

FUNCTIONS

Answers

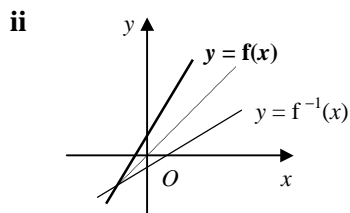
- 1**
- a** $y = 10x + 3$
 swap $x = 10y + 3$
 $y = \frac{x-3}{10}$
 $f^{-1}(x) = \frac{x-3}{10}, x \in \mathbb{R}$
- b** $y = 9 + 2x$
 swap $x = 9 + 2y$
 $y = \frac{x-9}{2}$
 $f^{-1}(x) = \frac{x-9}{2}, x \in \mathbb{R}$
- c** $y = 5 - 6x$
 swap $x = 5 - 6y$
 $y = \frac{5-x}{6}$
 $f^{-1}(x) = \frac{5-x}{6}, x \in \mathbb{R}$
- d** $y = \frac{x+3}{4}$
 swap $x = \frac{y+3}{4}$
 $y = 4x - 3$
 $f^{-1}(x) = 4x - 3, x \in \mathbb{R}$
- e** $y = \frac{1}{3}(2x - 5)$
 swap $x = \frac{1}{3}(2y - 5)$
 $y = \frac{3x+5}{2}$
 $f^{-1}(x) = \frac{3x+5}{2}, x \in \mathbb{R}$
- f** $y = 8 - \frac{3}{5}x$
 swap $x = 8 - \frac{3}{5}y$
 $y = \frac{40-5x}{3}$
 $f^{-1}(x) = \frac{40-5x}{3}, x \in \mathbb{R}$
- 2**
- a** $y = \ln x$
 swap $x = \ln y$
 $y = e^x$
 $f^{-1}(x) = e^x, x \in \mathbb{R}$
- b** $y = \frac{1}{x}$
 swap $x = \frac{1}{y}$
 $y = \frac{1}{x}$
 $f^{-1}(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$
- c** $y = \sqrt[4]{x}$
 swap $x = \sqrt[4]{y}$
 $y = x^4$
 $f^{-1}(x) = x^4, x \in \mathbb{R}, x > 0$
- d** $y = 3x - 4$
 swap $x = 3y - 4$
 $y = \frac{x+4}{3}$
 $f(0) = -4, f(3) = 5$
 $f^{-1}(x) = \frac{x+4}{3}, x \in \mathbb{R}, -4 \leq x < 5$
- e** $y = \frac{1}{x-5}$
 swap $x = \frac{1}{y-5}$
 $y = \frac{1}{x} + 5$
 $f^{-1}(x) = \frac{1}{x} + 5, x \in \mathbb{R}, x \neq 0$
- f** $y = 2 + \frac{1}{x}$
 swap $x = 2 + \frac{1}{y}$
 $y = \frac{1}{x-2}$
 $f^{-1}(x) = \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$

3 a i $y = 2x + 1$

swap $x = 2y + 1$

$$y = \frac{x-1}{2}$$

$$f^{-1}: x \rightarrow \frac{x-1}{2}, x \in \mathbb{R}$$

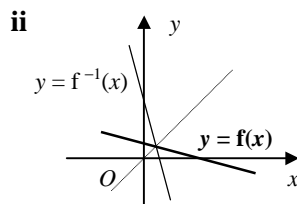


b i $y = \frac{1-x}{5}$

swap $x = \frac{1-y}{5}$

$$y = 1 - 5x$$

$$f^{-1}: x \rightarrow 1 - 5x, x \in \mathbb{R}$$

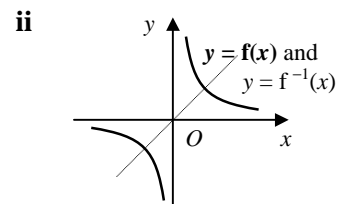


c i $y = \frac{10}{x}$

swap $x = \frac{10}{y}$

$$y = \frac{10}{x}$$

$$f^{-1}: x \rightarrow \frac{10}{x}, x \in \mathbb{R}, x \neq 0$$



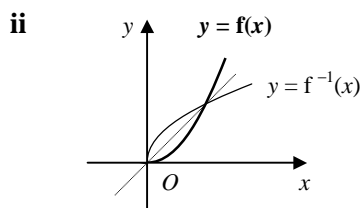
d i $y = x^2$

swap $x = y^2$

$$y = \pm\sqrt{x}$$

(domain of $f \Rightarrow +$)

