1. $$f(x) = (2 - 5x)^{-2}, \qquad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction.

(5)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1 - 9x .$$

(2)

3. Given that $y = 3x^2 + 4\sqrt{x}, \quad x > 0$, find

(a) $\dfrac{dy}{dx}$,

(2)

(b) $\dfrac{d^2y}{dx^2}$,

(2)

(c) $\int y\,dx$.

(3)

4. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.

(3)

(b) Calculate her total savings over the complete 200 week period.

(3)

5. $$f(x) = x^3 + 4x^2 + x - 6.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of f(x).

(2)

(b) Factorise f(x) completely.

(4)

(c) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)

6. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(b) Write down the range of f^{-1}.

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)

7.

$$f(x) = x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.

(3)

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(c) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants, a, b and c.

(3)

(d) In the space provided on page 21, sketch the graph of $y = f(x)$.

(3)

(e) Hence sketch the graph of $y = |f(x)|$.

(1)

8. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant \pi,$$

(a) express $\arcsin x$ in terms of y.

(2)

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)

9.

Figure 2

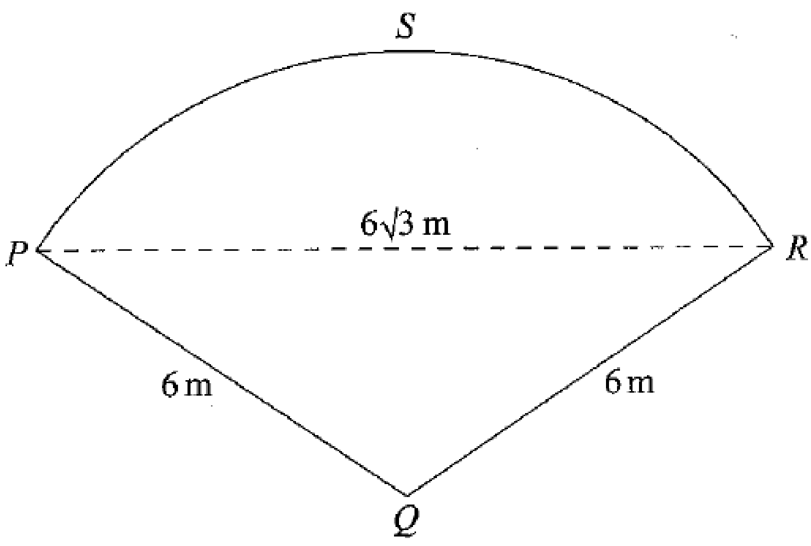


Figure 2 shows a plan of a patio. The patio *PQRS* is in the shape of a sector of a circle with centre *Q* and radius 6 m.

Given that the length of the straight line *PR* is $6\sqrt{3}$ m,

(a) find the exact size of angle *PQR* in radians.
(3)

(b) Show that the area of the patio *PQRS* is 12π m^2.
(2)

(c) Find the exact area of the triangle *PQR*.
(2)

(d) Find, in m^2 to 1 decimal place, the area of the segment *PRS*.
(2)

(e) Find, in m to 1 decimal place, the perimeter of the patio *PQRS*.
(2)

10. A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \ldots.$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

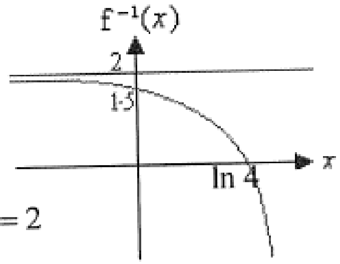
1.	$f(x) = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$.	B1
	$= \frac{1}{4}\left\{\underline{1+(-2)(**x); + \frac{(-2)(-3)}{2!}(**x)^2 + \frac{(-2)(-3)(-4)}{3!}(**x)^3 + ...}\right\}$	Expands $(1+**x)^{-2}$ to give an unsimplified $1+(-2)(**x)$;	M1
		A correct unsimplified {..........} expansion with candidate's $(**x)$	A1
	$= \frac{1}{4}\left\{\underline{1+(-2)(\frac{-5x}{2}); + \frac{(-2)(-3)}{2!}(\frac{-5x}{2})^2 + \frac{(-2)(-3)(-4)}{3!}(\frac{-5x}{2})^3 + ...}\right\}$		
	$= \frac{1}{4}\left\{1 + 5x; + \frac{75x^2}{4} + \frac{125x^3}{2} + ...\right\}$		
	$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + ...$	Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$;	A1;
		Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$	A1
	$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + ...$		**[5]**

