2nd Year Assignment 15

1. a) Prove that $\tan\theta + \cot\theta \equiv 2\text{cosec } 2\theta, \quad \theta \neq \frac{n\pi}{3}, n\epsilon\mathbb{Z}$
   b) Hence explain why the equation $\tan\theta + \cot\theta = 1$ does not have any real solutions

2. a) Solve, for $-180° \leq \theta \leq 180°$, the equation $5\sin 2\theta = 9\tan\theta$, giving your answer, where necessary, to 1 d.p.
   b) Deduce the smallest positive solution to the equation $5\sin(2x - 50°) = 9\tan(x - 25°)$

3. a) Express $10\cos\theta - 3\sin\theta$ in the form $\text{R}\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
   Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

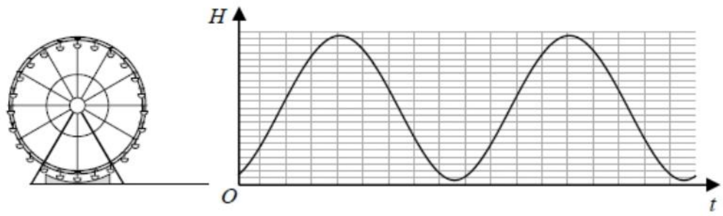


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$H = k - 10\cos(80t)° + 3\sin(80t)°$

where k is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) Find a complete equation for the model,

(ii) Hence find the maximum height of the passenger above the ground.

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

4. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

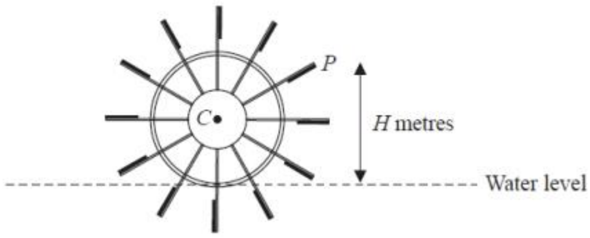


Figure 6

Figure 6 shows the cross-section of a water wheel.
The wheel is free to rotate about a fixed axis through the point C.
The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.
The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find
(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,
(c) find the value of T giving your answer to 3 significant figures.

(Solutions based entirely on calculator technology are not acceptable.)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

5. Sara is investigating the variation in the daily maximum gust, t kn, for Camborne in June and July 1987.
She used the Large Data Set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.
(a) State the sampling technique used by Sara.

(b) From your knowledge of the large data set explain why this process may not generate a sample of size 20.

The data Sara collected are summarised as follows $n = 20, \sum t = 374, \sum t^2 = 7600$
(c) Calculate the standard deviation. Give your answer to 3 s.f.

6. Sarah was studying the relationship between rainfall, *r* mm, and humidity, *h*%, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the Large Data Set. She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

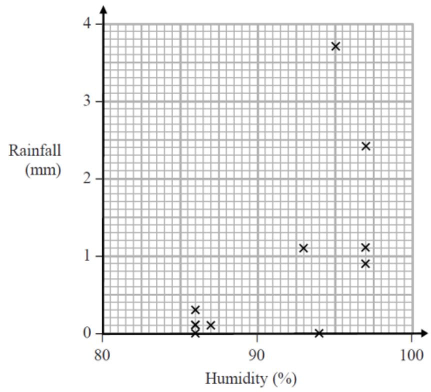
Sarah examined the rainfall figures and found

$$Q_1 = 0.1, \quad Q_2 = 0.9, \quad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.
(a) Show that *r* = 20.6 is an outlier.

(b) Give a reason why Sarah might:
(i) include this day's reading
(ii) exclude this day's reading.

Sarah decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of *r* and *h*.



c) Give an interpretation of the correlation between rainfall and humidity.

The equation of the regression line of *r* on *h* for these 10 days is *r* = −12.8 + 0.15*h*
(d) Give an interpretation of the gradient of this regression line.

(e) (i) Comment on the suitability of Sarah's sampling method for this study.
(ii) Suggest how Sarah could make better use of the large data set for her study.

7. Lauren wants to find the average daily mean windspeed in Hurn in 1987.
She only has access to the Large Data Set. She uses it to obtain a random sample of the daily mean windspeeds, t knots, on n days in Hurn in 1987.
The data collected by Lauren are summarised as follows

$$\sum(t-5) = 55, \qquad \bar{t} = 10$$

(a) Find n.