$$f^{-1}: x \rightarrow \sqrt{x}, x \in \mathbb{R}, x > 0$$

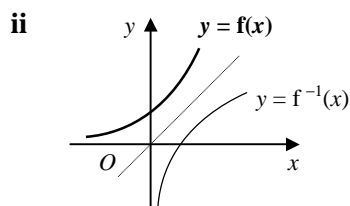


e i $y = e^x$

swap $x = e^y$

$$y = \ln x$$

$$f^{-1}: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$$

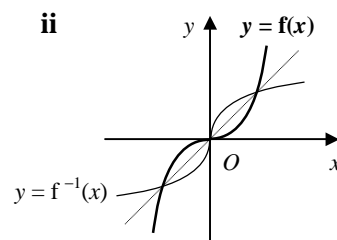


f i $y = x^3$

swap $x = y^3$

$$y = \sqrt[3]{x}$$

$$f^{-1}: x \rightarrow \sqrt[3]{x}, x \in \mathbb{R}$$



4 a $y = 5x + 1$

swap $x = 5y + 1$

$$f^{-1}(x) = y = \frac{x-1}{5}$$

$$\frac{x-1}{5} = 2$$

$$x - 1 = 10$$

$$x = 11$$

d $y = \sqrt{x+2}$

swap $x = \sqrt{y+2}$

$$f^{-1}(x) = y = x^2 - 2$$

$$x^2 - 2 = 3x - 4$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

b $y = \frac{2x-4}{3}$

swap $x = \frac{2y-4}{3}$

$$f^{-1}(x) = y = \frac{3x+4}{2}$$

$$\frac{3x+4}{2} = 7 - x$$

$$3x + 4 = 14 - 2x$$

$$x = 2$$

e $y = \frac{4}{x+3}$

swap $x = \frac{4}{y+3}$

$$f^{-1}(x) = y = \frac{4}{x} - 3$$

$$\frac{4}{x} - 3 = 5(x+1)$$

$$4 - 3x = 5x(x+1)$$

$$5x^2 + 8x - 4 = 0$$

$$(5x-2)(x+2) = 0$$

$$x = -2, \frac{2}{5}$$

c $y = e^x + 2$

swap $x = e^y + 2$

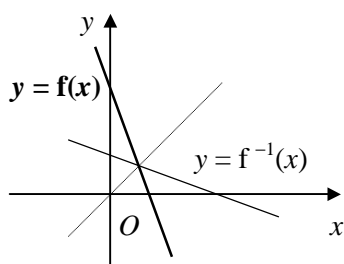
$$f^{-1}(x) = y = \ln(x-2)$$

$$\ln(x-2) = \ln(3x-8)$$

$$x-2 = 3x-8$$

$$x = 3$$

5 a



b $4 - 2x = x$

$x = \frac{4}{3}$

$\therefore \left(\frac{4}{3}, \frac{4}{3}\right)$

6 a $g \Rightarrow y = \frac{1}{2x+4}$

swap $x = \frac{1}{2y+4}$

$y = \frac{1}{2} \left(\frac{1}{x} - 4 \right) = \frac{1}{2x} - 2$

$g^{-1}(x) = \frac{1}{2x} - 2, x \in \mathbb{R}, x \neq 0$

range: $g^{-1}(x) \in \mathbb{R}, g^{-1}(x) \neq -2$

b $= g(3 - 2x)$

$= \frac{1}{2(3-2x)+4} = \frac{1}{10-4x}$

$gf(x) = \frac{1}{10-4x}, x \in \mathbb{R}, x \neq \frac{5}{2}$

c $f \Rightarrow y = 3 - 2x$

swap $x = 3 - 2y$

$f^{-1}(x) = y = \frac{3-x}{2}$

$\therefore \frac{1}{10-4x} = \frac{3-x}{2}$

$2 = (3-x)(10-4x)$

$2x^2 - 11x + 14 = 0$

$(2x-7)(x-2) = 0$

$x = 2, \frac{7}{2}$

7 a i $y = 5x + 2$

swap $x = 5y + 2$

$y = \frac{x-2}{5}$

