Inverse Trig Functions

6 Given that x satisfies $\arcsin x = k$, where $0 < k < \frac{\pi}{2}$,

a state the range of possible values of x **(1 mark)**

b express, in terms of x,

i $\cos k$ **ii** $\tan k$ **(4 marks)**

Given, instead, that $-\frac{\pi}{2} < k < 0$,

c how, if at all, are your answers to part **b** affected? **(2 marks)**

7 Sketch the graphs of:

a $y = \frac{\pi}{2} + 2\arcsin x$ **b** $y = \pi - \arctan x$

c $y = \arccos(2x + 1)$ **d** $y = -2\arcsin(-x)$

8 The function f is defined as $\mathrm{f}: x \mapsto \arcsin x$, $-1 \leqslant x \leqslant 1$, and the function g is such that $\mathrm{g}(x) = \mathrm{f}(2x)$.

a Sketch the graph of $y = \mathrm{f}(x)$ and state the range of f. **(3 marks)**

b Sketch the graph of $y = \mathrm{g}(x)$. **(2 marks)**

c Define g in the form $\mathrm{g}: x \mapsto \ldots$ and give the domain of g. **(3 marks)**

d Define g^{-1} in the form $\mathrm{g}^{-1}: x \mapsto \ldots$ **(2 marks)**

9 a Prove that for $0 \leqslant x \leqslant 1$, $\arccos x = \arcsin\sqrt{1 - x^2}$ **(4 marks)**

b Give a reason why this result is not true for $-1 \leqslant x \leqslant 0$. **(2 marks)**

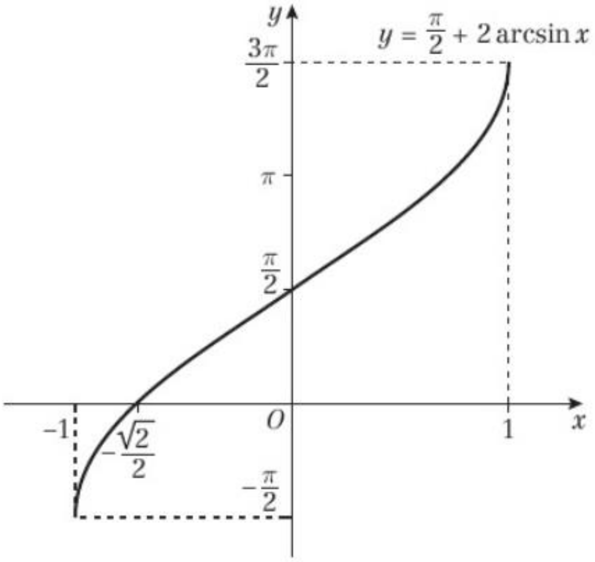
5 $\alpha, \pi - \alpha$

6 **a** $0 < x < 1$

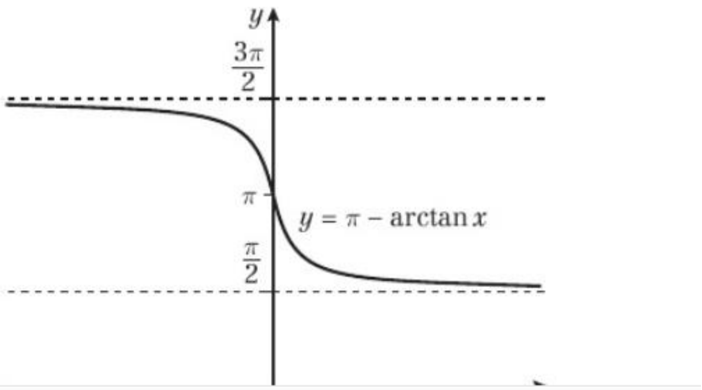
b **i** $\sqrt{1 - x^2}$ **ii** $\dfrac{x}{\sqrt{1 - x^2}}$

c **i** no change **ii** no change

7 **a**



b



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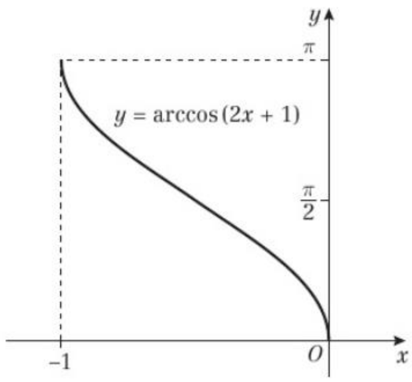

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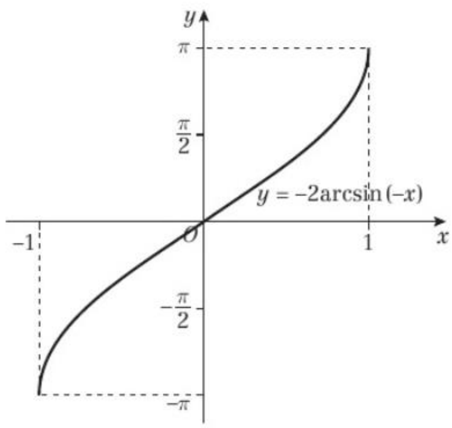
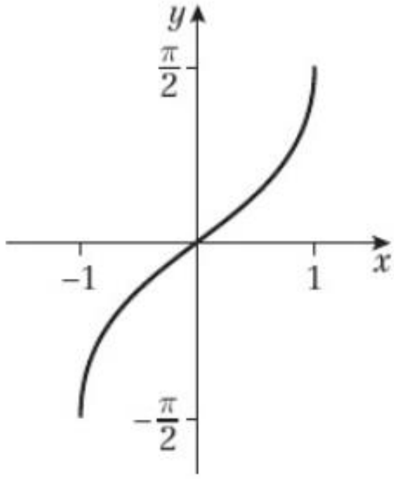


8 **a**



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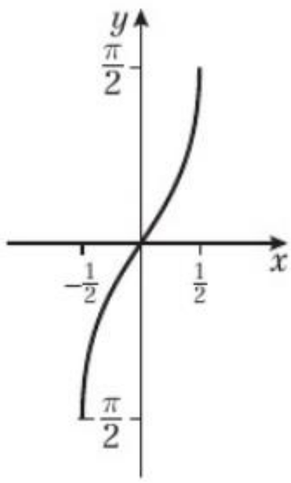

Range: $-\frac{\pi}{2} \leqslant \mathrm{f}(x) \leqslant \frac{\pi}{2}$

c g: $x \to \arcsin 2x$, $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$

d g^{-1}: $x \to \frac{1}{2}\sin x$, $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

9 **a** Let $y = \arccos x$. $x \in [0,1] \Rightarrow y \in \left[0, \frac{\pi}{2}\right]$

$\cos y = x$, so $\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$

(Note, $\sin y \neq -\sqrt{1 - x^2}$ since $y \in \left[0, \frac{\pi}{2}\right]$, so $\sin y \geqslant 0$)

$y = \arcsin\sqrt{1 - x^2}$

Therefore, $\arccos x = \arcsin\sqrt{1 - x^2}$ for $x \in [0,1]$.

b For $x \in [-1,0]$, $\arccos x \in \left(\frac{\pi}{2}, \pi\right)$, but arcsin only has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.