2nd Year Assignment 22

1. (a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that $\cos 2A = 1 - 2\sin^2 A$

HINT: Put A = B

The curves C_1 and C_2 have equations C_1: $y = 3\sin 2x$, C_2: $y = 4\sin^2 x - 2\cos 2x$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

HINT: Use $\cos 2x = 1 - 2\sin^2 x$ (Rearrange to make $\sin x$ the subject)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

HINT: Look at this example

Example 5: Express $cos2x - 2sin2x$ in the form $Rcos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Proving the double-angle sine formula:	$1cos2x - 2sin2x = Rcos(2x + \alpha) \equiv Rcos2x\ cos\alpha - Rsin2x\ sin\alpha$
Equating coefficients:	$1 = Rcos\alpha$ (1) (equating $cos2x$ coefficients) $-2 = -Rsin\alpha$ (2) (equating $sin2x$ coefficients)
Solving simultaneously.: We divide equation [2] by [1].	$tan\alpha = \frac{Rsin\alpha}{Rcos\alpha} = \frac{-2}{1} = -2$ $\therefore \alpha = \arctan(-2) = 1.11$
Finding R: Square equations [1] and [2] then add them together. We also use the identity $cos^2\alpha + sin^2\alpha \equiv 1$	$(1)^2 + (2)^2 \Rightarrow R^2cos^2\alpha + R^2sin^2\alpha = (1)^2 + (-2)^2$ $\Rightarrow R^2(cos^2\alpha + sin^2\alpha) = 5$ $\Rightarrow R^2 = 5 \therefore R = \sqrt{5}$
Putting everything together:	So $cos2x - 2sin2x = \sqrt{5}cos(2x + 1.11)$

A shortcut for finding R is to use $R = \sqrt{a^2 + b^2}$

(d) Hence find, for $0 \le x < 180°$, all the solutions of $4\cos 2x + 3\sin 2x = 2$, giving your answers to 1 decimal place.

HINT: Use your answer to part (c)

2. Solve, for $-180° < \theta < 180°$, $(1 + tan\theta)(5sin\theta - 2) = 0$

HINT: If two things multiply together to make zero, one of them must be zero.

So you need to solve two equations: $1 + tan\theta = 0$ and $5sin\theta - 2 = 0$

3. Integrate the following

a) $\int 3\sin^2 x \; dx$

b) $\int 4\cos^2 x \, dx$

c) $\int 3\sin x \cos x \, dx$

d) $\int (2 - 3\sin x)^2 \, dx$

e) $\int (1 - \cos 2x)^2 \, dx$

f) $\int 2\tan^2 x \, dx$

g) $\int 5\cot^2 x \, dx$

h) $\int (2\tan x - \cot x)^2 \, dx$

i) $\int \frac{4\sin x}{\cos^2 x} dx$

j) $\int \frac{\cos x}{3\sin^2 x} dx$

4. $f(x) = \frac{12x+5}{(1+4x)^2}, \quad |x| < \frac{1}{4}$

For $x \neq \frac{1}{4}, \frac{12x+5}{(1+4x)^2} = \frac{A}{1+4x} + \frac{B}{(1+4x)^2}$, where A and B are constants

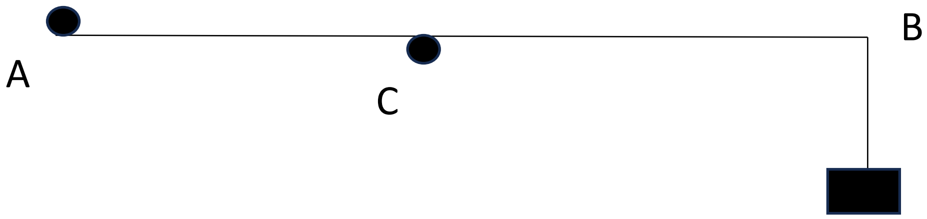
a) Find the values of A and B
b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term x^2, simplifying each term.

5. A small company decided to import fine Chinese porcelain. They believed that in the long term this would prove to be an increasingly profitable arrangement with profits increasing proportionally to sales. Over the next six years their sales and profits were as shown in the table below.

Year	1994	1995	1996	1997	1998	1999
Sale in thousands	165	165	170	178	178	175
Profits in £1000	65	72	75	76	80	83

Using a 1% significance level, test to see if there is any evidence that the company's beliefs were correct, and that profit and sales were positively correlated.

6.



A box of mass 76 kg is attached by a string to one end B of a uniform rod AB of length 5 m and mass 24 kg .

The rod is held horizontally in equilibrium by two smooth cylindrical pegs, one at A and one at C , where AC = 2 m, as shown in the figure above.

Calculate the magnitude of the forces exerted by each of the pegs onto the rod.

7. The functions f and g are defined by

$f : x \to ln\,(3x - 2), x \in \mathbb{R}, x > \frac{2}{3}$

$g : x \to \frac{3}{x-2}, x \in \mathbb{R}, x \neq 2$

(a) Find the exact value of $fg(3)$

(b) Find an expression for $f^{-1}(x)$ and state its domain

(c) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same diagram

(d) Sketch the graph of $y = |g(x)|$

(e) Solve the equation $|\frac{3}{x-2}| = 4$, giving exact values for your answer

8. A variable x was measured to the nearest whole number. 50 observations are given in the table below.

x	5 – 8	9 – 13	14 – 20	21 – 24
Frequency	10	9	14	17

A histogram was drawn and the bar representing the 9 – 13 class has a width of 2 cm and a height of 2.7 cm. Find the width and height of the bar representing the 14 – 20 class.

9. The points D, E and F have coordinates (-3, 2), (4, -1) and (1, -8) respectively.

(a) Show that angle DEF is a right angle.

Given that D, E and F all lie on the circle C.

(b) Find the coordinates of the centre of C.

(c) Find the equation of the circle C.

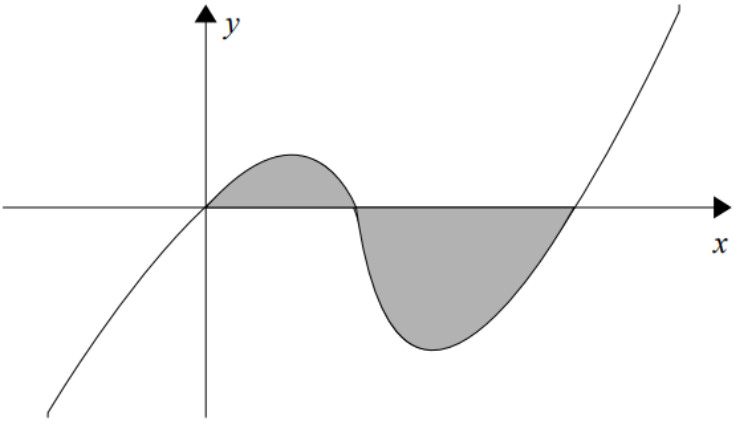
10. (a) Prove that $n^2 + n + 11$ is prime for all integers between 1 and 5.

(b) Prove that $n^2 + n + 11$ is not prime for all values of n.

Test Yourself

Give yourself 20 minutes to answer these questions.

A)



The sketch shows the curve $y = x(x - 2)(x - 5)$

(a) Write down the values of x where the curve crosses the x axis.

(b) Find the area of the shaded region.

B)

Given that the point A has position vector $-5\mathbf{i} + 7\mathbf{j}$ and the point B has position vector $-8\mathbf{i} + 2\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$

(b) Find $|\overrightarrow{AB}|$