Second Year Assignment 5

1. A sequence is given by $x_1 = 1, x_{n+1} = x_n(p + x_n)$, where p is a constant $(p \neq 0)$
   a) Find x_2 in terms of p
   b) Show that $x_3 = 1 + 3p + 2p^2$

   Given that $x_3 = 1$,
   c) Find the value of p
   d) Write down the value of x_{2008}

2. Solve these equations by writing them in the form $a^x = a^y$

(a) $4(2^x)^2 = 8^x\sqrt{2}$ (b) $\frac{5^{2x+1}}{\sqrt{5}} = 25(5^x)$ (c) $5(5^{x+3})^2 = \frac{1}{125}$

3. Write each of the following in the form $y = A(x + B)^2 + C$ and state the minimum (or maximum) value of *y*

(a) $y = 2x^2 - 4x - 7$
(b) $y = 2x^2 + 12x - 15$
(c) $y = -x^2 + 6x + 2$

4. Find the set of values of *k* which satisfy the conditions below.
Give answers in both set and interval notation if appropriate.

(a) $0 = x^2 - kx + 9$ has no real solutions
(b) $0 = x^2 + 2kx - k$ has repeated roots
(c) $0 = x^2 + 2k$ has no real solutions
(d) $0 = kx^2 - 2 + kx + x$ has discriminant equal to -24

5. Solve the equations, giving exact answers

(a) $|3x - 2| = |7x - 8|$
(b) $|x^2 + 7x + 2| = |5 - x|$

6. Simplify the following by writing the below as a single fraction in its simplest form:

(a) $\frac{3x}{(x-2)^2} - \frac{1}{x-4}$ * (b) $\frac{1}{2(x+3)} + \frac{1}{3(x-1)}$ (c) $\frac{2}{x^2+2x+1} + \frac{1}{x+1}$

7. Use the binomial theorem to expand $\sqrt{(4 - 9x)}$, $|x| < \frac{4}{9}$, in ascending powers of *x*, up to and including the term in x^3, simplifying each term.

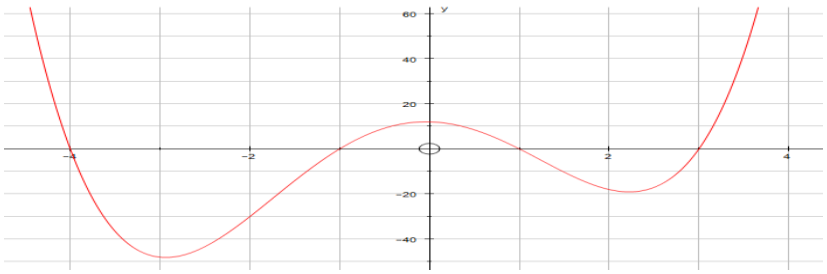
*I've changed the answer – it should be correct now!

8. $f(x) = \dfrac{3x^2+16}{(1-3x)(2+x)^2} = \dfrac{A}{(1-3x)} + \dfrac{B}{(2+x)} + \dfrac{C}{(2+x)^2}$, $|x| < \frac{1}{3}$.

(*a*) Find the values of *A* and *C* and show that *B* = 0.

(*b*) Hence, or otherwise, find the series expansion of f(*x*), in ascending powers of *x*, up to and including the term in x^3. Simplify each term.

9. The graph of $f(x) = x^4 + bx^3 + cx^2 + dx + e$ crosses the *x* axis at $(-4, 0), (-1, 0), (1, 0), (3, 0)$.
To 1 s.f., it has minima at $(-3, -50)$ and $(2, -20)$, and a maximum at $(0, 10)$
b, c, d and e are real constants



(a) Find the values of b, c, d *and* e

(b) Find the set of values of *p* for which the graph of $y = f(x) + p$ has no real roots

10. The second and fourth terms of a geometric series are 7.2 and 5.832 respectively.
The common ratio of the series is positive.
For this series, find
(*a*) the common ratio,
(*b*) the first term,
(*c*) the sum of the first 50 terms, giving your answer to 3 decimal places,
(*d*) the difference between the sum to infinity and the sum of the first 50 terms, giving your answer to 3 decimal places.

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

A) Sketch:

(a) $y = \frac{1}{(x-2)^2} + 1$ (b) $y = x(x-1)^3$ (c) $y = x(2-x)(1-x)$

State the intercepts with the axes and the equations of any asymptotes

(B) Susan is using surds to evaluate an expression. Explain what is wrong with her working and write out a complete solution

$3\sqrt{2} + 2\sqrt{3} - 5\sqrt{18} + 2\sqrt{24}$
$= \sqrt{18} + \sqrt{12} - \sqrt{450} + \sqrt{96}$
$= \sqrt{6}(\sqrt{3} + \sqrt{2} - \sqrt{75} + \sqrt{16})$
$= \sqrt{6}(\sqrt{3} + \sqrt{2} - 5\sqrt{3} + 4)$
$= -\sqrt{6}(4\sqrt{3} - \sqrt{2} - 4)$
$= -\sqrt{6}(4(\sqrt{3-2} - 4)$
$= -\sqrt{6}(4 - 4)$
$= 0$

Answers

1. a) $p+1$ c) $-\frac{3}{2}$ d) $-\frac{1}{2}$

2. (a) $x=\frac{3}{2}$ (b) $x=\frac{3}{2}$ (c) $x=-5$

3. (a) $2(x-1)^2-9;\ \ min-9$ (b) $2(x+3)^2-33;\ \ min-33$
(c) $-(x-3)^2+11;\ maximum\ 11$

4. (a) $\{k:-6<k<6\}$ (b) $k=-1, k=0$
(c) $\{k:k>0\}$ (d) $k=-5$

5. a) $\frac{3}{2}, 1$ b) $-3-\sqrt{2},\ \sqrt{2}-3,\ \ -4-\sqrt{19},\ \ \sqrt{19}-4$

6. (a) $\frac{2(x^2-4x-2)}{(x-4)(x-2)^2}$ (b) $\frac{5x+3}{6(x+3)(x-1)}$ (c) $\frac{x+3}{(x+1)^2}$

7. $2-\frac{9}{4}x-\frac{81}{64}x^2-\frac{729}{512}x^3$

8. a) A=3, C=4 b) $4+8x+\frac{111}{4}x^2+\frac{161}{2}x^3$

9. (a) b = 1, c = −13, d = −1, e = 12
(b) p > 50 (explain why)

10. a) 0.9 b) 8 c) 79588 d) 0.412