Second Year Assignment 9

1 The data below show the height above sea level, x metres, and the temperature, y °C, at 7.00 a.m., on the same day in summer at nine places in Europe.

Height, x (m)	1400	400	280	790	390	590	540	1250	680
Temperature, y (°C)	6	15	18	10	16	14	13	7	13

The product moment correlation coefficient is −0.975. Use this value to test for negative correlation at the 5% significance level. Interpret your result in context. **(3 marks)**

2 From the large data set, the daily total rainfall, x mm, and the daily total sunshine, y hours, were recorded for Camborne on seven consecutive days in May 2015.

Rainfall, x	2.2	tr	1.4	4.4	tr	0.2	0.6
Sunshine, y	5.2	7.7	5.6	0.3	5.1	0.1	8.9

a State the meaning of 'tr' in the table above. **(1 mark)**

b Calculate the product moment correlation coefficient for these 7 days, stating clearly how you deal with the entries marked 'tr'. **(2 marks)**

c With reference to your answer to part **b**, comment on the suitability of a linear regression model for these data. **(2 marks)**

3 Data are collected on the number of units (c) of a catalyst added to a chemical process, and the rate of reaction (r).

The data are coded using $x = \log c$ and $y = \log r$. It is found that a linear relationship exists between x and y and that the equation of the regression line of y on x is $y = 1.31x - 0.41$.

Use this equation to determine an expression for r in terms of c.

4 Members of a school book club read either murder mysteries (M), ghost stories (G) or epic fiction (E). $P(M) = 0.5$, $P(G) = 0.4$ and $P(E) = 0.6$. Given that no one reads both ghost stories and epic fiction and that $P(M \cap G) = 0.3$,

a draw a Venn diagram to illustrate these probabilities. **(4 marks)**

b Find:

i $P(M \cup G)$ **ii** $P((M \cap G) \cup (M \cap E))$ **(2 marks)**

c Are the events G' and M independent? You must justify your answer. **(2 marks)**

5 Given that events A and B are independent and that $P(A) = x$ and $P(B) = y$, find, in terms of x and y:

a $P(A \cap B)$ **(2 marks)**

b $P(A \cup B)$ **(2 marks)**

c $P(A \cup B')$ **(2 marks)**

6 A veterinary surgery has 750 registered pet owners. Of these 450 are female. 320 of the pet owners own a cat and 250 own a dog. Of the remaining pet owners, 25 are males who own another type of pet. No one owns more than one type of pet. 175 female owners have a cat. One owner is chosen at random. Given that:

F is the event that an owner is female

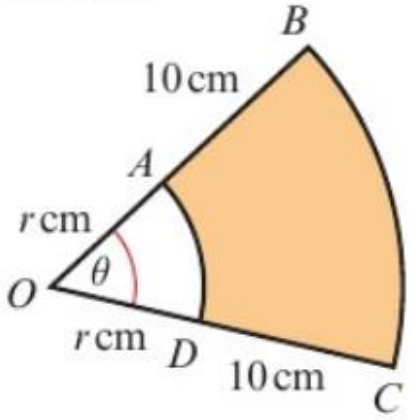
D is the event that an owner has a dog

C is the event that an owner has a cat.

Find:

a $P(D' \cap C')$ **b** $P(D|F')$ **c** $P(F'|C)$ **d** $P((D' \cap C')|F)$

7 In the diagram, AD and BC are arcs of circles with centre O, such that $OA = OD = r$ cm, $AB = DC = 10$ cm and $\angle BOC = \theta$ radians.



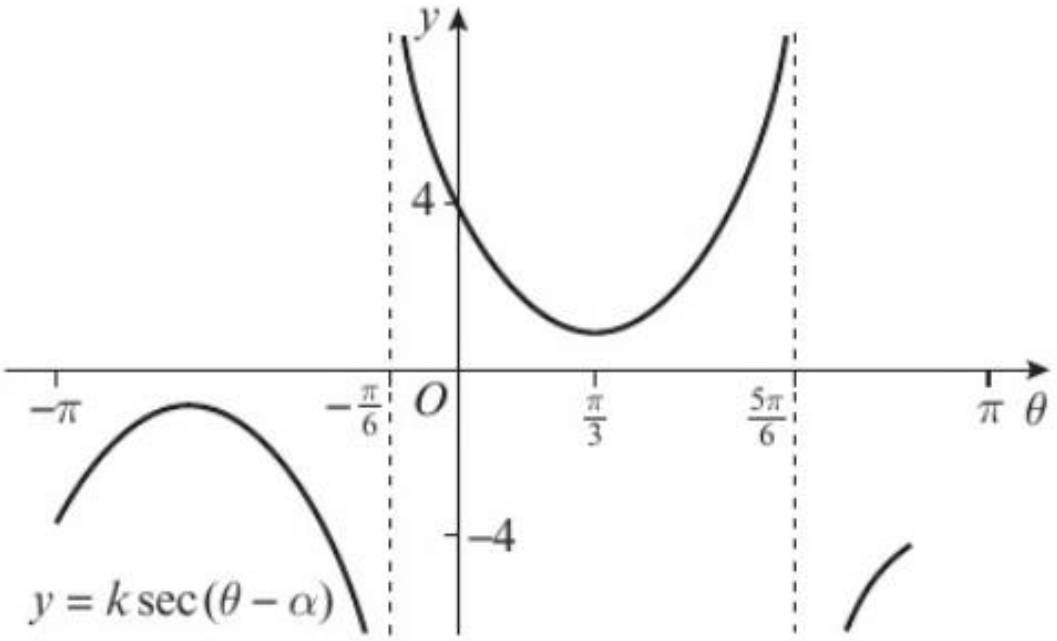
a Given that the area of the shaded region is 40 cm^2, show that $r = \frac{4}{\theta} - 5$. **(4)**

b Given also that $r = 6\theta$, calculate the perimeter of the shaded region. **(6)**

8 The diagram shows the graph of

$$y = k \sec(\theta - \alpha)$$

The curve crosses the y-axis at the point $(0, 4)$, and the θ-coordinate of its minimum point is $\frac{\pi}{3}$



a State, as a multiple of π, the value of α.

