Second Year Assignment 13

1. a) Prove that the sum of a geometric series is given by $S = \frac{a(1-r^n)}{1-r}$
   b) Prove that the sum of the first n terms of an arithmetic series is given by $S = \frac{n}{2}(2a + (n-1)d))$
   c) Prove that the sum of the first n natural numbers is given by $S = \frac{n}{2}(n+1)$

2. The functions f and g are given by

$$f(x) = \frac{x}{x-2}, \quad \{x \in \mathcal{R}, x \neq 2\}$$
$$g(x) = \frac{3}{x}, \quad \{x \in \mathcal{R}, x \neq 0\}$$

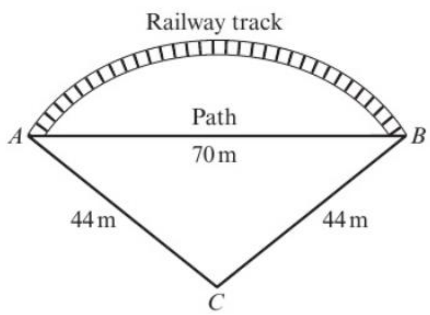
   a) Find an expression for $f^{-1}(x)$
   b) Write down the range for $f^{-1}(x)$
   c) Calculate $gf(1.5)$
   d) Use algebra to find the values of x for which $g(x) = f(x) + 4$

3. $q(x) = \frac{9x^2+26x+20}{(1+x)(2+x)}, |x| < 1$
   a) Show that the expansion of q(x) is ascending powers of x can be approximated to $10 - 2x + Bx^2 + Cx^3$, where B and C are constants to be found.
   b) Find the percentage error made in using the series expansion in part (a) to estimate the value of $q(0.1)$. Give your answer to 2 s.f.

4. a) Show that the equation $4\sin^2 x + 9\cos x - 6 = 0$, can be written as $4\cos^2 x - 9\cos x + 2 = 0$

   b) Hence solve for $0 \leq x \leq 4\pi$, $4\sin^2 x + 9\cos x - 6 = 0$, giving your answers to 1 d.p.

5. There is a straight path of length 70 cm from the point A to the point B. The points are also joined by a railway track in the form of an arc of a circle whose centre is C and whose radius is 44 cm, as shown in the diagram.
   Calculate
   a) The size of $\angle ACB$ in radians
   b) The length of the railway track
   c) The shortest distance from C to the path
   d) The area of the region bounded by the railway track and the path.



6. Given that $p = \sec\theta - \tan\theta$ and $q = \sec\theta + \tan\theta$, show that $p = \frac{1}{q}$

7. John and Kayleigh play darts in the same team. The events J and K are defined as follows:

J is the event that John wins his match; K is the event that Kayleigh wins her match

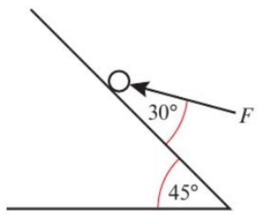
P(J) = 0.6, P(K) = 0.7 and P(J∪K) = 0.8

Find the probability that

a) Both John and Kayleigh win their matches
b) John wins his match, given that Kayleigh loses her
c) Kayleigh wins her match, given that John wins his
d) Determine whether the events J and K are statistically independent. You must show all your working.

8. A particle of mass 2 kg sits on a smooth slope that is inclined at 45° to the horizontal. A force of F N acts at an angle of 30° to the plane and causes the particle to acceleprate up the hill at $2\ m\ s^{-2}$.

Show that $F = \frac{2}{\sqrt{3}}(4 + \sqrt{2}g)N$



9. A shipping container of mass 15 000 kg is being pulled by a winch up a rough slope which is inclined at 10° to the horizontal. The winch line imparts a constant force of 42 000 N, which acts parallel to and up the slope, causing the shipping container to accelerate at a constant rate of $0.1\ m\ s^{-2}$.

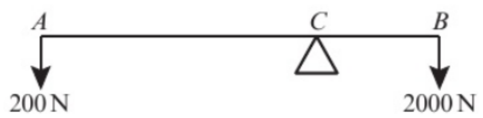
Calculate

a) The reaction between the shipping container and the slope
b) The coefficient of friction, μ, between the shipping container and the slope

When the shipping container is travelling at $2\ ms^{-1}$, the engine is turned off

c) Find the time taken for the shipping container to come to rest
d) Determine whether the shipping container will remain at rest, justifying your answer.

10. A lever consists of a uniform steel rod AB of weight 200 N and length 3 m, which rests on a pivot at C. A 2000 N weight is placed at B and is supported by a force of 200 N applied vertically downwards at A. Given that the lever is in equilibrium, calculate the length CB.



TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

Prove that:

a $(\tan\theta + \cot\theta)(\sin\theta + \cos\theta) \equiv \sec\theta + \operatorname{cosec}\theta$

b $\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$

c $(1 - \sin x)(1 + \operatorname{cosec} x) \equiv \cos x \cot x$

d $\dfrac{\cot x}{\operatorname{cosec} x - 1} - \dfrac{\cos x}{1 + \sin x} \equiv 2\tan x$