Lauren uses the same sampling method to estimate that the average daily mean windspeed in Hurn in 2015 was 11 mph.

(b) Convert 11 mph into knots.

(c) Hence, compare the average daily mean windspeed in Hurn in 1987 and 2015.

(d) With reference to the large data set, state **one** limitation of your conclusion in part (c).

(e) Explain how Lauren can
(i) improve her data collection method
(ii) improve her data processing
to allow for a more reliable comparison in part (c).

8. The table shows the mean daily temperatures at each of the eight weather stations in the Large Data Set for August 2015

	Camborne	Heathrow	Hurn	Leeming	Leuchars	Beijing	Jacksonville	Perth
Mean daily mean temp °C	15.4	18.1	16.2	15.6	14.7	26.6	26.4	13.6

a) Give a geographical reason why the temperature in August might be lower in Perth than in Jacksonville

b) Comment on whether this data supports the conclusion that coastal locations experience lower average temperatures than inland locations.

9. Harriet believes that the random sample S, representing total daily hours of sunshine from the Large Data Set, can be modelled by a discrete uniform distribution, once S has been rounded to the nearest integer.
a) Write down the probability distribution of S

b) Using this model, find the probability that the total daily hours of sunshine is less than 10

c) State what makes Harriet's assumption very unlikely

d) Suggest a refinement to Harriet's model.

10. Joshua compares the amount of rain in 2015 between Heathrow and the city X on the continent of Asia using the Large Data Set.
(a) Write down the name of the city X that Joshua compares with Heathrow.

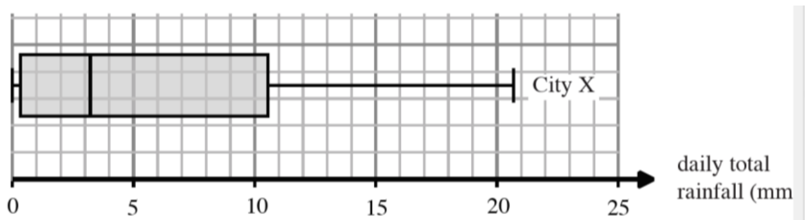
At random, he selects 8 data points about the daily total rainfall, in mm, in May 2015 for the two cities. These 8 data points are shown below.

Heathrow:	7.0	0.2	1.2	tr	0.8	6.8	0.2	4.2
City X:	6.0	0.0	20.7	9.0	14.3	0.5	0.0	0.4

(b) Explain what is meant by the reading 'tr'.

(c) State **one**
(i) advantage
(ii) disadvantage
of Joshua using 8 data points from the large data set for his comparisons.

The diagram below shows a box-plot for the data collected by Joshua on the rainfall in the city X in May 2015.



Draw another box-plot to represent the data collected by Joshua for Heathrow. In your data processing, take 'tr' to mean 0.0 mm of rainfall and ignore outliers.

(e) Compare the amount of rainfall in May 2015 between Heathrow and the city X.

11. A car has six forward gears.

The fastest speed of the car in 1st gear is $28\ km\ h^{-1}$

The fastest speed of the car in 6th gear is $115\ km\ h^{-1}$

Given that the fastest speed of the car in successive gears is modelled by an arithmetic sequence,

(a) find the fastest speed of the car in 3rd gear.

Given that the fastest speed of the car in successive gears is modelled by a geometric sequence,

(b) find the fastest speed of the car in 5th gear.

12. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form $\log_{10} T = b \log_{10} l + \log_{10} a$

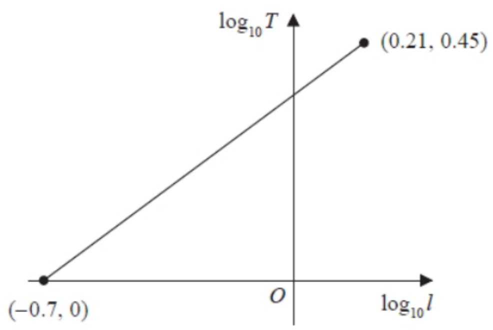


Figure 3

A student carried out an experiment to find the values of the constants *a* and *b*.
The student recorded the value of T for different values of *l*.
Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.
The straight line passes through the points (– 0.7, 0) and (0.21, 0.45)

(b) Using this information, find a complete equation for the model in the form $T = al^b$ giving the value of a and the value of b, each to 3 significant figures.

(c) With reference to the model, interpret the value of the constant a.