$f^{-1}: x \rightarrow \frac{x-2}{5}, x \in \mathbb{R}$

ii $= f\left(\frac{1}{x}\right)$

$= \frac{5}{x} + 2$

$fg: x \rightarrow \frac{5}{x} + 2, x \in \mathbb{R}, x \neq 0$

iii $y = \frac{5}{x} + 2$

swap $x = \frac{5}{y} + 2$

$y = \frac{5}{x-2}$

$(fg)^{-1}: x \rightarrow \frac{5}{x-2}, x \in \mathbb{R}, x \neq 2$

b $\frac{x-2}{5} = \frac{5}{x} + 2$

$x(x-2) = 25 + 10x$

$x^2 - 12x - 25 = 0$

$x = \frac{12 \pm \sqrt{144+100}}{2} = 6 \pm \sqrt{61} = -1.81, 13.81$

8 a $y = \frac{1}{2} \ln(4x - 9)$

swap $x = \frac{1}{2} \ln(4y - 9)$

$$4y - 9 = e^{2x}$$

$$y = \frac{1}{4}(e^{2x} + 9)$$

$$f^{-1}: x \rightarrow \frac{1}{4}(e^{2x} + 9), x \in \mathbb{R}$$

b $y = \frac{x-2}{x+5}$

swap $x = \frac{y-2}{y+5}$

$$xy + 5x = y - 2$$

$$y(1-x) = 5x + 2$$

$$y = \frac{5x+2}{1-x}$$

$$f^{-1}: x \rightarrow \frac{5x+2}{1-x}, x \in \mathbb{R}, x \neq 1$$

c $y = e^{0.4x-2}$

swap $x = e^{0.4y-2}$

$$0.4y - 2 = \ln x$$

$$y = \frac{5}{2}(2 + \ln x)$$

$$f^{-1}: x \rightarrow 5 + \frac{5}{2} \ln x, x \in \mathbb{R}, x > 0$$

d $y = \sqrt[3]{x^5 - 3}$

swap $x = \sqrt[3]{y^5 - 3}$

$$y^5 - 3 = x^3$$

$$y = \sqrt[5]{x^3 + 3}$$

$$f^{-1}: x \rightarrow \sqrt[5]{x^3 + 3}, x \in \mathbb{R}$$

e $y = \log_{10}(2 - 7x)$

swap $x = \log_{10}(2 - 7y)$

$$2 - 7y = 10^x$$

$$y = \frac{1}{7}(2 - 10^x)$$

$$f^{-1}: x \rightarrow \frac{1}{7}(2 - 10^x), x \in \mathbb{R}$$

f $y = \frac{4-x}{3x+2}$

swap $x = \frac{4-y}{3y+2}$

$$3xy + 2x = 4 - y$$

$$y(3x+1) = 4 - 2x$$

$$y = \frac{4-2x}{3x+1}$$

$$f^{-1}: x \rightarrow \frac{4-2x}{3x+1}, x \in \mathbb{R}, x \neq -\frac{1}{3}$$

9 a i $y = e^{2x}$

swap $x = e^{2y}$

$$2y = \ln x$$

$$y = \frac{1}{2} \ln x$$

$$f^{-1}: x \rightarrow \frac{1}{2} \ln x, x \in \mathbb{R}, x > 0$$

b i $y = x^2 + 4$

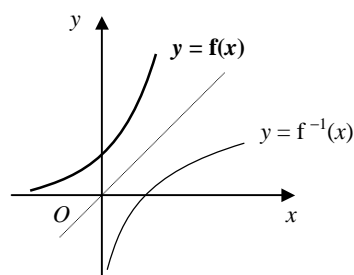
swap $x = y^2 + 4$

$$y = \pm \sqrt{x-4}$$

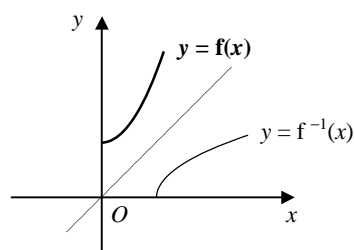
(domain of $f \Rightarrow +$)

