

RADIANS

You need to learn the exact values of the trigonometric ratios of these angles measured in radians:

■ $\sin \frac{\pi}{6} = \frac{1}{2}$	■ $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$	■ $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
■ $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	■ $\cos \frac{\pi}{3} = \frac{1}{2}$	■ $\tan \frac{\pi}{3} = \sqrt{3}$
■ $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	■ $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	■ $\tan \frac{\pi}{4} = 1$

You can use these rules to find sin, cos or tan of any positive or negative angle measured in radians using the corresponding acute angle made with the x -axis, θ .

- $\sin (\pi - \theta) = \sin \theta$
- $\sin (\pi + \theta) = -\sin \theta$
- $\sin (2\pi - \theta) = -\sin \theta$
- $\cos (\pi - \theta) = -\cos \theta$
- $\cos (\pi + \theta) = -\cos \theta$
- $\cos (2\pi - \theta) = \cos \theta$
- $\tan (\pi - \theta) = -\tan \theta$
- $\tan (\pi + \theta) = \tan \theta$
- $\tan (2\pi - \theta) = -\tan \theta$

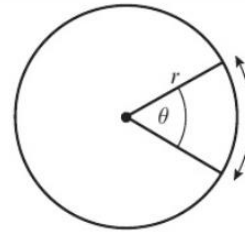
Sketch the following graphs

- 1) $y = \sin (\theta - \pi)$
- 2) $y = \tan (2\theta + \pi)$
- 3) $y = 3 \cos(\theta - 2\pi) + 4$

5.2 Arc length

Using radians greatly simplifies the formula for **arc length**.

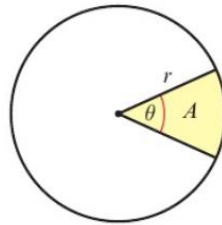
- To find the arc length l of a sector of a circle use the formula $l = r\theta$, where r is the radius of the circle and θ is the angle, in radians, contained by the sector.



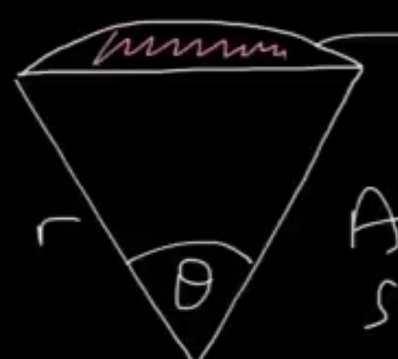
5.3 Areas of sectors and segments

Using radians also greatly simplifies the formula for the area of a **sector**.

- To find the area A of a sector of a circle use the formula $A = \frac{1}{2}r^2\theta$, where r is the radius of the circle and θ is the angle, in radians, contained by the sector.



Notation A sector of a circle is the portion of a circle enclosed by two radii and an arc. The smaller area is known as the **minor** sector and the larger is known as the **major** sector.



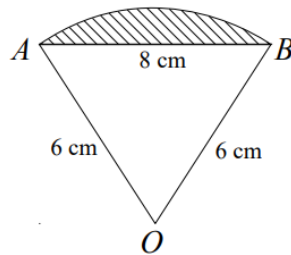
segment

Area of a segment =
area of sector
- area of triangle

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$
$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

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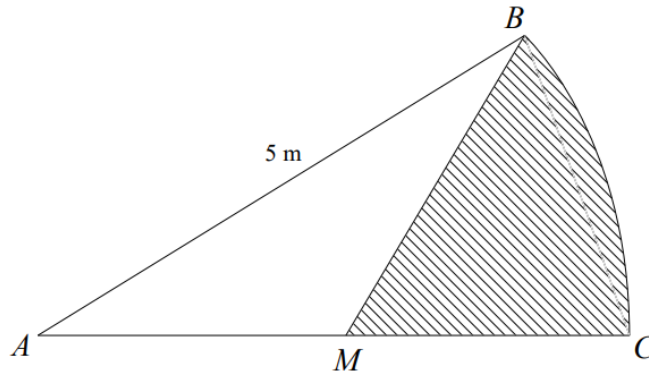
- 1 Sector AOB is a sector of a circle, radius 6cm. The chord AB is 8cm long.