13. $\frac{3x^2+4x-5}{(x+3)(x-2)} = A + \frac{B}{x+3} + \frac{C}{x-2}$

a) Find the values of the constants A, B and C

b) Hence, or otherwise, expand $\frac{3x^2+4x-5}{(x+3)(x-2)}$ in ascending powers of x, as far as the term in x^2.

Give each coefficient as a simplified fraction.

14. Figure 2 shows a sketch of the graph with equation
$y = 2|x+4| - 5$
The vertex of the graph is at the point P, shown in Figure 2.
(a) Find the coordinates of P. (2)

(b) Solve the equation $3x + 40 = 2|x+4| - 5$

A line l has equation $y = ax$, where a is a constant.
Given that l intersects $y = 2|x+4| - 5$ at least once,

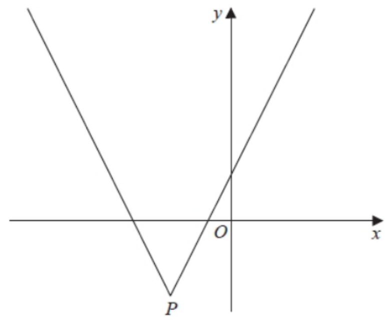
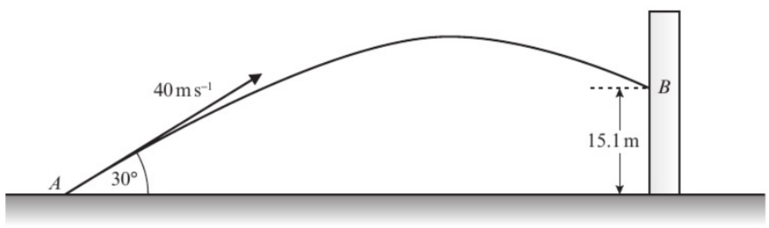


Figure 2

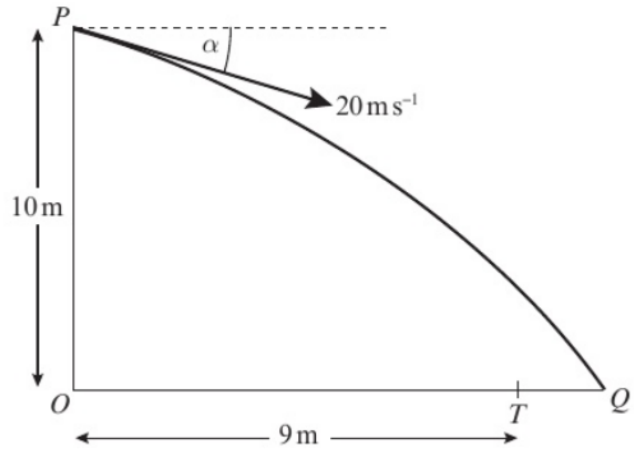
(c) find the range of possible values of a, writing your answer in set notation

15. A golf ball is driven from point A with a speed of $40\ ms^{-1}$ at an angle of elevation of $30°$. On its downwards flight, the ball hits an advertising hoarding at a height $15.1\ m$ above the level of A, at point B, as shown in the diagram.



Find
a) The time taken by the ball to reach its greatest height above A
b) The time taken by the ball to travel from A to B
c) The speed with which the ball hits the hoarding

16. In this question, use $g = 10ms^{-2}$
A stone is thrown from a point, P, at a target which is on horizontal ground. The point P is $10\ m$ above O on the ground. The stone is thrown from P at a speed of $20ms^{-1}$ at an angle of α below the horizontal, where $\tan\alpha = \frac{3}{4}$
The stone is modelled as a particle moving freely under gravity and the target as a point T.
The distance $OT = 9m$. The stone misses the target and hits the ground at the point Q, where OTQ is a straight line, as shown in the diagram.



Find

a) The time taken by the ball to travel from P to Q
b) The distance TQ

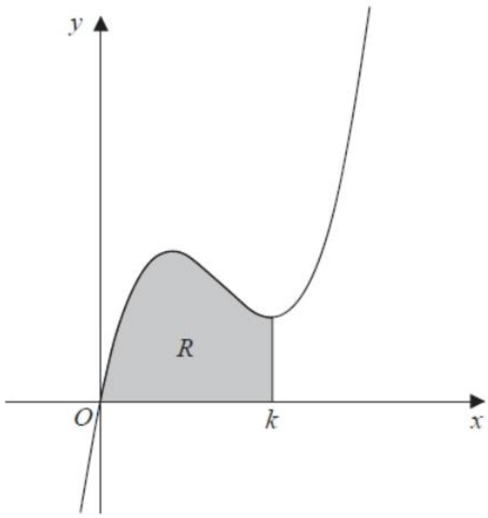
The point A is on the path of the ball, vertically above T.

c) Find the speed of the ball at A.

