2nd Year Assignment 20

1. $f(x) = (3x-2)(x-k)-8$, where *k* is a constant.

(a) Write down the value of $f(k)$

When $f(x)$ is divided by $(x-2)$ the remainder is 4

(b) Find the value of *k*.

(c) Factorise f (x) completely.

HINT: Use the remainder theorem:

If a polynomial, $f(x)$, is divided by $(ax - b)$, the remainder is f($\frac{b}{a}$)

e.g. when $f(x) = x^2 + 7x + 12$ is divided by $(x-3)$, the remainder is f(3).
Since $f(3) = 0$, the remainder is 0.
This means that $(x-3)$ is a factor of $x^2 + 7x + 12$

2.

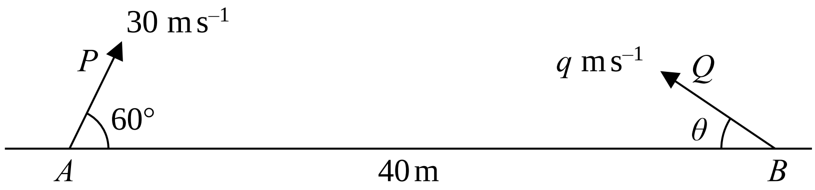


Figure 4

The points *A* and *B* lie 40 m apart on horizontal ground. At time *t* = 0 the particles *P* and *Q* are projected in the vertical plane containing *AB* and move freely under gravity. Particle *P* is projected from *A* with speed 30 m s^{-1} at 60° to *AB* and particle *Q* is projected from *B* with speed *q* m s^{-1} at angle θ to *BA*, as shown in Figure 4.
At *t* = 2 seconds, *P* and *Q* collide.

(*a*) Find

(i) the size of angle θ,

(ii) the value of *q*.

(*b*) Find the speed of *P* at the instant before it collides with *Q*.

HINT: Use suvat in two dimensions

A	**Vertical**	**Horizontal**
s	**y**	$\boldsymbol{x}$
u	**30 sin 60°**	**30 cos 60°**
v		
a	**-9.8**	**0**
t	**2**	**2**

B	**Vertical**	**Horizontal**
s	**y**	$\boldsymbol{40 - x}$
u	**q sin $\boldsymbol{\theta}$**	**-q cos $\boldsymbol{\theta}$**
v		
a	**-9.8**	**0**
t	**2**	**2**

3. For each of the following, find $\frac{dy}{dx}$

a) $y = \frac{1}{2}x^2 + 6\cos(3x^3)$

b) $y = \ln(5x^2) - e^{3cos2x}$

c) $y = \ln(5\cos(6x^3))$

d) $y = e^{7x^2}\sin(5x^4)$

e) $y = \frac{e^{7x^2}}{\sin(5x^4)}$

f) $y = \sec(8x^3)$

g) $x = 3t^2\sin(6x)$, $y = \ln(3\cot x)$

h) $5x^3 - 4x^2y^3 - 3\mathrm{cosec}(4y^5) = 7$

4. A curve has equation $y = e^x(x^2 - 2x + 2)$

a) Find the exact co-ordinates of the stationary point on C and determine its nature

b) Find the co-ordinates of any non-stationary points of inflection on C

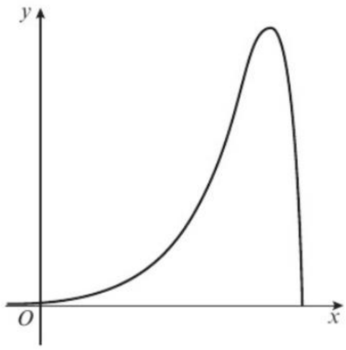
5. At time, t seconds, the surface area of a cube is $A\ cm^2$ and the volume is $V\ cm^3$. The surface area of the cube is expanding at a constant rate of $2\ cm^2s^{-1}$

a) Write an expression for V in terms of A

b) Find an expression for $\frac{dV}{dA}$

c) Find an expression for $\frac{dV}{dt}$ in terms of V

6. A computer model for the shape of a rollercoaster is given by the parametric equations



$x = 5 + \ln t, \quad y = 5sin2t, \quad 0 < t \leq \frac{\pi}{2}$

a) Find the coordinates of the point where $t = \frac{\pi}{6}$

Given that one unit on the model represents 5m in real life,

b) Find the maximum height of the rollercoaster

c) Find the horizontal distance covered during the descent of the rollercoaster

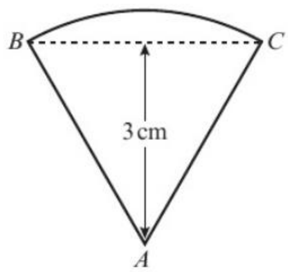
d) Hence find the average gradient of the descent.

7.a) Express $5\sin^2\theta - 3\cos^2\theta + 6\,sin\theta cos\theta$ in the form $asin2\theta + bcos2\theta + c$, where a, b and c are constants to be found.

b) Hence find the maximum and minimum values of $5\sin^2\theta - 3\cos^2\theta + 6\,sin\theta cos\theta$

c) Solve $5\sin^2\theta - 3\cos^2\theta + 6\,sin\theta cos\theta = -1$ for $0 \leq \theta < 180°$, rounding your answer to 1 decimal place.

8. The shape of a badge is a sector ABC of a circle with centre A and radius AB as shown in the diagram. The triangle ABC is equilateral and has perpendicular height 3 cm.



a) Find, in surd form, the length AB

b) Find, in terms of π, the area of the badge

c) Prove that the perimeter of the badge is $a(\pi + b)cm$, where a and b are exact values to be found.

9. A convergent geometric series has first term *a* and common ratio *r*. The second term of the series is -3 and the sum to infinity of the series is 6.75

a) Show that $27r^2 - 27r - 12 = 0$

b) Given that the series is convergent, find the value of *r*

c) Find the sum of the first 5 terms of the series, giving your answer to 2 decimal places.

10. $f(x) = 4cotx - 8x + 3,\ \ 0 < x < \pi$, where *x* is in radians.

a) Show that there is a root, α, of $f(x) = 0$ in the interval [0.8, 0.9]

b) Show that the equation $f(x) = 0$ can be written in the form $x = \frac{(cosx)}{(2sinx)} + \frac{3}{8}$

c) Use the iterative formula $x_{n+1} = \frac{cosx_n}{2sinx_n} + \frac{3}{8},\ \ x_0 = 0.85$, to calculate the values of x_1, x_2 and x_3, giving your answer to 4 decimal places

d) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 0.831$, correct to 3 decimal places.

Test Yourself

Give yourself 20 minutes to answer these questions.

Find the following integrals, using

a) your knowledge of differentiation and

b) understanding that integration is the inverse of differentiation

1. $\int \cos x\, dx$
2. $\int -\sin x\, dx$
3. $\int \frac{1}{x} dx$
4. $\int e^x dx$
5. $\int \cos 2x\, dx$
6. $\int e^{4x} dx$
7. $\int e^x + xe^x dx$
8. $\int 12x(x^2 + 1)^5 dx$