2nd Year Assignment 16

1. a) Use the double angle formulae to prove that $\frac{1-cos2x}{1+cos2x} \equiv \tan^2 x$

   b) Hence find for $-\pi \leq x \leq \pi$ all the solutions of $\frac{1-cos2x}{1+cos2x} = 3$, leaving your answer in terms of π

2. a) Prove, by counter example, that the statement $\sec(\mathrm{A} + \mathrm{B}) \equiv \sec \mathrm{A} + \sec \mathrm{B}$ for all A and B is false.

   b) Prove that $\tan \theta + \cot \theta \equiv 2 \operatorname{cosec} 2\theta, \quad \theta \pm \frac{n\pi}{2}, \quad n \epsilon \mathrm{Z}$

3. a) Sketch the curve with equation $y = (x - 4)(2x + 3)$

   b) The curve cuts the x-axis at A and B and the y-axis at C. State the co-ordinates of A, B and C.

   c) Find the area of the region bounded by the curve and the x-axis

4. A particle of mass 0.3 kg is on a rough plane which is inclined at an angle of $30°$ to the horizontal. The particle is held at rest on the plane by a force of magnitude 3 N acting up the plane, in a direction parallel to a line of greatest slope of the plane. The particle is on the point of slipping up the plane. Find the coefficient of friction between the particle and the plane.

5. A plank of wood *AB* has length 4 m and mass 40 kg . The plank is smoothly supported at *A* and at *C* , where *AC* = 3 m, as shown in the figure above.
A man of mass 80 kg stands on the plank at a distance *d* m from *A* .
The plank with the man standing on it remains in equilibrium with *AB* horizontal, and the reactions on the rod at *A* and at *C* equal.
The plank is modelled as a uniform rod and the man as a particle.
Determine the value of *d* .

6. A particle is projected vertically upwards from a point O with speed $u\ m/s$. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u\ m/s$.
Find
(a) the value of u
(b) the time from the instant that the particle leaves O to the instant that it returns to O.

7. Solve the inequalities, giving your answer in set notation:

(a) $2x^2 - 9x + 4 \leq 0$ (b) $2 - p - 3p^2 \geq 0$ (c) $9x - 2x^2 \leq 10$

8. Estimate the standard deviation and 30% – 70% interpercentile range of each distribution:

a)

Score	100-106	107-113	114-120	121-127	128-134
Frequency	6	11	22	9	2

b)

x	2-4	4-7	7-10	10-15
f	2	10	22	6

9. Given the mean $\bar{x}$ and the variance, σ_x^2 of the distribution *X*, find the mean $\bar{y}$ and the variance, σ_y^2 of the coded data *Y*:

(a) $\bar{x} = 231, \sigma_x^2 = 19.5, \quad Y = \frac{1}{3}X - 1$
(b) $\bar{x} = 9.08, \sigma_x^2 = 1.5, \quad Y = 5 + \frac{1}{2}X$
(c) $\bar{x} = -7.13, \sigma_x^2 = 1.85, \quad Y = 7 - X$
(d) $\bar{x} = 36, \sigma_x^2 = 12, \quad Y = X - 10$

10. a) Sketch, in the interval $-2\pi \leq x \leq 2\pi$, the graph of $y = 3 + 5\ cosec\ x$.

b) Hence deduce the range of values of *k* for which the equation $3 + 5\ cosec\ x = k$ has no solutions.

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

1. A curve C is defined by the parametric equations $x = \sqrt{t}, y = t(9 - t)$

a) Find a Cartesian equation of the curve in the form $y = f(x)$ and determine the domain and range of $f(x)$

b) Sketch C, showing clearly the coordinates of any turning points, endpoints and intersections with the coordinate axes.

2. A box of mass 5 kg sits on a smooth slope that is inclined at an angle of $30°$ to the horizontal. It is attached to alight scale-pan by a light inextensible string which passes over a smooth pulley, as shown in the diagram. The scale-pan carries two masses A and B. The mass of A is 2 kg and the mass of B is 5 kg. Work out the force exerted by A on B.