Second Year Assignment 7

1. The functions f and g are given by
$f: x \to 4x - 1, \quad \{x \epsilon \mathcal{R}\}$
$g: x \to \frac{3}{2x-1}, \quad \left\{x \epsilon \mathcal{R}, x \neq \frac{1}{2}\right\}$
Find in its simplest form:
a) the inverse function f^{-1}

b) the composite function *gf*, stating its domain

c) the values of *x* for which $2\, f(x) = g(x)$, giving your answer to 3 decimal places.

2. Prove that $\sec^2 \theta + cosec^2 \theta \equiv \sec^2 \theta\, cosec^2 \theta$

3. A train engine of mass 6400 kg is pulling a carriage of mass 1600 kg along a straight horizontal railway track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The shunt is modelled as a light rod. The engine provides a constant driving force of 12 000 N. the resistances to motion of the engine and the carriage are modelled as constant forces of magnitude R N and 2000 N respectively. Given that the acceleration of the engine and the carriage is $0.5\ ms^{-2}$:
a) find the value of R

b) show that the tension in the shunt is 2800 N.

4. a) Express $\frac{8x+4}{(1-x)(2+x)}$ as partial fractions.

b) Hence or otherwise expand $\frac{8x+4}{(1-x)(2+x)}$ in ascending powers of x as far as the term in x^2

c) State the set of values of *x* for which the expansion is valid.

5. Find the constant term in the expansion of $\left(\frac{x^2}{2} - \frac{2}{x}\right)^9$

6. a) Sketch the graphs of $y = \arcsin(x)$ and $y = \arccos(x)$ on the same diagram, $-1 \leq x \leq 1$

b) Use your sketch to state an approximate solution to the equation

$$\arcsin(x) = \arccos(x), \qquad -1 \leq x \leq 1$$

c) Find an exact solution to the equation

$$\arcsin(x) = \arccos(x), \qquad -1 \leq x \leq 1$$

7. Prove that the sum of a geometric series, with first term a, and common ration r, is given by $S = \frac{a(r^n - 1)}{r - 1}$

8. The shape BCD is a design for a logo.

The straight lines DB and DC are equal in length.

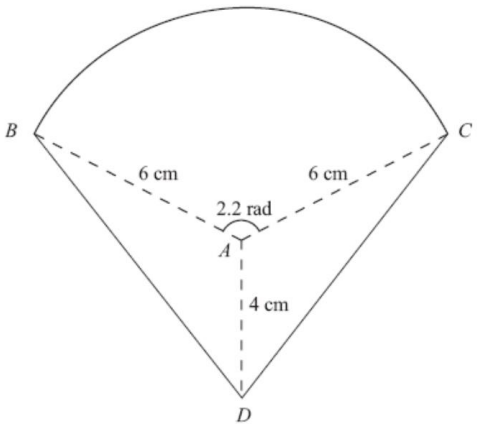
The curve BC is an arc of a circle with centre A and radius 6 cm.

The size of ∠ BAC is 2.2 radians and AD = 4 cm.

Find (a) the area of the sector BAC, in cm^2 ,

(b) the size of ∠ DAC, in radians to 3 significant figures,

(c) the complete area of the logo design, to the nearest cm^2



9. Solve the equation $\operatorname{cosec}^2 2x - \cot 2x = 1$ for $0 \leq x \leq 180°$

10. a) When θ is small, show that the equation $\frac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$ can be written as $9\theta + 2$

b) Hence write down the value of $\frac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$ when θ is small

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

1. Solve, for $-180 \leq \theta \leq 180$, the equation, $sec\,(2\theta + 10) = -1.3$
   Give your answers to 1 decimal place.

2. $f(x) = arccos\, x - \frac{\pi}{3}, \qquad x \epsilon \mathcal{R}, -1 \leq x \leq 1.$
   a) State the value of $f(-\frac{1}{2})$ in terms of π.
   b) Solve the equation $f(x) = 0$.
   c) Define the inverse function $f^{-1}(x)$ and state its domain.

Answers

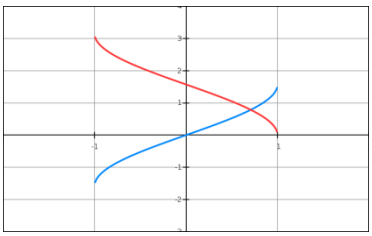
1. a) $\frac{x+1}{4}$ b) $\frac{3}{8x-3}$, , $\left\{x\epsilon\mathcal{R}, x \neq \frac{3}{8}\right\}$ c) -0.076 and 0.826

3. a) 6000 N

4 a) $\frac{4}{1-x} - \frac{4}{2+x}$ b) $2 + 5x + \frac{7}{2}x^2$ c) $|x| < 1$

5) 672

6. a)



b) ≈ 0.7

c) $\frac{\sqrt{2}}{2}$

8 a) $39.6\ cm^2$ b) 2.04 c) $61\ cm^2$

9. $22.5°, 45°, 112.5°, 135°$

10. 2