# 2nd Year Assignment 18

1. A curve has the equation $y = sin\ 5x + \cos 3x$. Find the equation of the tangent to the curve at the point $(\pi, -1)$

2. A curve has the equation $y = 2x^2 - sin\ x$. Show that the equation of the normal to the curve at the point with x-coordinate $\pi$ is $x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0$

3. A student is attempting to differentiate $\ln kx$.
   The student writes: $\boldsymbol{y = \ln kx,\ so \frac{dy}{dx} = k \ln kx}$
   Explain the mistake made by the student and state the correct derivative.

4. Prove, from first principles that the derivative of $\sin x$ is $\cos x$

5. Differentiate
   a. $\sin^2 3x$ b. $e^{(x+1)^2}$ c. $\ln(\cos x)^2$ d. $\frac{1}{3+\cos 2x}$ e. $\sin\left(\frac{1}{x}\right)$

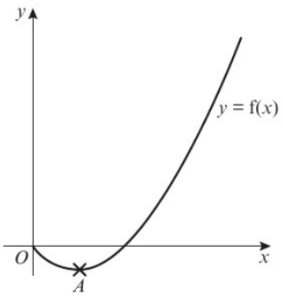
6. A curve C has equation $y = (x + 3)^2 e^{3x}$
   a. Find $\frac{dy}{dx}$ b. Find the gradient of C at the point where $x = 2$

7. Differentiate
   a. $(2 \sin x - 3 \cos x) \ln 3x$ b. $x^4 e^{7x-3}$

8. A curve C has equation $\frac{e^{2x}}{(x-2)^2}$, $x \neq 2$
   a. Show that $\frac{dy}{dx} = \frac{Ae^{2x}(Bx-C)}{(x-2)^3}$, where A, B and C are integers to be found
   b. Find the equation of the tangent of C at the point $x = 1$

9. Given that $f(x) = \frac{2x}{x+5} + \frac{6x}{x^2+7x+10}$, $x > 0$
   a. Show that $f(x) = \frac{2x}{x+2}$
   b. Hence find $f'(3)$

10. The diagram shows part of the curve with equation $y = f(x)$, where $f(x) = x(1 + x) \ln x$, $x > 0$
    The point A is the minimum point of the curve
    a. Find $f'(x)$
    b. Hence show that the x-coordinate of A is the solution to the equation $x = e^{-\frac{1+x}{1+2x}}$



# Test Yourself

Time yourself for 20 minutes for these two questions.

A. $p(x) = \frac{9-3x-12x^2}{(1-x)(1+2x)}$. Show that $p(x)$ can be written in the form $A + \frac{B}{1-x} + \frac{C}{1+2x}$, where A, B and C are constants to be found.

B. i) The 4th, 5th and 6th terms in an arithmetic sequence are $12 - 7k, 3k^2, k^2 - 10k$

Given that $k$ is an integer, find the first term and the common difference.

ii) The 4th term of an arithmetic sequence is 72. The 11th term is 51. The sum of the first $n$ terms is 1125. Find the two possible values of $n$.