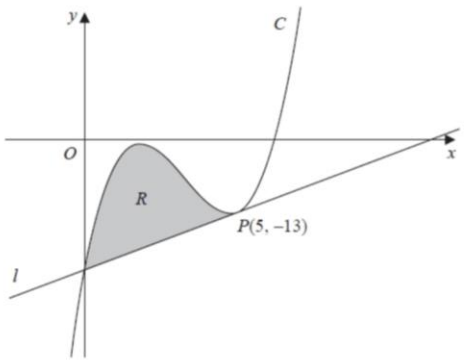
$(x-2)(3x-8)\frac{dy}{dx} = (8x-18)y$

c) hence find the particular solution to this differential equation that satisfies $y = 8$ at $x = 3$, giving your answer in the form $y = f(x)$

4. Prove the following trigonometric identities
   a) $\sec\theta\, cosec\ \theta \equiv 2cosec\ 2\theta$
   b) $\tan(\frac{\pi}{4}+x) - \tan\left(\frac{\pi}{4}-x\right) \equiv 2\tan 2x$
   c) $\sin(x+y)\sin(x-y) \equiv \cos^2 y - \cos^2 x$
   d) $1 + \cos 2\theta + \cos 4\theta \equiv 4\cos^2\theta\cos 2\theta$

5. The diagram shows a sketch of part of the curve C with equation $y = x^3 - 10x^2 + 27x - 23$

   The point $P(5, -13)$ lies on C The line l is the tangent to C at P

   (a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

   (b) Hence verify that l meets C again on the y-axis.

   The finite region R, shown shaded in the diagram, is bounded by the curve C and the line l.

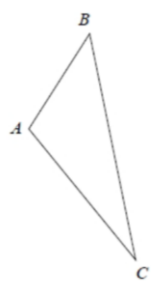
   (c) Use algebraic integration to find the exact area of R.

6. (a) Given that $2\log(4 - x) = \log(x + 8)$ show that $x^2 - 9x + 8 = 0$

   (b) (i) Write down the roots of the equation $x^2 - 9x + 8 = 0$

   (ii) State which of the roots in (b)(i) is not a solution of $2\log(4 - x) = \log(x + 8)$ giving a reason for your answer.

7. The diagram shows a sketch of a triangle ABC. Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$, show that ∠BAC = 105.9° to one decimal place.



8. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

9. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y, -\frac{\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(b) (i) Use the small angle approximation for $sin\ 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i)

(c) Show that, for all points (x, y) lying on C, $\frac{dy}{dx} = \frac{1}{a\sqrt{b-x^2}}$

where a and b are constants to be found.

10. A brick of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal.

A force of 12 newtons acts on the brick acting up the plane and the brick is in equilibrium on the point of slipping up the plane.

Find (a) The coefficient of friction between the brick and the plane

The 12 N force is removed.

(b) Determine whether or not the brick moves.

Test Yourself

How many of these formulae can you remember?

Pythagoras

1. $\cos^2 A + \sin^2 A =$
2. $\sec^2 A =$ (don't write $\frac{1}{\cos^2 A}$)
3. $\text{cosec}^2 \text{A} =$ (don't write $\frac{1}{\sin^2 A}$)

Double angles

4. $\cos 2A =$
5. $\cos 2A =$
6. $\cos 2A =$

7. $\sin 2A =$
8. $\tan 2A =$

Differentiate

9. $\sin x$
10. $\cos x$
11. $\tan x$
12. $\text{cosec } x$
13. $\sec x$
14. $\cot x$

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A

15. $s =$
16. $A =$