2nd Year Assignment 19

1. A curve has the equation $y = 2sin2x + cos2x$. Find the stationary points of the curve in the interval $0 \leq x \leq \pi$
2. The population of Cambridge was 37 000 in 1900, and was about 109 000 in 2000. Given that the population, P, at a time t years after 1900 can be modelled using the equation$P = P_o k^t$
    a. Find the values of P_o and k
    b. Evaluate $\frac{dP}{dt}$ in the year 2000
    c. Interpret your answer to part b) in the context of the model.
3. The curve C has equation $x = 4cos2y$
    a. Show that the point Q(2,$\frac{\pi}{6}$) lies on C
    b. Show that $\frac{dy}{dx} = -\frac{1}{4\sqrt{3}}$ at Q
    c. Find an equation of the normal to C at Q. Give your answer in the form $ax + by + c = 0$, where a,b and c are exact constants.
4. Given that $y = 3x^2(5x - 3)^3$, show that $\frac{dy}{dx} = Ax(5x - 3)^n(Bx + C)$, where n, A, B and C are constants to be determined.
5. Differentiate $\frac{x^4}{cos3x}$ with respect to *x*
6. Show that if $y = secx$ then $\frac{dy}{dx} = secx\ tanx$
7. Find the points of zero gradient on the curve with parametric equations $x = \frac{t}{1-t}$, $y = \frac{t^2}{1-t}$, $t \neq 1$.
8. A curve C has equation $3^x = y - 2xy$. Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (2,-3)
9. The curve C has equation $y = xe^x$
    a. Find the exact coordinates of the stationary point on C and determine its nature
    b. Find the coordinates of any non-stationary points of inflection on C
    c. Hence sketch the graph of $y = xe^x$
10. The volume of a cube is decreasing at a constant rate of $4.5\ cm^3$ per second
    a. Find the rate at which the length of one side of the cube is decreasing when the volume is $100\ cm^3$
    b. Find the volume of the cube when the length of one side is decreasing at the rate of 2 mm per second.

Test Yourself

Time yourself for 20 minutes to answer these questions.

1. A curve has equation $f(x) = (x^3 - 2x)e^{-x}$

a) Find $f'(x)$

The normal to C at the origin intersects C again at P.

b) Show that the x-coordinate of P is the solution to the equation $2x^2 = e^x + 4$

2. The curve C has parametric equations $x = 4cos\ 2t, y = 3\sin t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

A is the point $\left(2, \frac{2}{3}\right)$, and lies on C

a) Find the value of t at the point A.

b) Find $\frac{dy}{dx}$ in terms of t

c) Show that an equation of the normal to C at A is $6y - 16x + 23 = 0$

The normal at A cuts C again at the point B

d) Find the y-coordinate of the point B