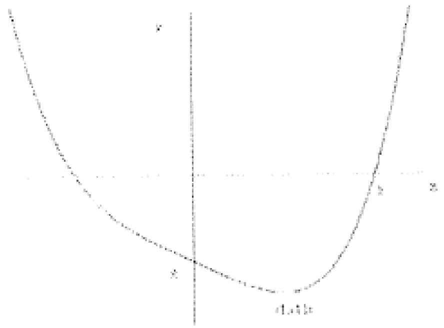
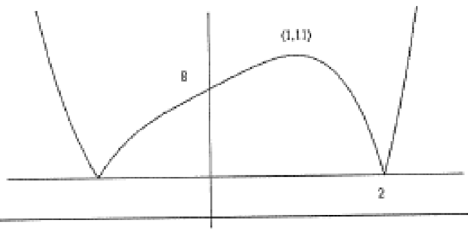
2. (a)	$(1-2x)^5 = 1 + 5\times(-2x) + \frac{5\times4}{2!}(-2x)^2 + \frac{5\times4\times3}{3!}(-2x)^3 + ...$	
	$= 1 - 10x + 40x^2 - 80x^3 + ...$	B1, M1, A1, A1
		(4)

3.	(a) $\left(\frac{dy}{dx}\right) = 6x^1 + \frac{4}{2}x^{-\frac{1}{2}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$	M1 A1	(2)
	(b) $6 + -x^{-\frac{3}{2}}$ or $6 + -1 \times x^{-\frac{3}{2}}$	M1 A1ft	(2)
	(c) $x^3 + \frac{8}{3}x^{\frac{3}{2}} + C$ A1: $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ A1: both, simplified and $+\,C$	M1 A1 A1	(3)
			7

4.	(a) Identify $a = 5$ and $d = 2$ (May be implied)	B1	
	$(u_{200} =) a + (200-1)d$ (= 5 + (200 – 1)×2)	M1	
	= 403(p) or (£) 4.03	A1	(3)
	(b) $(S_{200} =) \frac{200}{2}[2a + (200-1)d]$ or $\frac{200}{2}(a + \text{"their 403"})$	M1	
	$= \frac{200}{2}[2 \times 5 + (200-1) \times 2]$ or $\frac{200}{2}(5 + \text{"their 403"})$	A1	
	= 40 800 or £408	A1	(3)
			6

5. (a)	$f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$	M1
	$\{ = -8 + 16 - 2 - 6\}$	
	$= 0,\ \therefore\ x + 2$ is a factor	A1 (2)
(b)	$x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3)$	M1, A1
	$= (x + 2)(x + 3)(x - 1)$	M1, A1 (4)
(c)	–3, –2, 1	B1 (1) **(7)**

