

Second Year Assignment 6

1 a) Sketch the curve with equation $y = f(x)$ where $f(x) = x^2 - 4$

b) Sketch the graphs of i) $y = f(4x)$, ii) $\frac{1}{3}y = f(x)$, (iii) $y = f(-x)$ (iv) $y = -f(x)$

2. a) When θ is small, show that $\frac{\sin 3\theta}{\theta \sin 4\theta} = \frac{3}{4\theta}$

b) i) When θ is small, show that the equation $\frac{4\cos 3\theta - 2 + 5\sin \theta}{1 - \sin 2\theta}$ can be written as $9\theta + 2$

ii) Hence write down the value of $\frac{4\cos 3\theta - 2 + 5\sin \theta}{1 - \sin 2\theta}$ when θ is small

3. The first three terms in the binomial expansion of $\frac{1}{\sqrt{a+bx}}$ are $3 + \frac{1}{3}x + \frac{1}{18}x^2 + \dots$

a) Find the values of the constants a and b

b) Find the coefficient of the x^3 term in the expansion.

4. A boat A has a position vector of $(2\mathbf{i} + \mathbf{j})$ km and a buoy B has a position vector of $(6\mathbf{i} - 4\mathbf{j})$ km, relative to a fixed origin O.

a) Find the distance of the boat from the buoy.

b) Find the bearing of the boat from the buoy.

The boat travels with constant velocity $(8\mathbf{i} - 10\mathbf{j})$ km/h.

c) Verify that the boat is travelling directly towards the buoy.

d) Find the speed of the boat.

e) Work out how long it will take the boat to reach the buoy.

5. a) The 1st term of an Arithmetic Sequence is 12 and the 5th term is 44. Find the 7th term.

b) The 3rd term of an Arithmetic Sequence is 25 and the common difference is 9. Find the 7th term.

c) The 1st term of an Arithmetic Sequence is 2 and the common difference is 3. Find the 1000th term.

d) The 18th term of an Arithmetic Sequence is 78 and the 25th term is 106. Find the 208th term.

6. Find the values of the constants A, B, C, D and E in the following identity:

$$3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + Bx + C)(x^2 - 3) + Dx + E$$

7. a) Find the coordinates of the points at which the line with equation $y = x - 4$ intersects the curve with equation $y^2 = 2x^2 - 17$

b) Prove that the equation $\frac{4x+3}{2x-1} + \frac{6x+1}{2x+3} = 3$ has no real roots

c) A point P lies on the line with equation $y = 4 - 3x$. The point P is a distance $\sqrt{34}$ from the origin. Find the two possible positions of point P.

8. A sector of a circle of radius 28 cm has perimeter P cm and area A cm². Given that A = 4P, find the value of P.

9. Given that $2x^2 + 24x + 2 = a(x + b)^2 + c$,

(a) Find the values of the constants a, b and c .

(b) Hence sketch the graph of $y = 2x^2 + 24x + 2$, giving the coordinates of the points where the graph crosses the x and y axis and the coordinates of the turning point.

(c) State the equation of the line of symmetry.

(d) State the least value of $2x^2 + 24x + 2$ and state the value of x for which this occurs.

10. A particle A is projected vertically upwards from the ground with speed $u \text{ ms}^{-1}$. The greatest height reached by the particle is 62.5 m above the ground.

(a) Find the total time for which the particle is 50 m or more above the ground.

One second after the particle A is projected upwards, a second particle B is also projected upwards from the ground with speed 40 ms^{-1} (B is projected from a position that ensures that A and B do not collide).

(b) After how many seconds does B overtake A?

(c) Will B also overtake A during their descents?

TEST YOURSELF

Give yourself 20 minutes to answer these questions.

If you finish early, check your answers.

I will mark your answers. Set your work out carefully.

1. $g(x) = \frac{3+5x}{(1+3x)(1-x)}$

Given that $g(x)$ can be expressed in the form $\frac{A}{1+3x} + \frac{B}{1-x}$,

- Find the values of A and B
- Hence, or otherwise, find the series expansion of $g(x)$, in ascending powers of x , up to and including the x^2 term. Simplify each term.

2. At the end of Year 1, a company employs 2400 people. A model predicts that the number of employees will increase by 6% each year, forming a geometric sequence.

- Find the predicted number of employees after 4 years, giving your answer to the nearest 10.

The company expects to expand in this way until the total number of employees first exceeds 6000 at the end of a year, N

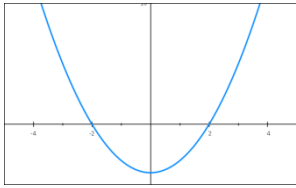
- Show that $(N - 1) \log 1.06 > \log 2.5$
- Find the value of N

The company has a charity scheme whereby they match any employee charity contribution exactly.

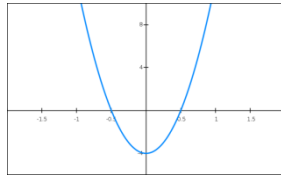
- Given that the average employee charity contribution is £5 each year, find the total charity donation over a 10-year period from the end of Year 1 to the end of Year 10. Give your answer to the nearest £1000

Answers

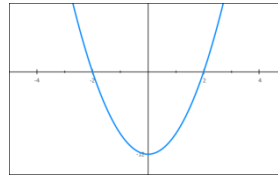
1a



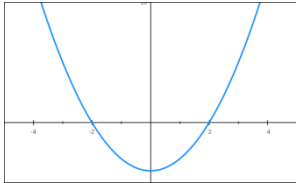
1b



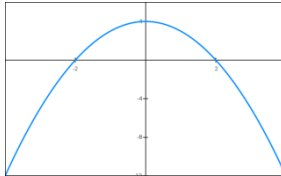
1c



1d



1e



2. b) ii) 2

3. a) $a = \frac{1}{9}, b = -\frac{2}{81}$

b) $\frac{5}{486}$

4. a) $\sqrt{41}$ km

b) 321.3°

d) $2\sqrt{41}$ km h⁻¹

e) 30 mins

5. a) 60

b) 61

c) 2999

d) 838

6. A = 3, B = -4, C = 1, D = 4, E = 1

7. a) 5. (-11, -15) (3, -1)

c) 4. (3, -5) and $(-\frac{3}{5}, \frac{29}{5})$

8. 78.4

9. (a) $a = 2, b = 6, c = -70$

(b) U shaped parabola Turning Point (-6, -70), y-intercept (0, 2), x-intercepts $(-6 \pm \sqrt{35}, 0)$

(c) $x = -6$

(d) least value is -70 when $x = -6$

10. (a) 3.2 s (2sf)

(b) 3.03 s (3 sf)

(c) During their descents, both A and B start from $v = 0$ and gain 9.8 m/s per second, so B could not overtake A on the way down