Second Year Assignment 12

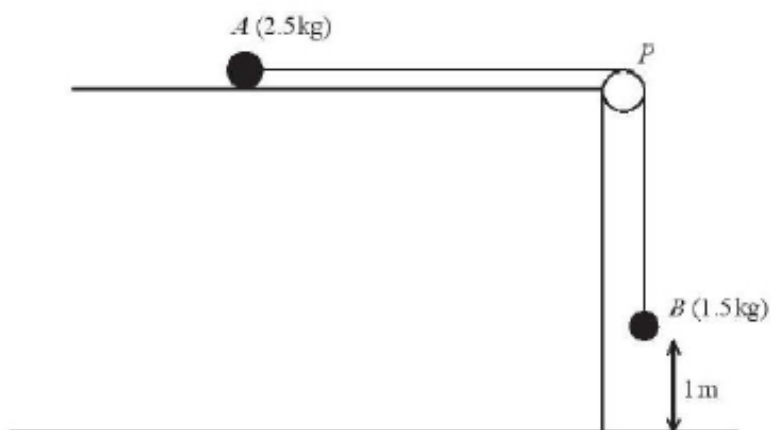Q1.

A (2.5kg)

P

B (1.5kg)

1 m

Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

It was found that it actually took 2.3 seconds for ball B to reach the floor.

(d) Using this information

(i) comment on the appropriateness of using the model to find the time it takes ball B to reach the floor, justifying your answer.
(ii) suggest one improvement that could be made in the model.

(2)

(Total for question = 10 marks)

Q2.

Unless otherwise indicated, wherever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.
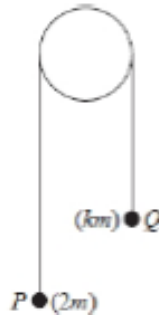

Figure 1

Two small balls, P and Q, have masses $2m$ and km respectively, where $k < 2$.
The balls are attached to the ends of a string that passes over a fixed pulley.
The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

The system is released from rest and, in the subsequent motion, P moves downwards with an acceleration of magnitude $\dfrac{5g}{7}$

The balls are modelled as particles moving freely.
The string is modelled as being light and inextensible.
The pulley is modelled as being small and smooth.

Using the model,

(a) find, in terms of m and g, the tension in the string,

(3)

(b) explain why the acceleration of Q also has magnitude $\dfrac{5g}{7}$

(1)

(c) find the value of k.

(4)

(d) Identify one limitation of the model that will affect the accuracy of your answer to part (c).

(1)
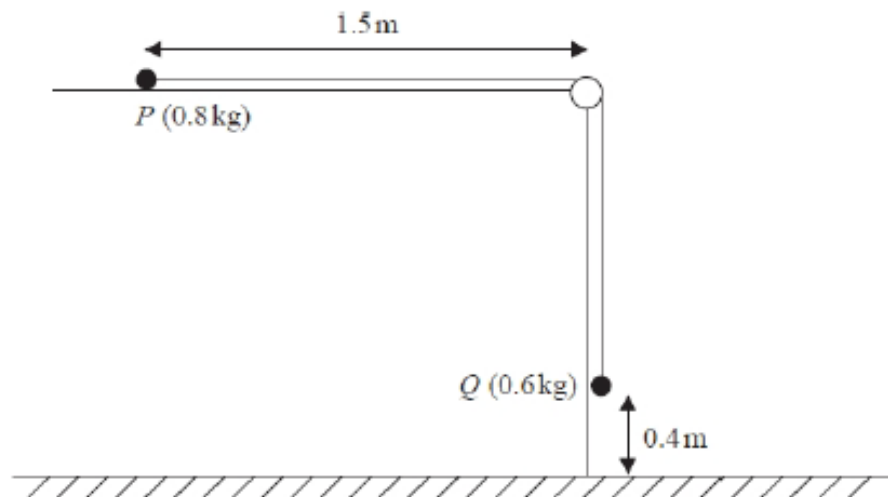
(Total for question = 9 marks)

Q3.

Figure 1

A small ball, *P*, of mass 0.8 kg, is held at rest on a smooth horizontal table and is attached to one end of a thin rope.

The rope passes over a pulley that is fixed at the edge of the table.

The other end of the rope is attached to another small ball, *Q*, of mass 0.6 kg, that hangs freely below the pulley.

Ball *P* is released from rest, with the rope taut, with *P* at a distance of 1.5 m from the pulley and with *Q* at a height of 0.4 m above the horizontal floor, as shown in Figure 1.

Ball *Q* descends, hits the floor and does not rebound.

The balls are modelled as particles, the rope as a light and inextensible string and the pulley as small and smooth.

Using this model,

(a) show that the acceleration of *Q*, as it falls, is 4.2 m s^{-2}

(5)

(b) find the time taken by *P* to hit the pulley from the instant when *P* is released.