6. (a) $y = \ln(4-2x)$			
$e^y = 4-2x$ leading to $x = 2-\frac{1}{2}e^y$	Changing subject and removing ln	M1 A1	
$y = 2-\frac{1}{2}e^x \quad \Rightarrow \quad f^{-1} \mapsto 2-\frac{1}{2}e^x$ *	cso	A1	
Domain of f^{-1} is $\mathbb{R}$		B1	(4)
(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \mathbb{R}$)		B1	(1)
(c)  $y = 2$	Shape	B1	
	1.5	B1	
	ln 4	B1	
		B1	(4)
(d) $x_1 \approx -0.3704, \quad x_2 \approx -0.3452$	cao	B1, B1	(2)
If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.			
(e) $x_3 = -0.354\,030\,19\ \ldots$			
$x_4 = -0.350\,926\,88\ \ldots$			
$x_5 = -0.352\,017\,61\ \ldots$			
$x_6 = -0.351\,633\,86\ \ldots$	Calculating to at least x_6 to at least four dp	M1	
$k \approx -0.352$	cao	A1	(2)
			[13]

7.	(a) $f(-2) = 16 + 8 - 8 \, (= 16) > 0$	B1
	$f(-1) = 1 + 4 - 8 \, (= -3) < 0$	B1
	Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$	B1ft **(3)**
	ft their calculation as long as there is a sign change	
	(b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \quad \Rightarrow \quad x = 1$	M1 A1
	Turning point is $(1, -11)$	A1 **(3)**
	(c) $a = 2, b = 4, c = 4$	B1 B1 B1 **(3)**
	(d) 	
	Shape	B1
	ft their turning point in correct quadrant only	B1 ft
	2 and −8	B1 **(3)**
	(e) 	
	Shape	B1 **(1)**
		[13]

8.	(i) $\sec^2 x - \text{cosec}^2 x = (1+\tan^2 x) - (1+\cot^2 x)$	M1 A1
	$= \tan^2 x - \cot^2 x$ * cso	A1 **(3)**
	(ii)(a) $y = \arccos x \Rightarrow x = \cos y$	B1
	$x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$	B1 **(2)**
	Accept $\arcsin x = \arcsin \cos y$	
	(b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	B1 **(1)**
		[6]

Question Number	Scheme	Marks
9. (a)	$\cos PQR = \frac{6^2 + 6^2 - (6\sqrt{3})^2}{2 \times 6 \times 6}$ $\left\{= -\frac{1}{2}\right\}$	M1, A1
	$PQR = \frac{2\pi}{3}$	A1 (3)
(b)	$\text{Area} = \frac{1}{2} \times 6^2 \times \frac{2\pi}{3}\ \text{m}^2$	M1
	$= 12\pi\ \text{m}^2$ (*)	A1cso (2)
(c)	$\text{Area of } \Delta = \frac{1}{2} \times 6 \times 6 \times \sin\frac{2\pi}{3}\ \text{m}^2$	M1
	$= 9\sqrt{3}\ \text{m}^2$	A1cso (2)
(d)	$\text{Area of segment} = 12\pi - 9\sqrt{3}\ \text{m}^2$	M1
	$= 22.1\ \text{m}^2$	A1 (2)
(e)	$\text{Perimeter} = 6 + 6 + \left[6 \times \frac{2\pi}{3}\right]\ \text{m}$	M1
	$= 24.6\ \text{m}$	A1ft (2) **(11)**

Number		
10. (a)	$\{S_n = \}\ a + ar + \ldots + ar^{n-1}$	B1
	$\{rS_n = \}\ ar + ar^2 + \ldots + ar^n$	M1
	$(1-r)S_n = a(1-r^n)$	dM1
	$S_n = \dfrac{a(1-r^n)}{1-r}$ (✱)	A1cso (4)
(b)	$a = 200, r = 2, n = 10, \quad S_{10} = \dfrac{200(1-2^{10})}{1-2}$	M1, A1
	$= 204{,}600$	A1 (3)
(c)	$a = \dfrac{5}{6},\ r = \dfrac{1}{3}$	B1
	$S_\infty = \dfrac{a}{1-r}, \qquad S_\infty = \dfrac{\frac{5}{6}}{1-\frac{1}{3}}$	M1
	$= \dfrac{5}{4}$ o.e.	A1 (3)
(d)	$-1 < r < 1$ (or $\lvert r \rvert < 1$)	B1 (1) **(11)**