2nd Year Assignment 23

1. Carry out the following integrations. (For the definite integrals, you must carry out the integration yourself – don't just put the expression into your calculator. By all means, check your work like this).

Remember that if you have to integrate a fraction, ask yourself whether if you differentiate the denominator, does it equal the numerator. If the answer is yes, the answer is $ln\ (denominator)$

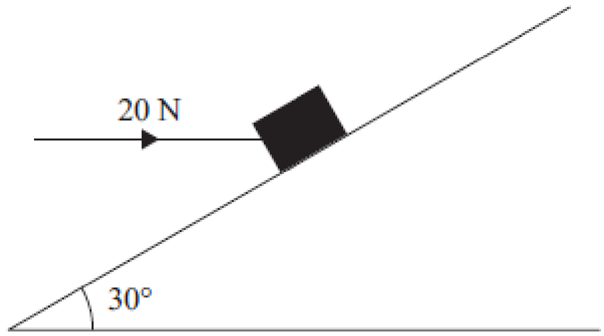
a. $\int \frac{1}{2} x\, e^{4x} dx$
b. $\int 2x^2 \sec^2 x \tan x\, dx$
c. $\int \frac{3x-1}{\sqrt{4x-1}} dx$ (use $u = 4x - 1$)
d. $\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}} dx$ (use $x = \sec\theta$)
e. $\int \sin^2 3x\, dx$
f. $\int \frac{2^x}{2^x+1} dx$
g. $\int \frac{x+2}{x+5} dx$ (divide $(x+2)$ by $(x+1)$ and then integrate)
h. $\int \frac{x}{x^2+9} dx$
i. $\int \frac{\ln x^2}{x} dx$
j. $\int \frac{1}{\sqrt{x}\, cos^2\sqrt{x}}\ dx$
k. $\int_1^2 \frac{32x^2+4}{(4x+1)(4x-1)} dx$ (give your answer in the form $2 + k \ln m$)
l. $\int \frac{x}{\sqrt{x+1}} dx$ (use $t^2 = x + 1$)
m. $\int_{\frac{\pi}{12}}^{\frac{\pi}{3}} (\cos x + \sin x)(\cos x - \sin x)\, dx$
n. $\int_1^4 \frac{4}{16x^2+8x-3} dx$

2. a) $f(x) = (1-x)^{\frac{1}{3}}, \quad -1 < x < 1$
Find the binomial expansion of *f* (*x*) in ascending powers of *x* up and including the term in x^2
$g(x) = (8-3x)^{\frac{1}{3}}, \quad -\frac{8}{3} < x < \frac{8}{3}$
b) Use the result of part (a) to find the binomial expansion of *g* (*x*) in ascending powers of *x* up and including the term in x^2.

c) Hence, show that $\sqrt[3]{7} \approx \frac{551}{288}$

3. By considering the compound angle identity for $\tan(A+B)$, with suitable values for A and B, show that $\cot 75° = 2 - \sqrt{3}$.

4. A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in the diagram. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.

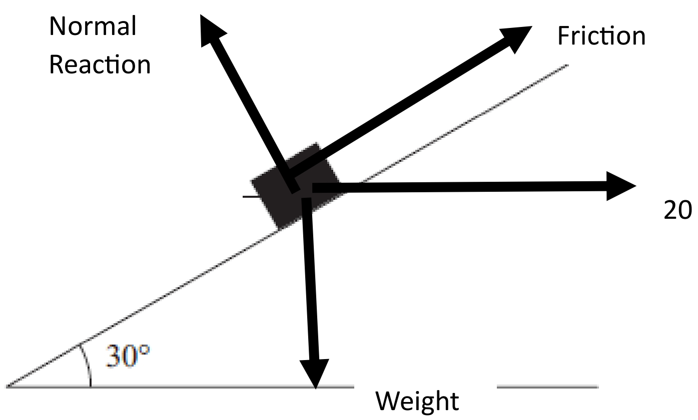


The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle. Find

(*a*) the magnitude of the normal reaction of the plane on the box,

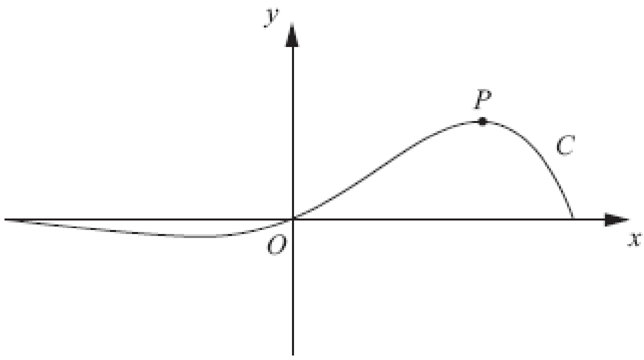
(*b*) the coefficient of friction between the box and the plane.

Hint: Start by drawing a diagram showing all the forces.



Then resolve parallel and perpendicular to the plane

5. The diagram shows a sketch of the curve C which has equation $y = e^{x\sqrt{3}} \sin 3x$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.



(*a*) Find the *x*-coordinate of the turning point P on C, for which $x > 0$. Give your answer as a multiple of π.

(*b*) Find an equation of the normal to C at the point where $x = 0$.

Hint: To differentiate $y = e^{x\sqrt{3}} \sin 3x$, use the product rule. For (b) find y and $\frac{dy}{dx}$ when $x = 0$ and then use $y - y_1 = m(x - x_1)$

6. Use the trapezium rule with 8 strips to estimate the value of $\int_{-2}^{2} e^{x^2} dx$

7. $f(x) = x^3 + 3x^2 - 2\sqrt{x}\ ,\ \ x > 0$

(a) Show that f(x) = 0 has a root in the interval [0.6 , 0.7]

(b) Find f'(x)

(c) Staring with $x_0 = 0.65$, apply the Newton-Raphson procedure once to find an approximate solution to the equation $f(x) = 0$ giving your answer to 3 decimal places.

8. A curve has the equation $x^2 + 4xy - x + y^2 = 35$

(a) Find an expression for $\frac{dy}{dx}$

(b) Find an equation for the tangent to the curve at the point $P\ (2, 3)$