(6)

(c) State one limitation of the model that will affect the accuracy of your answer to part (a).
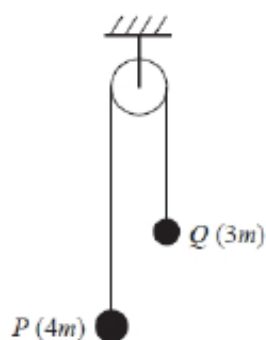
(1)

(Total for question = 12 marks)

Q4.

Figure 1

One end of a string is attached to a small ball P of mass 4m.

The other end of the string is attached to another small ball Q of mass 3m.

The string passes over a fixed pulley.

Ball P is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

Ball P is released.

The string is modelled as being light and inextensible, the balls are modelled as particles, the pulley is modelled as being smooth and air resistance is ignored.

(a) Using the model, find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while P is falling and before Q hits the pulley.

(8)

(b) State one limitation of the model, apart from ignoring air resistance, that will affect the accuracy of your answer to part (a).
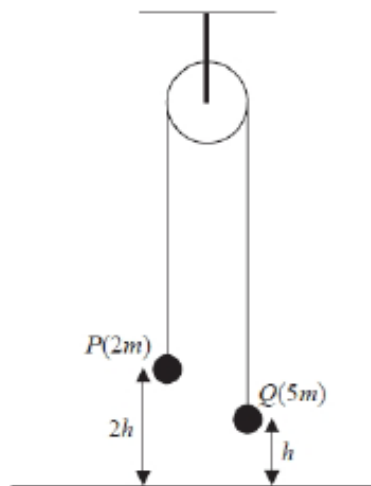
(1)

(Total for question = 9 marks)

Q5.

Figure 1

A ball *P* of mass 2*m* is attached to one end of a string.

The other end of the string is attached to a ball *Q* of mass 5*m*.

The string passes over a fixed pulley.

The system is held at rest with the balls hanging freely and the string taut.

The hanging parts of the string are vertical with *P* at a height 2*h* above horizontal ground and with *Q* at a height *h* above the ground, as shown in Figure 1.

The system is released from rest.

In the subsequent motion, *Q* does not rebound when it hits the ground and *P* does not hit the pulley.

The balls are modelled as particles.

The string is modelled as being light and inextensible.

The pulley is modelled as being small and smooth.

Air resistance is modelled as being negligible.

Using this model,

(a) (i) write down an equation of motion for *P*,

   (ii) write down an equation of motion for *Q*,

(4)

(b) find, in terms of *h* only, the height above the ground at which *P* first comes to instantaneous rest.

(7)

Q6.

In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c
are constants.

s	0	1	2	3	4
$P(S=s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

(Total for question = 6 marks)

Q7.

Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call

that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

(Total for question = 8 marks)

Q8.
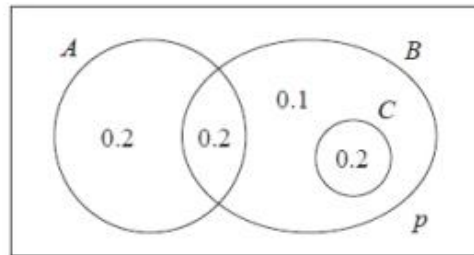
The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p.

(1)

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

(Total for question = 2 marks)

Q9.

Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(Total for question = 5 marks)

Q10.

Helen believes that the random variable C, representing cloud cover from the large data set, can be modelled by a discrete uniform distribution.

(a) Write down the probability distribution for C.

(2)

(b) Using this model, find the probability that cloud cover is less than 50%

(1)

Helen used all the data from the large data set for Hurn in 2015 and found that the proportion of days with cloud cover of less than 50% was 0.315

(c) Comment on the suitability of Helen's model in the light of this information.

(1)

(d) Suggest an appropriate refinement to Helen's model.

(1)

(Total for question = 5 marks)

TEST YOURSELF

Give yourself 20 minutes to answer this question.
If you finish early, check your answers.
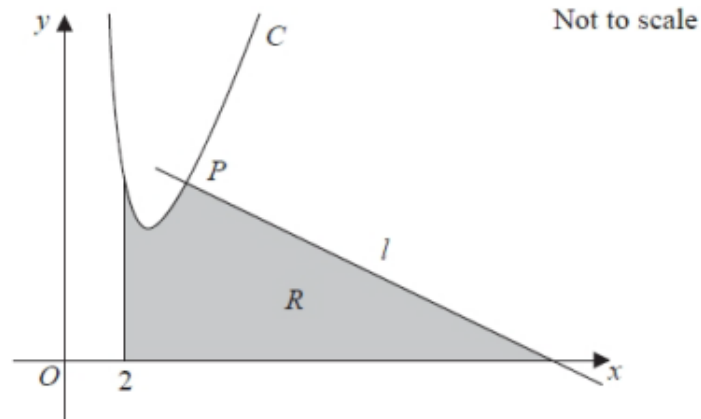*I will mark your answers. Set your work out carefully.*

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)