Second Year Assignment 14

Q1.

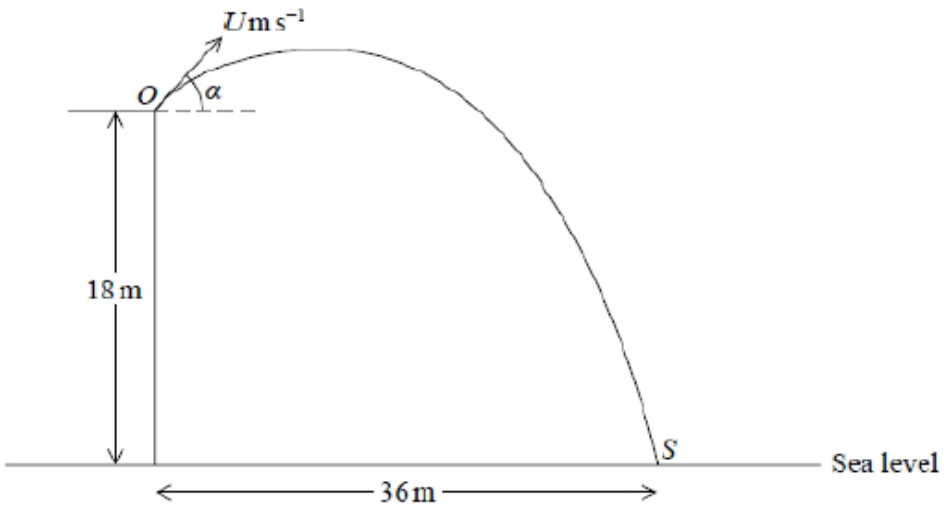

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with g = 10 m s^{-2}

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

(Total for question = 13 marks)

Q2.

Unless otherwise stated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

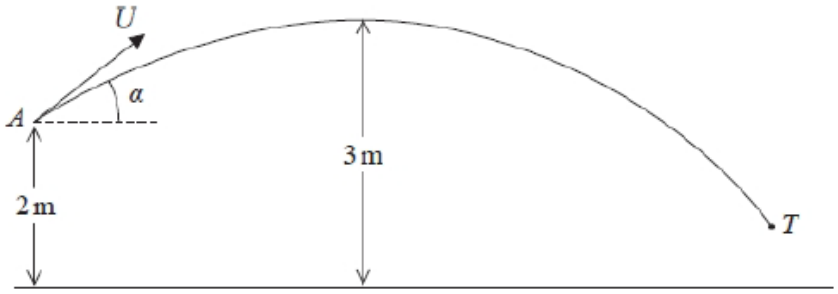

Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

(Total for question = 15 marks)

Q3.

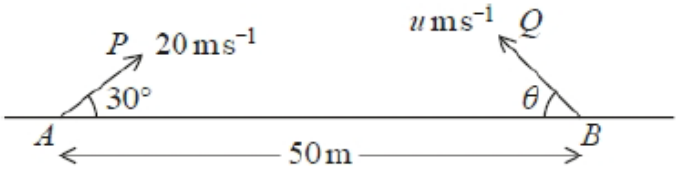

Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB.

Ball Q is projected from B with speed u m s^{-1} at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,
(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

(Total for question = 13 marks)

Q4.

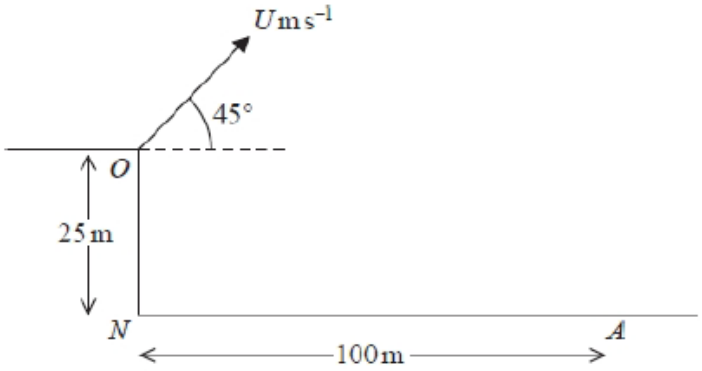

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where AN = 100 m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

(Total for question = 11 marks)

Q5.

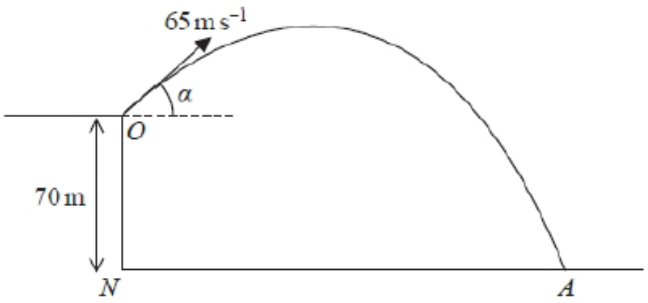

Figure 3

A small stone is projected with speed 65 m s^{-1} from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan \alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

Q6 Prove by contradiction that the sum of a rational number and an irrational number is irrational

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Q7 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3, u_2 = 5$$
$$u_n = u_{n-1} + u_{n-2}, \qquad n \geq 3$$

Find u_3, u_4 and u_5

Q8 The functions f and g are defined by

$$f : x \to \ln(3x - 2), \quad x \in \mathbb{R}, \ x > \frac{2}{3}$$

$$g : x \to \frac{3}{x - 2}, \quad x \in \mathbb{R}, \ x \neq 2$$

(a) Find the exact value of fg(3)

(b) Find an expression for $f^{-1}(x)$ and state its domain

(c) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same diagram

(d) Sketch the graph of $y = |g(x)|$

(e) Find the exact values of $\left| \frac{3}{x - 2} \right| = 4$

Q9 (a) Given that θ is small, use the small angle approximation of $\cos\theta$ to show that

$$4\cos(\theta) + \cos^2(2\theta) \approx 5 - 6\theta^2 + 4\theta^4$$

(b) Hence find an approximation of $4\cos(\theta) + \cos^2(2\theta)$ when $\theta = 3°$

(c) Calculate the percentage error in your approximation

Q10 (a) Expand $(9 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to and including x^3

(b) Write down the range of values of x for which the expansion is valid.

(c) Use your expansion, with a suitable value of x, to find the value of $\sqrt{8.9}$ correct to 5 significant figures

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

A Leroy asks 50 people which sports they watch. The can chose from football, golf and hockey.

5 people watch all three sports.
8 people watch football and golf
7 people watch golf and hockey
9 people watch football and hockey
31 people watch football
13 people watch golf
17 people watch hockey.

(a) Draw a Venn diagram for this information.

One of the people is selected at random.

(b) Given they watch football, find the probability they do not watch hockey.

B The temperature and the number of hours of sunshine on 12 days is recorded.
The product moment correlation coefficient is calculated to be $r = 0.636$.

Stating your hypotheses clearly test, at the 1% significance level, whether there is a positive correlation between temperature and hours of sunshine