(1)

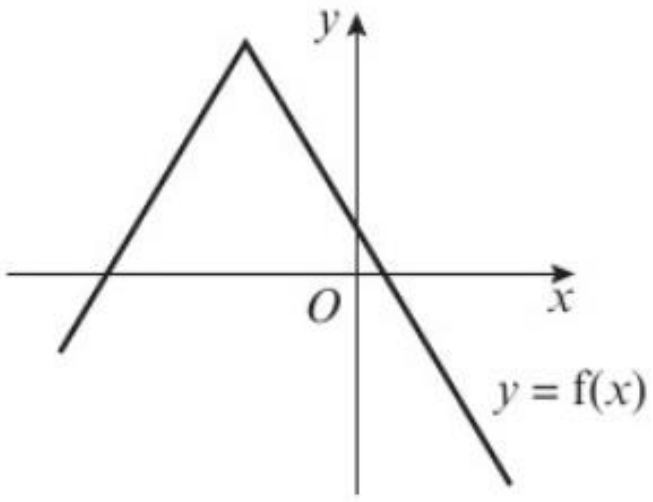
b Find the value of k. **(2)**

c Find the exact values of θ at the points where the graph crosses the line $y = -2\sqrt{2}$. **(3)**

9 The function f is defined by

$f(x) = -\frac{5}{3}|x + 4| + 8, \ x \in \mathbb{R}$

The diagram shows a sketch of the graph $y = f(x)$.



a State the range of f. **(1)**

b Give a reason why $f^{-1}(x)$ does not exist. **(1)**

c Solve the inequality $f(x) > \frac{2}{3}x + 4$. **(5)**

d State the range of values of k for which the equation $f(x) = \frac{5}{3}x + k$ has no solutions. **(2)**

10. Solve the following equations in the given intervals

a) $(\sec\theta - \cos\theta)^2 = \tan\theta - \sin^2\theta, \quad 0 \leq \theta \leq \pi$

b) $3\sec\frac{1}{2}\theta = 2\tan^2\frac{1}{2}\theta, \quad 0 \leq \theta \leq 360°$

c) $\tan^2 2\theta = \sec 2\theta - 1, \quad 0 \leq \theta \leq 180°$

d) $\sec^2\theta - \left(1+\sqrt{3}\right)\tan\theta + \sqrt{3} = 1, \quad 0 \leq \theta \leq 2\pi$

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

A When $(1 + ax)^n$ is expanded as a series in ascending powers of x, the coefficients of x and x^2 are -6 and 45 respectively.

a Find the value of a and the value of n.

b Find the coefficient of x^3.

c Find the set of values of x for which the expansion is valid.

B A geometric series is given by

$$6 - 24x + 96x^2 - \ldots.$$

The series is convergent.

a Write down a condition on x.

Given that $\sum_{r=1}^{\infty} 6 \times (-4x)^{r-1} = \frac{24}{5}$

b Calculate the value of x.

Answers

1)

H_0: $\rho = 0$, H_1: $\rho < 0$, critical value = –0.5822. Reject H_0. There is evidence that the greater the altitude, the lower the temperature.

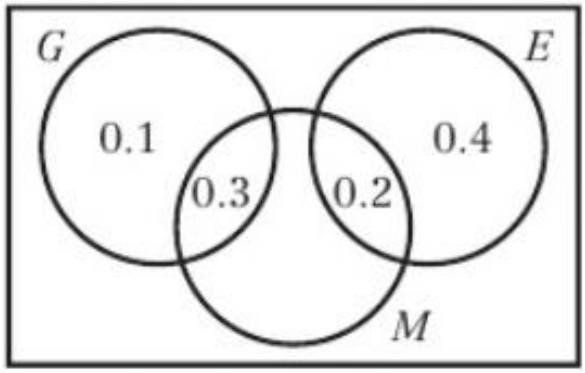
2) a) A trace is an amount less than 0.05 mm

b) -0.473 (3 s.f.) (treat "tr" as 0)

c) The data shows a weak negative correlation so a linear model may not be best. There may be other variables affecting the relationship or a different model might be a better fit.

3) $r = 0.389c^{1.31}$

4 **a**



b **i** 0.6 **ii** 0.5

c Not independent.

$P(G' \cap M) = 0.2$, $P(G') \times P(M) = 0.6 \times 0.5 = 0.3$

5 **a** xy **b** $x + y - xy$ **c** $1 - y + xy$

6) a) $\frac{6}{25}$ $b) \frac{13}{30}$ $c) \frac{29}{64}$ $d) \frac{31}{90}$

7) b) 28 cm

Answers

8 **a** $\frac{\pi}{3}$ **b** $k = 2$ **c** $-\frac{11\pi}{12}, -\frac{5\pi}{12}$

9 **a** $f(x) \leqslant 8$

b The function is not one-to-one.

c $-\frac{32}{3} < x < -\frac{8}{7}$

d $k > \frac{44}{3}$

10. a) $0, \frac{\pi}{4}, \pi$ b)120° c) 0°, 180°, d) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{4}, \frac{4\pi}{3}$