Second Year Assignment 11

1. a) Simplify $(4x^2)^3 \div 2x^5$
b) Expand and simplify $(2x+3)(x-2)(3x-1)$
c) Evaluate $\left(\frac{8}{27}\right)^{\frac{2}{3}}$
d) Rationalise the denominator and simplify $\frac{\sqrt{23}-\sqrt{37}}{\sqrt{23}+\sqrt{37}}$
e) Given that $x^3 - x^2 - 17x - 15 = (x+3)(x^2+bx+c)$, where b and c are constants,
i) find the values of b and c
ii) Fully factorise $x^3 - x^2 - 17x - 15$

2. The simultaneous equations $2x - y = 1, \quad x^2 + 4ky + 5k = 0$ have only one solution. Find this solution.

3. The circle with equation $(x-4)^2 + (y+7)^2 = 50$ meets the straight line with equation $x - y - 5 = 0$ at points A and B.
a) Find the coordinates of the points A and B
b) Find the equation of the perpendicular bisector of the line segment AB
c) Show that the perpendicular bisector of AB passes through the centre of the circle.
d) Find the area of triangle OAB.

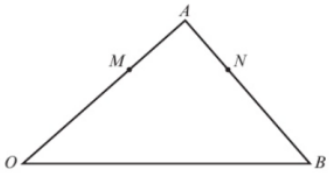
4. Sketch the following graphs. Label any asymptotes and show the coordinates where the graphs cross the axes.
a) $y = 3\sin 2x, \quad 0 \le x \le 2\pi$
b) $y = 2^x, \quad -4 \le x \le 4$
c) $y = 3(x^4 - 1) - 45, \quad -3 \le x \le 3$

5. a) Solve the equation $7^{2x+1} = 1000$
b) Given that $\log_a(pq) = 5$ and $\log_a(p^2q) = 9$, find the values of $\log_a p$ and $\log_a q$
c) Given that $p = \log_q 16$, express in terms of p:
i) $\log_q 2$ ii) $\log_q(8q)$
d) Sketch the graph of $y = \log_{10}(3x-2)$, stating the equation of the asymptote

6. The price of a computer system can be modelled by the formula $P = 100 + 850e^{-\frac{t}{2}}$ where P is the price of the system in £s and t is the age of the computer in years after being purchased.
a) Calculate the price of the new system
b) Calculate its price after 3 years, giving your answer to the nearest £
c) When will it be worth less than £200?
d) Find its price as $t \to \infty$
e) Sketch the graph showing P against t
f) Comment on the appropriateness of this model

7. $g(x) = (4 + kx)^5$, where k is a constant
Given that the coefficient of x^3 in the binomial expansion of $g(x)$ is 20, find the value of k.

8. OAB is a triangle. $\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**. The point M divides OA in the ratio 3:2. MN is parallel to OB.



a) Express the vector $\overrightarrow{ON}$ in terms of **a** and **b**.

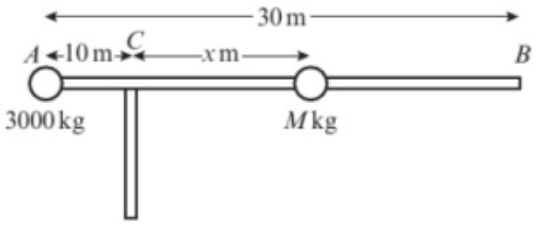
b) Find the vector $\overrightarrow{MN}$.

c) Show that AN : NB = 2 : 3

9. $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 20$
a) Find the coordinates of the stationary points of $f(x)$ and determine the nature of each of them.

b) Sketch the graph of $y = f(x)$

10. The beam of a crane is modelled as a uniform rod AB, of length 30 m and weight 4000 kg, resting in horizontal equilibrium. The beam is supported bya tower at C, where AC = 10 m. A counterbalance mass of weight 3000 kg is placed at A and a load of mass M is placed at a variable distance x m from the supporting tower, where $x \geq 5$.



a) Find an expression for M in terms of x.

b) Hence determine the maximum and minimum loads that can be lifted by the crane.

c) Criticise this model in relation to the beam.

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

A. A non-uniform rod AB, of length 5 m and mass 15 kg, rests horizontally suspended from the ceiling by two vertical strings attached to C and D, where AC = 1 m and AD = 3.5 m
a) Given that the centre of mass is at E where AE = 3 m, find the magnitudes of the tensions in the strings.
When a particle of mass 9 kg is attached to the rod at F the tension in the string at D is twice the tension in the string at C.
b) Find the distance AF

B. A plank AB, of mass 12 kg and length 3 m, is in equilibrium in a horizontal position resting on supports at C and D where AC = 0.7 m and DB = 1.1 m. A boy of mass 32 kg stands on the plank at point E. The plank is about to tilt about D. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance AE.