Exercise 1A

Online Explore the original and coded data graphically using technology.

1 Data are coded using $Y = \log y$ and $X = \log x$ to give a linear relationship. The equation of the regression line for the coded data is $Y = 1.2 + 0.4X$.

a State whether the relationship between y and x is of the form $y = ax^n$ or $y = kb^x$.

b Write down the relationship between y and x and find the values of the constants.

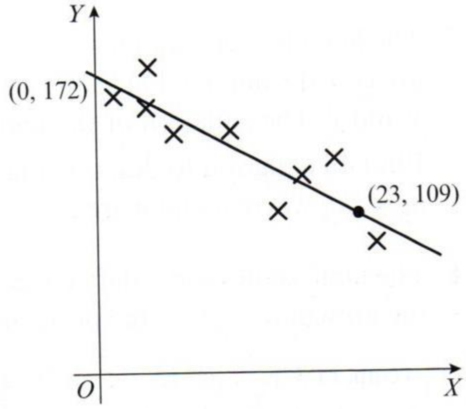
2 Data are coded using $Y = \log y$ and $X = x$ to give a linear relationship. The equation of the regression line for the coded data is $Y = 0.4 + 1.6X$.

a State whether the relationship between y and x is of the form $y = ax^n$ or $y = kb^x$.

b Write down the relationship between y and x and find the values of the constants.

(P) **3** The scatter diagram shows the relationship between two sets of coded data, X and Y, where $X = \log x$ and $Y = \log y$. The regression line of Y on X is shown, and passes through the points (0, 172) and (23, 109).

The relationship between the original data sets is modelled by an equation of the form $y = ax^n$. Find, correct to 3 decimal places, the values of a and n.



(P) **4** The size of a population of moles is recorded and the data are shown in the table. T is the time, in months, elapsed since the beginning of the study and P is the number of moles in the population.

T	2	3	5	7	8	9
P	72	86	125	179	214	257

a Plot a scatter diagram showing $\log P$ against T.

b Comment on the correlation between $\log P$ and T.

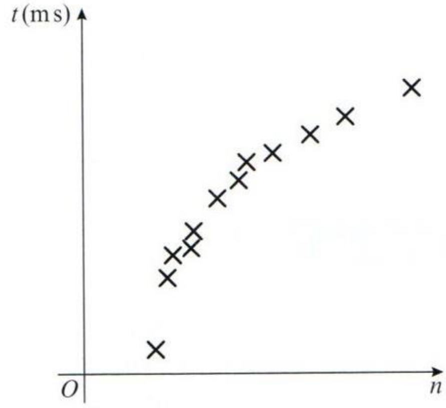
c State whether your answer to **b** supports the fact that the original data can be modelled by a relationship of the form $P = ab^T$.

d Calculate the values of a and b for this model.

e Give an interpretation of the value of b you calculated in part **d**.

Hint Think about what happens when the value of T increases by 1. When interpreting coefficients, refer in your answer to the context given in the question.

5 The time, t m s, needed for a computer algorithm to determine whether a number, n, is prime is recorded for different values of n. A scatter graph of t against n is drawn.



a Explain why a model of the form $t = a + bn$ is unlikely to fit these data.

The data are coded using the changes of variable $y = \log t$ and $x = \log n$. The regression line of y on x is found to be $y = -0.301 + 0.6x$.

b Find an equation for t in terms of n, giving your answer in the form $t = an^k$, where a and k are constants to be found.

6 Data are collected on the number of units (c) of a catalyst added to a chemical process, and the rate of reaction (r).

The data are coded using $x = \log c$ and $y = \log r$. It is found that a linear relationship exists between x and y and that the equation of the regression line of y on x is $y = 1.31x - 0.41$.

Use this equation to determine an expression for r in terms of c.

7 The heights, h cm, and masses, m kg, of a sample of Galapagos penguins are recorded. The data are coded using $y = \log m$ and $x = \log h$ and it is found that a linear relationship exists between x and y. The equation of the regression line of y on x is $y = 0.0023 + 1.8x$.

Find an equation to describe the relationship between m and h, giving your answer in the form $m = ah^n$, where a and n are constants to be found.

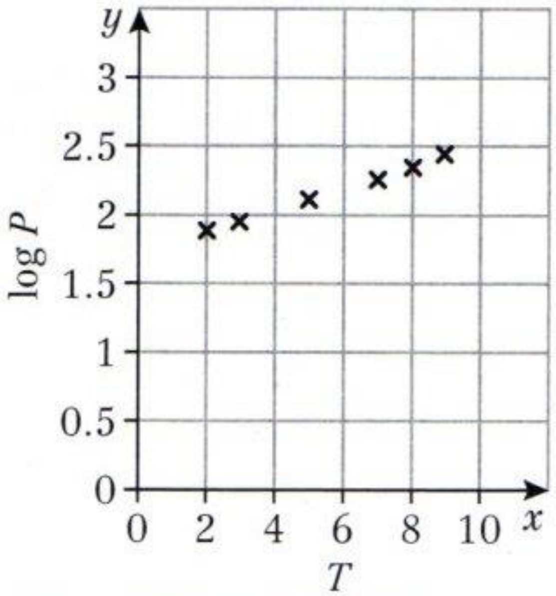
Exercise 1A

1 **a** $y = ax^n$ **b** $a = 15.8$ (3 s.f.), $n = 0.4$

2 **a** $y = kb^x$ **b** $k = 2.51$, $b = 39.8$ (3 s.f.)

3 $a = 1 \times 10^{172}$, $n = -2.739$ (3 d.p.)

4 **a**



T	2	3	5	7	8	9
log *P*	1.86	1.93	2.10	2.25	2.33	2.41

b Strong positive correlation

c Yes – the variables show a linear relationship wher log P is plotted against T.

d $a = 50.1$ (3 s.f.), $b = 1.2$

e For every month that passes, the population of moles increases by 20%.

5 **a** $t = a + bn$ would show a linear relationship. This graph is not a straight line.

b $a = 0.5$, $k = 0.6$

6 $r = 0.389c^{1.31}$

7 $a = 1.0$, $n = 1.8$