$$f^{-1}: x \rightarrow \sqrt{x-4}, x \in \mathbb{R}, x > 4$$

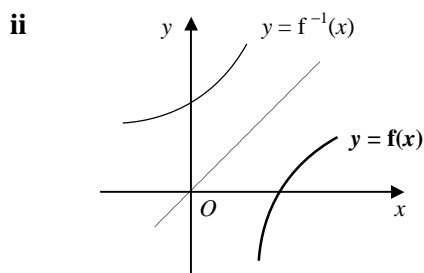
ii



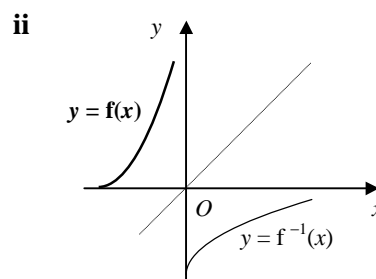
ii



c i $y = \ln(x - 3)$
 swap $x = \ln(y - 3)$
 $y - 3 = e^x$
 $y = e^x + 3$
 $f^{-1}: x \rightarrow e^x + 3, x \in \mathbb{R}$



d i $y = x^2 + 6x + 9 = (x + 3)^2$
 swap $x = (y + 3)^2$
 $y = -3 \pm \sqrt{x}$
 (domain of $f \Rightarrow +$)
 $f^{-1}: x \rightarrow -3 + \sqrt{x}, x \in \mathbb{R}, x > 0$



10 a i $f(x) = (x + 3)^2 - 6$
 $x < -3 \therefore \text{range: } f(x) > -6$
ii $y = (x + 3)^2 - 6$
 swap $x = (y + 3)^2 - 6$
 $y = -3 \pm \sqrt{x + 6}$
 (domain of $f \Rightarrow -$)
 $f^{-1}(x) = -3 - \sqrt{x + 6}, x \in \mathbb{R}, x > -6$

b i $f(x) = (x - 2)^2 + 1$
 $x \geq 2 \therefore \text{range: } f(x) \geq 1$
ii $y = (x - 2)^2 + 1$
 swap $x = (y - 2)^2 + 1$
 $y = 2 \pm \sqrt{x - 1}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = 2 + \sqrt{x - 1}, x \in \mathbb{R}, x \geq 1$

c i $f(x) = (x + \frac{5}{2})^2 - \frac{33}{4}$
 $x < -\frac{5}{2} \therefore \text{range: } f(x) > -8\frac{1}{4}$
ii $y = (x + \frac{5}{2})^2 - \frac{33}{4}$
 swap $x = (y + \frac{5}{2})^2 - \frac{33}{4}$
 $y = -\frac{5}{2} \pm \sqrt{x + \frac{33}{4}}$
 (domain of $f \Rightarrow -$)
 $f^{-1}(x) = -\frac{5}{2} - \sqrt{x + \frac{33}{4}}, x \in \mathbb{R}, x > -8\frac{1}{4}$

d i $f(x) = (x - \frac{3}{2})^2 + \frac{11}{4}$
 $2 < x < 4, f(2) = 3, f(4) = 9$
 $\therefore \text{range: } 3 < f(x) < 9$
ii $y = (x - \frac{3}{2})^2 + \frac{11}{4}$
 swap $x = (y - \frac{3}{2})^2 + \frac{11}{4}$
 $y = \frac{3}{2} \pm \sqrt{x - \frac{11}{4}}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = \frac{3}{2} + \sqrt{x - \frac{11}{4}}, x \in \mathbb{R}, 3 < x < 9$

e i $f(x) = 8 - 2x - x^2 = 9 - (x + 1)^2$
 $x \geq -1 \therefore \text{range: } f(x) \leq 9$
ii $y = 9 - (x + 1)^2$
 swap $x = 9 - (y + 1)^2$
 $y = -1 \pm \sqrt{9 - x}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = -1 + \sqrt{9 - x}, x \in \mathbb{R}, x \leq 9$