- (a) Find the angle AOB in radians, giving your answer to 3 decimal places (3)
 (b) Calculate the area of the sector AOB (2)
 (c) Calculate the shaded area. (3)

(Total for question 1 is 8 marks)

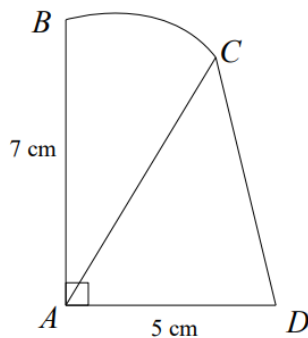
- 2 Sector ABC is a sector of a circle, centre A and radius 5m. Angle BAC = 0.5 radians



- (a) Find the length of the arc BC (2)
 (b) Calculate the area of the sector ABC (2)
 Given that M is the midpoint of AC
 (c) Find the perimeter of the shaded region (4)
 (d) Find the area of the shaded region (4)

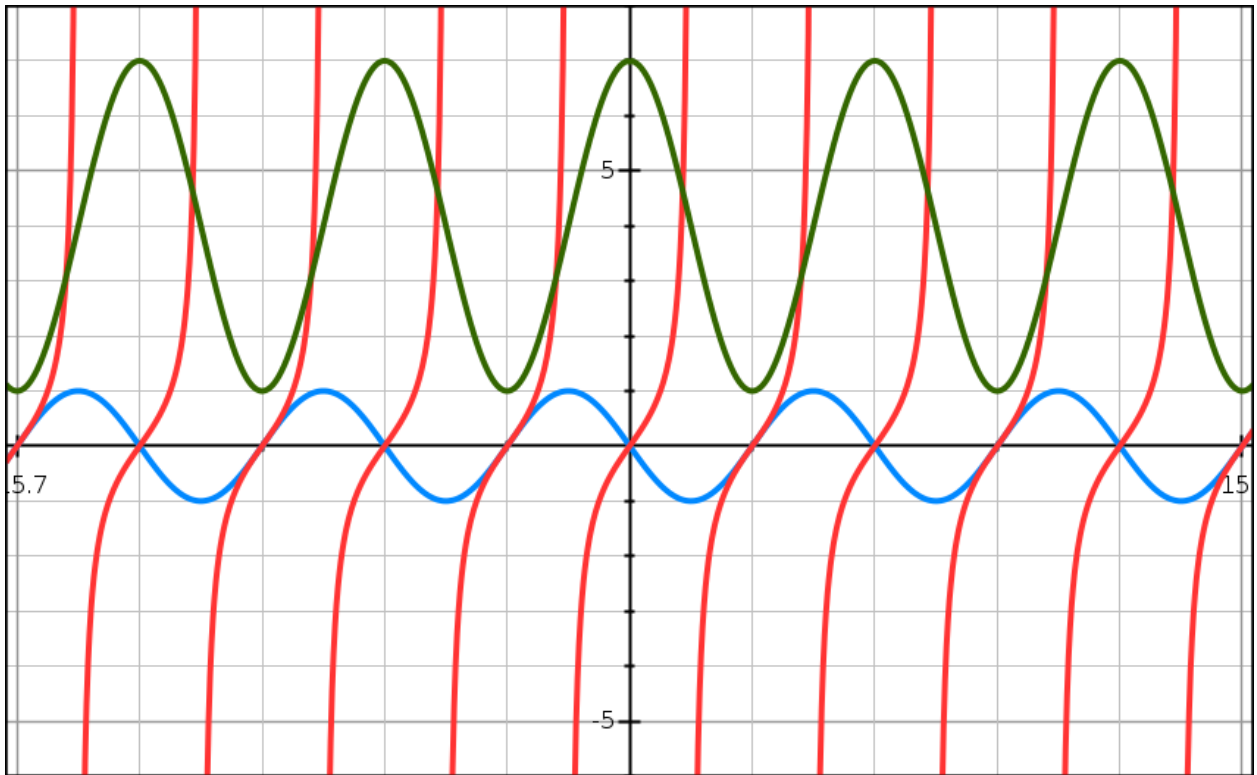
(Total for question 2 is 12 marks)

- 3 Sector ABC is a sector of a circle, centre A and radius 7 cm. Angle BAC = 0.6 radians



- (a) Find the length of the arc BC (2)
 (b) Calculate the area of the sector ABC (2)
 (c) Find the size of angle CAD, in radians (1)
 (d) Find the total area of the shape ABCD (3)

(Total for question 3 is 8 marks)



Answers (to 3 s.f. where appropriate)

1a) 1.459

b) 26.3

c) 8.38

2a) 2.5

b) $\frac{25}{4}$

c) 8.05

d) 3.25

3a) 4.2

b) 14.7

c) 0.971

d) 29.1

