Second Year Assignment 3

1. a) Given that the function $f(x) = x^2 + px$ is increasing on the interval $[-1,1]$, find one possible value for p.

b) State with justification whether this is the only possible value for p

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2. By completing the square, show that the solutions to the equation $x^2 + 2bx + c = 0$ are given by the formula $x = -b \pm \sqrt{b^2 - c}$

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3. a) Where is Hurn? (*A copy of the Large Data Set is on my website*).

b) Where is Leuchars?

c) Would you expect it to be windier in Hurn or Leuchars?

d) Here is the daily mean windspeed for Hurn and Leuchars in May 2015

	May 1,	May 2,	May 3,	May 4,	May 5,	May 6,	May 7,	May 8,	May 9,	May 10,
Hurn	9	11	14	8	19	17	8	8	12	7
Leuchars	8	13	15	10	12	7	11	7	8	11
	May 11,	May 12,	May 13,	May 14,	May 15,	May 16,	May 17,	May 18,	May 19,	May 20,
Hurn	7	8	4	8	7	11	7	14	11	10
Leuchars	18	20	10	8	11	16	16	8	9	10
	May 21,	May 22,	May 23,	May 24,	May 25,	May 26,	May 27,	May 28,	May 29,	May 30,
Hurn	6	5	7	6	6	6	7	10	11	7
Leuchars	16	13	8	14	8	10	10	15	9	11

Investigate which town is windier.

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4. A car of mass 1200 kg is moving along a level road. The car's engine provides a constant driving force. The motion of the car is opposed by a constant resistance. Given that the car is accelerating at $2\ ms^{-1}$ and that the magnitude of the driving force is three times the magnitude of the resistance force, show that the magnitude of the driving force is 3600 N.

5. The masses of arctic foxes are found and the mean mass was 6.1 kg. The variance was 4.2 An outlier is an observation which lies ± 2 standard deviations from the mean.

a) Which of these arctic fox masses are outliers? 2.4 kg, 10.1 kg, 3.7 kg, 11.5 kg

b) What are the smallest and largest masses that an arctic fox can be without being an outlier?

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6. A particle P is moving on the x-axis with constant deceleration $4\ ms^{-2}$. At time t = 0, P passes through the origin O with velocity 14 ms^{-1} in the positive direction. The point A lies on the axis and OA = 22.5 m. Find

a) the difference between the times when P passes through A

b) the total distance travelled by P during the interval between these times.

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7. a) Show that the equation $3\sin^2 x - \cos^2 x = 2$ can be written as $4\sin^2 x = 3$

b) Hence solve the equation $3\sin^2 x - \cos^2 x = 2$ in the interval $-180° \leq x \leq 180°$, giving your answer to 1 decimal place

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8. A curve C is described by the equation $2x^2 + 3y^2 - x + 6xy + 5 = 0$
Find an equation of the tangent to C at the point (1,-2), giving your answer in the form $ax + by + c = 0$, where *a, b* and *c* are integers.

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9. A pebble is projected vertically upwards with speed $21\ ms^{-1}$ from a point $32\ m$ above the ground. Find

a) the speed with which the pebble hits the ground

b) the total time for which the pebble is more than 40 m above the ground.

c) Sketch a velocity-time graph for the motion of the pebble from the instant it is projected to the instant it hits the ground, showing the values of *t* at any points where the graph intercepts the horizontal axis.

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10. A train, travelling on a straight track, is slowing down with constant deceleration $0.6\ ms^{-2}$. The train passes one signal with speed $72\ km\ h^{-1}$ and a second signal 25 s later. Find:
a) the velocity in $km\ h^{-1}$, of the train as it passes the second signal.
b) the distance between the signals.

TEST YOURSELF

Give yourself 20 minutes to answer these questions.
If you finish early, check your answers.
I will mark your answers. Set your work out carefully.

(A) $f(x) = \frac{9x^2+4}{9x^2-4}, \quad x \neq \pm\frac{2}{3}$

Given that $f(x) = A + \frac{B}{3x-2} + \frac{C}{3x+2}$, find the values of the constant A, B and C.

(B) Solve these equations

a) $|5x - 2| = -\frac{1}{4}x + 8$
b) $|12 - 5x| = -2x + 3$

Answers

1 a) $p \geq 2$ b) No. Can be any $p \geq 2$

2) Proof

3. a) Dorset in Southern England b) On the North-East coast of Scotland
c) Your opinion…
d) You must give some mathematical analysis to your argument. Some statistical calculations and diagrams are essential.

4. Proof

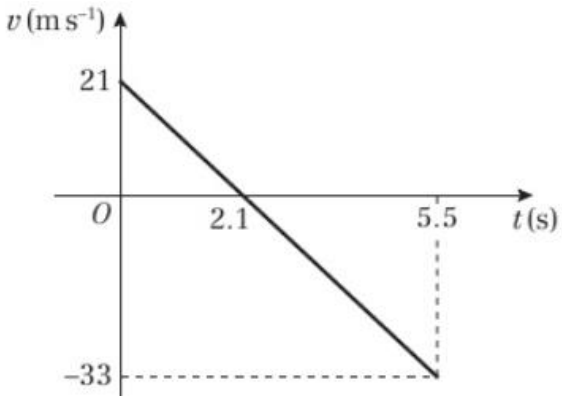
5 a) 11.5 kg b) Smallest 2.0 kg, Largest 10.2 kg

6 a) 2 s b) 16 m

7.a) Proof b) $-120°,\ -60°,\ 60°,\ 120°$

8. $3x + 2y + 1 = 0$

9. a) $33\ ms^{-1}$ b) 3.4 s (2 s.f.) c)



10. a) 18 $km\ h^{-1}$ b) 312.5 m