f i $f(x) = -5(x^2 - 4x) = 20 - 5(x - 2)^2$
 $x > 2 \therefore \text{range: } f(x) < 20$
ii $y = 20 - 5(x - 2)^2$
 swap $x = 20 - 5(y - 2)^2$
 $y = 2 \pm \sqrt{\frac{20 - x}{5}}$
 (domain of $f \Rightarrow +$)
 $f^{-1}(x) = 2 + \sqrt{\frac{20 - x}{5}}, x \in \mathbb{R}, x < 20$

11 a $y = \frac{1}{3}(2x - 5)$

swap $x = \frac{1}{3}(2y - 5)$

$$f^{-1}(x) = y = \frac{3x+5}{2}$$

$$\therefore \frac{3x+5}{2} = \frac{4}{2-x}$$

$$(3x+5)(2-x) = 8$$

$$3x^2 - x - 2 = 0$$

$$(3x+2)(x-1) = 0$$

$$x = -\frac{2}{3}, 1$$

b $y = \ln \frac{x+3}{5}$

swap $x = \ln \frac{y+3}{5}$

$$f^{-1}(x) = y = 5e^x - 3$$

$$\therefore 5e^x - 3 = 10 - 6e^{-x}$$

$$5e^{2x} - 13e^x + 6 = 0$$

$$(5e^x - 3)(e^x - 2) = 0$$

$$e^x = \frac{3}{5}, 2$$

$$x = \ln \frac{3}{5}, \ln 2$$

$$x = -0.511, 0.693 \text{ (3sf)}$$

c $y = x^2 - 4$

swap $x = y^2 - 4$

$$y = \pm\sqrt{x+4}$$

(domain of $f \Rightarrow +$)

$$f^{-1}(x) = \sqrt{x+4}$$

$$\therefore \sqrt{x+4} = \frac{x+6}{3}$$

$$x+4 = \frac{(x+6)^2}{9}$$

$$9(x+4) = x^2 + 12x + 36$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = -3, 0$$

12 a -2

b $\frac{6+b}{6-2} = 4$

$$6+b = 16$$

$$b = 10$$

c $y = \frac{x+10}{x-2}$

swap $x = \frac{y+10}{y-2}$

$$xy - 2x = y + 10$$

$$y(x-1) = 2x + 10$$

$$y = \frac{2x+10}{x-1}$$

$$f^{-1}(x) = \frac{2x+10}{x-1}, x \in \mathbb{R}, x \neq 1$$

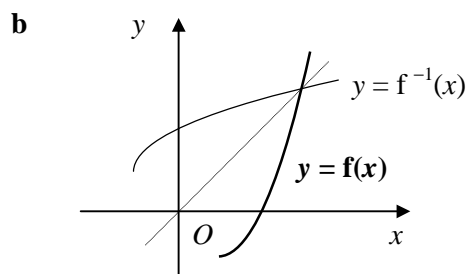
13 a $y = x^2 - 3x = (x - \frac{3}{2})^2 - \frac{9}{4}$

swap $x = (y - \frac{3}{2})^2 - \frac{9}{4}$

$$y = \frac{3}{2} \pm \sqrt{x + \frac{9}{4}}$$

(domain of $f \Rightarrow +$)

$$f^{-1}: x \rightarrow \frac{3}{2} + \sqrt{x + \frac{9}{4}}, x \in \mathbb{R}, x \geq -\frac{9}{4}$$



c $g \Rightarrow y = 2x + 3$

swap $x = 2y + 3$

$$g^{-1}(x) = y = \frac{x-3}{2}$$

$$g^{-1}(12) = \frac{9}{2}$$

$$f^{-1}g^{-1}(12) = f^{-1}\left(\frac{9}{2}\right)$$

$$= \frac{3}{2} + \sqrt{\frac{27}{4}}$$

$$= \frac{3}{2} + \frac{3}{2}\sqrt{3}$$

$$= \frac{3}{2}(1 + \sqrt{3})$$

$$\therefore a = 1\frac{1}{2}$$