Modelling with Trigonometric Functions

1

a Express $0.3 \sin\theta - 0.4 \cos\theta$ in the form $R \sin(\theta - \alpha)°$, where $R > 0$ and $0 < \alpha < 90°$. Give the value of α to 2 decimal places. **(4 marks)**

b i Find the maximum value of $0.3 \sin\theta - 0.4 \cos\theta$. **(2 marks)**

ii Find the value of θ, for $0 < \theta < 180$ at which the maximum occurs. **(1 mark)**

Jack models the temperature in his house, T°C, on a particular day by the equation

$$T = 23 + 0.3 \sin(18x)° - 0.4 \cos(18x)°, \; x \geqslant 0$$

where x is the number of minutes since the thermostat was adjusted.

c Calculate the minimum value of T predicted by this model, and the value of x, to 2 decimal places, when this minimum occurs. **(3 marks)**

d Calculate, to the nearest minute, the times in the first hour when the temperature is predicted, by this model, to be exactly 23 °C. **(4 marks)**

2

a Express $65 \cos\theta - 20 \sin\theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α correct to 4 decimal places. **(4 marks)**

A city wants to build a large circular wheel as a tourist attraction. The height of a tourist on the circular wheel is modelled by the equation

$$H = 70 - 65 \cos 0.2t + 20 \sin 0.2t$$

where H is the height of the tourist above the ground in metres, t is the number of minutes after boarding and the angles are given in radians. Find:

b the maximum height of the wheel **(2 marks)**

c the time for one complete revolution **(2 marks)**

d the number of minutes the tourist will be over 100 m above the ground in each revolution. **(4 marks)**

3 **a** Express $200\sin\theta - 150\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 4 decimal places. **(4 marks)**

The electric field strength, E V/m, in a microwave of width 25 cm can be modelled using the equation

$$E = 1700 + 200\sin\left(\frac{4\pi x}{25}\right) - 150\cos\left(\frac{4\pi x}{25}\right)$$

where x is the distance in cm from the left hand edge of the microwave oven.

b i Calculate the maximum value of E predicted by this model.

ii Find the values of x, for $0 \leqslant x < 25$, where this maximum occurs. **(3 marks)**

c Food in the microwave will heat best when the electric field strength at the centre of the food is above 1800 V/m. Find the range of possible locations for the centre of the food. **(5 marks)**

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4 **a** Express $1.4\sin\theta - 5.6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Round R and α to 3 decimal places. **(4 marks)**

b Hence find the maximum value of $1.4\sin\theta - 5.6\cos\theta$ and the smallest positive value of θ for which this maximum occurs. **(3 marks)**

The length of daylight, $d(t)$ at a location in northern Scotland can be modelled using the equation

$$d(t) = 12 - 5.6\cos\left(\frac{360t}{365}\right)^{\circ} + 1.4\sin\left(\frac{360t}{365}\right)^{\circ}$$

where t is the numbers of days into the year.

c Calculate the minimum number of daylight hours in northern Scotland as given by this model. **(2 marks)**

d Find the value of t when this minimum number of daylight hours occurs. **(1 mark)**

5 **a** Express $12\sin x + 5\cos x$ in the form $R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Round α to 1 decimal place. **(4 marks)**

A runner's speed, v in m/s, in an endurance race can be modelled by the equation

$$v(x) = \frac{50}{12\sin\left(\frac{2x}{5}\right)^{\circ} + 5\cos\left(\frac{2x}{5}\right)^{\circ}}, 0 \leqslant x \leqslant 300$$

where x is the time in minutes since the beginning of the race.

b Find the minimum value of v. **(2 marks)**

c Find the time into the race when this speed occurs. **(1 mark)**

5 **a** $R = 0.5, \alpha = 53.13°$

b **i** 0.5 **ii** $\theta = 143.1°$

c Minimum value is 22.5, occurs at 17.95 minutes

d 3, 13, 23, 33, 43, 53 minutes

6 **a** $R = 68.0074, \alpha = 0.2985$ **b** 138.0 m

c 31.4 minutes **d** 11.1 minutes

7 **a** $R = 250, \alpha = 0.6435$

b **i** 1950 V/m **ii** $x = 4.41$ cm, $x = 16.91$ cm

c $2.10 \leqslant x \leqslant 6.71, 14.60 \leqslant x \leqslant 19.21$

26 **a** $R = 5.772, \alpha = 75.964°$ **b** 5.772 when $\theta = 166.0°$

c 6.228 hours **d** 350.8 days

27 **a** $13 \sin(x + 22.6°)$ **b** 3.8 m/s

c 168.5 minutes