

Second Year Assignment 4

1. Sketch the following graphs, stating the asymptotes and axis intercepts:

(a) $y = 3 + \frac{4}{x+1}$ (b) $y = \frac{2}{x+1} - 4$

(c) $y = 3 + \frac{4}{(x+1)^2}$ (d) $y = 3 - \frac{4}{x^2}$

2. Find the coordinates of the centre and radius of each of the following circles:

(a) $x^2 + y^2 - 4y + 3 = 0$ (b) $x^2 + y^2 - 2x - 10y - 23 = 0$

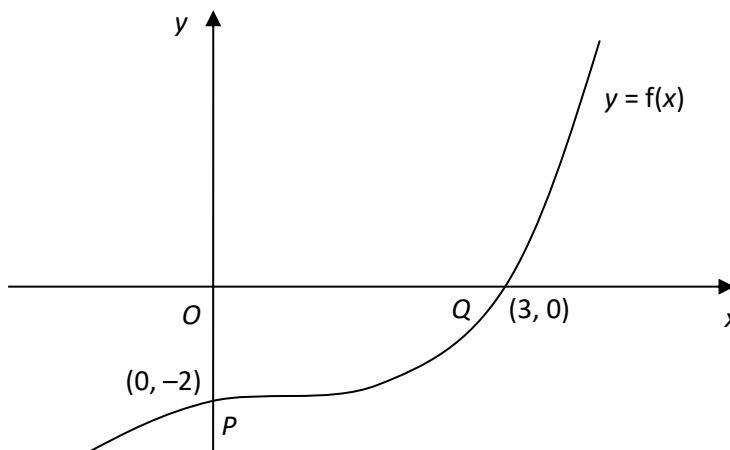
(c) $x^2 + y^2 = 8x - 6y$ (d) $x^2 + y^2 - 2x + 16y = 35$

3. (a) A particle moves in a straight line. When $t = 0$ its velocity is $3m/s$. When $t = 4$ its velocity is $12m/s$. Find its acceleration, assumed to be constant.

(b) A car is approaching traffic lights at $15m/s$ when the driver applies the brake and comes to a stop in $45m$. Find the deceleration, assumed constant, and the time taken to stop.

4.

The diagram shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.



In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$,

(b) $y = f^{-1}(x)$,

(c) $y = \frac{1}{2} f(3x)$.

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

5. Find the value or set of values of k such that:

(a) $x^2 - 6x + k = 0$ has equal roots

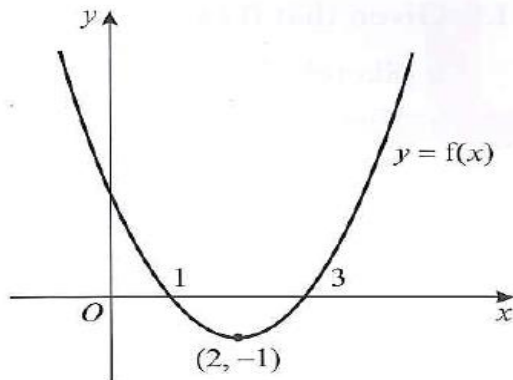
(b) $x^2 + kx + 4 = 0$ has real roots

6. Writing your answers in set notation form, find the set of values of x for which: both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$ are satisfied.

7. (a) A particle has constant acceleration $6m/s^2$ whilst travelling in a straight line between points A and B. It passes A at $2m/s$ and B at $5m/s$. Calculate the distance AB.

(b) A person on the top of a tower of height 45m holds their arm out over the side of the building and drops a stone vertically downwards. The stone takes 3.03 s to reach the ground. Use this information to prove that the value of acceleration due to gravity is 9.8 to 2s.f.

8. The diagram shows the graph of the quadratic function $f(x)$. The graph meets the x -axis at $(1, 0)$ and $(3, 0)$ and the minimum point is $(2, -1)$



(a) find the equation of the graph in the form $y = ax^2 + bx + c$

(b) On separate axes, sketch the graphs of

(i) $y = f(x + 2)$

(ii) $y = f(2x)$

(c) On each graph label the coordinates of the points at which the graph meets the x -axis and label the coordinates of the minimum point.

9. Using proof by contradiction, prove that there are infinitely many prime numbers.

10. Sophie will be paid a salary of £35000 in 2018. Each year Sophie will get a 3% pay rise, the first increase being in 2019, so that her salaries form a geometric sequence (a) Find, to the nearest £100, Sophie's salary in 2020. Sophie will receive a salary each year until she retires at the end of 2037. (b) Find, to the nearest £100, the total amount Sophie will have earned from 2018 until she retires in 2037.

TEST YOURSELF

Give yourself 20 minutes to answer these questions.

If you finish early, check your answers.

I will mark your answers. Set your work out carefully.

(A) A sequence u_1, u_2, u_3, \dots defined by $u_1 = 1, u_{n+1} = (u_n - 1)^2, n \geq 1$

(a) Find u_2, u_3 and u_4

(b) Write down the value of u_{10}

(B) The function f is defined by $f(x) = \frac{1}{x+2}, x \in \mathbb{R}, x \neq -2$

(a) Write down the range of $f(x)$

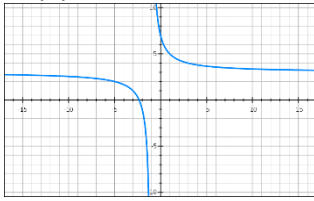
(b) Find an expression for $f^{-1}(x)$ and state its domain

$g(x) = x^2 - 5, x \in \mathbb{R}$

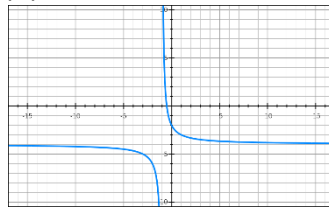
(c) Solve $fg(x) = \frac{1}{2}$

Answers

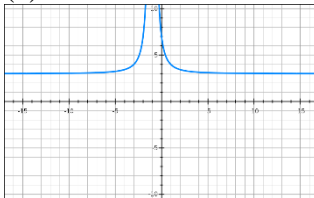
1. (a)



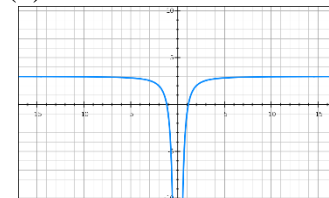
(b)



(c)



(d)



2. (a) $C(0, 2)$, $r = 1$

(b) $C(1, 5)$, $r = 7$

(c) $C(4, -3)$, $r = 5$

(d) $C(1, -8)$, $r = 10$

3. (a) $2.25m/s^2$

(b) $2.5m/s^2$ and 6 seconds to stop

5. (a) $k = 9$

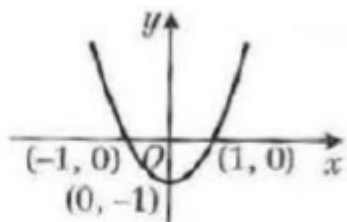
(b) $k \leq -4$, $k \geq 4$

6. $\left\{x: \frac{1}{2} < x < \frac{5}{2}\right\}$

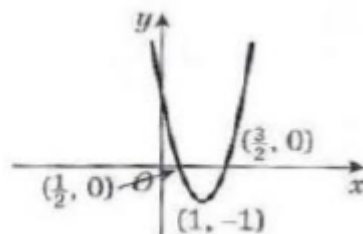
7. (a) 1.75m

8. (a) $y = x^2 - 4x + 3$

(b) (i)



(ii)



9.

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10. a) £37100 b) £940500