17. A car of mass 2150 kg is travelling down a rough road that is inclined at $10°$ to the horizontal. The engine of the car applies a constant driving force of magnitude $700\ N$, which acts in the direction of travel of the car. Any friction between the road and the tyres is initially ignored and air resistance is modelled as a single constant force of magnitude $F\ N$ that acts to oppose the motion of the car.
a) Given that the car is travelling in a straight line at a constant speed of $22\ ms^{-1}$, find the magnitude of F.
The driver brakes suddenly. In the subsequent motion, the car continues to travel in a straight line, and the tyres skid along the road, bringing the car to a standstill after $40\ m$. The driving force is removed and the force due to air resistance is modelled as remaining constant.
b) Find the coefficient of friction between the tyres and the road.
c) State one limitation of the model used for air resistance.

18. Jean always goes to work by bus or she takes a taxi. If one day she goes to work by bus, the probability that she goes to work by taxi the next day is 0.4. If one day she goes to work by taxi, the probability that she goes to work by bus the next day is 0.7.
Given that jean takes the bus to work on Monday, find the probability that she takes a taxi to work on Wednesday.

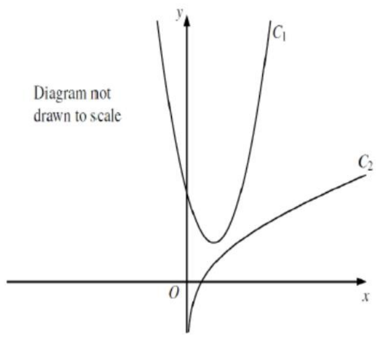
19. Figure 3 shows a sketch of part of the curve with equation $y = 2x^3 - 17x^2 + 40x$
The curve has a minimum turning point at $x = k$.
The region R, shown shaded in the diagram, is bounded by the curve, the x-axis and the line with equation x = k.



Show that the area of R is $\frac{256}{3}$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

20. The curve C_1, shown in the diagram, has equation $y = 4x^2 - 6x + 4$.



The point P lies on C_1 The curve C_2, also shown in the diagram, has equation $y = \frac{1}{2}x + ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

TEST YOURSELF 1

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

A. Prove from first principles that the derivative of $3x^2$ is $6x$

B. A lorry is driven between London and Newcastle.

   In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is $C = \frac{1500}{v} + \frac{2v}{11} + 60$

   (a) Find, according to this model,
   (i) the value of v that minimises the cost of the journey,
   (ii) the minimum cost of the journey.
   (Solutions based entirely on graphical or numerical methods are not acceptable.)

   (b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

   (c) State one limitation of this model

TEST YOURSELF 2

Give yourself 20 minutes to answer this question.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

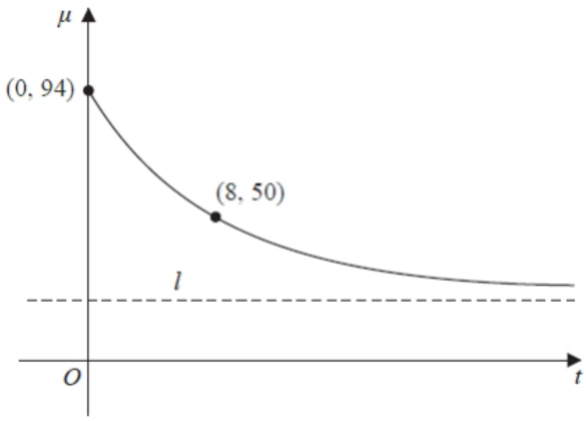
The temperature, $\theta°C$, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation $\theta = 15 + 68e^{-0.114t}$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35°C.

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15°C



The temperature, $\mu°C$, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$\mu = A + Be^{-\frac{t}{8}} \qquad t \geq 0$

where A and B are constants.

The diagram shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown in the diagram, is the asymptote to the curve.

Using the equation of this model and the information given in the diagram, find

(d) the equation for the asymptote l..